Heat Production and Tidally Driven Fluid Flow in the Permeable Core of Enceladus

Yang Liao, Francis Nimmo, and Jerome A. Neufeld

1Department of Geology and Geophysics, Woods Hole Oceanographic Institution, Woods Hole, MA, USA, 2Department of Earth and Planetary Sciences, University of California, Santa Cruz, CA, USA, 3BP Institute, University of Cambridge, Cambridge, UK, 4Department of Earth Sciences, University of Cambridge, Cambridge, UK, 5Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, UK

Abstract Saturn’s moon Enceladus has a global subsurface ocean and a porous rocky core in which water-rock reactions likely occur; it is thus regarded as a potentially habitable environment. For icy moons like Enceladus, tidal heating is considered to be the main heating mechanism, which has generally been modeled using viscoelastic solid rheologies in existing studies. Here we provide a new framework for calculating tidal heating based on a poroviscoelastic model in which the porous solid and interstitial fluid deformation are coupled. We show that the total heating rate predicted for a poroviscoelastic core is significantly larger than that predicted using a classical viscoelastic model for intermediate to large (>10^{14} \text{ Pa}\cdot\text{s}) rock viscosities. The periodic deformation of the porous rock matrix is accompanied by interstitial pore fluid flow, and the combined effects through viscous dissipation result in high heat fluxes particularly at the poles. The heat generated in the rock matrix is also enhanced due to the high compressibility of the porous matrix structure. For a sufficiently compressible core and high permeability, the total heat production can exceed 10 GW—a large fraction of the moon’s total heat budget—without requiring unrealistically low solid viscosities. The partitioning of heating between rock and fluid constituents depends most sensitively on the viscosity of the rock matrix. As the core of Enceladus warms and weakens over time, pore fluid motion likely shifts from pressure-driven local oscillations to buoyancy-driven global hydrothermal convection, and the core transitions from fluid-dominated to rock-dominated heating.

Plain Language Summary Discoveries made by the Cassini spacecraft revealed that we may find life even in our own cosmic neighborhood in the ocean of Saturn’s moon Enceladus. Recent studies infer that the fluid-saturated rocky core of Enceladus harbors some of the processes that likely gave rise to life on Earth. Here we develop a physics-based model to understand the heating in the water-saturated porous core of Enceladus during periodic deformation under tidal forces. We find that the periodically loaded moon distributes the heat generated during its deformation in both its rocky and fluid part. When the core is strong and rigid (likely in the early history), heating in the fluid could be hundreds or thousands of times larger than that in the solid rock; when the core is severely weakened (such as in its later history), more heat is distributed in the solid rock than in the fluid. We also find that the coupling between pore fluid and solid rock may enhance the heating of the core for a larger range of material properties, in contrast with earlier studies. The enhanced heating could provide explanation for the high heat flux measured for Enceladus, and a potential hot period in its early history.

1. Introduction

In nearly 15 years after the discovery of the enigmatic polar plumes by Cassini (Porco et al., 2006), the interior structure and orbital history of Enceladus has been made clearer based on analysis of its gravity, topography and libration data, and numerical simulation (Iess et al., 2014; Neveu & Rhoden, 2019; Nimmo et al., 2018; Taubner et al., 2015; Vance et al., 2018). It is now believed that under the ice shell of the moon, there is a global, salty, liquid ocean (Čadek et al., 2016; Postberg et al., 2009, 2011, 2018; Thomas et al., 2016) overlying a highly porous, silicate-based rocky core (Hemingway et al., 2018; Iess et al., 2014; Roberts, 2015), where water-rock hydration reactions and hydrothermal circulation are supported (Choblet et al., 2017; Hsu et al., 2015; Sekine et al., 2015; Travis & Schubert, 2014; Waite et al., 2017)—a picture that presents great similarities to Earth (Kelley et al., 2005; Russell et al., 2010).
Enceladus is thought to consist of an ice shell about 20 km thick on average, overlying an ocean about 40 km deep and a low-density (≈2,500 kg m$^{-3}$), high-porosity rocky core (Cadek et al., 2016; Hemingway & Mittal, 2019; Iess et al., 2014). The shell is thinnest at the active south pole and also exhibits reduced thickness at the north pole (e.g., Hemingway & Mittal, 2019). Based on these thickness estimates, the total conductive heat loss across the whole of Enceladus is about 25–40 GW, while the measured excess thermal emission at the south pole is 16 ± 3 GW (Howett et al., 2011).

The source of the heat is Enceladus's tides, which transfer energy from Saturn's rotation. But the distribution of the tidal heating is currently uncertain. As previously reviewed (Nimmo et al., 2018), initial models (Roberts & Nimmo, 2008; Ross & Schubert, 1989; Shoji et al., 2013; Tobie et al., 2008) focused on dissipation in the ice shell. However, since the ice shell is thought to be conductive and the surface is cold, only a relatively thin layer at the base of the shell is warm enough to be dissipative. As a consequence, viscoelastic models have found it difficult to generate sufficient heat within the ice shell (Souček et al., 2019). Turbulent dissipation of water in south polar fractures could be an additional energy source (Kite & Rubin, 2016) but cannot explain the survival of relatively thin ice elsewhere, where fractures are absent (e.g., the northern polar region). Dissipation in the ocean driven by obliquity tides has been suggested as a major heat source (Tyler, 2009). However, for the inferred obliquity, and ocean and shell thicknesses, the magnitude of the predicted heat production is many orders of magnitude too small (Beuthe et al., 2016; Chen & Nimmo, 2011; Hay & Matsuyama, 2017; Matsuyama et al., 2018). Other mechanisms for driving turbulent ocean dissipation have also been suggested (Lemasquerier et al., 2017; Wilson & Kerswell, 2018) but have not so far received much scrutiny.

Recently, the silicate core has become more popular as a source of heat. A sufficiently weak silicate core can generate significant amounts of heat (Choblet et al., 2017; Efroimsky, 2018; Roberts, 2015). Below we consider an additional possibility: That tidal pumping of water through the porous core results in significant heat production. Tidally driven viscous dissipation in pore fluids can generate heat (Al-Hadhrami et al., 2003; Jupp & Schultz, 2004; Vance et al., 2007). For Enceladus, this mechanism has only been studied once in a simplified fashion (Vance et al., 2007) and was found to be insignificant in the context of heating and maintaining a liquid ocean. However, as that model omitted the interaction between porous fluid flow and tidal deformation, the resulting heating rates are likely underestimated.

Besides providing an additional heat source, heating of the fluid in the core of Enceladus is also relevant to geochemical processes crucial for understanding the habitability of the moon. On Earth, rock-water reactions and potential metabolic reactions under the seafloor need to occur in pore fluids and along the rock-fluid interface, where the motion and heating of pore water play a nonnegligible role (Mayhew et al., 2013; Schrenk et al., 2013; Schwarzenbach, 2016). Predicting the habitability beneath the seafloor of Enceladus therefore requires more knowledge of the thermal and kinetic history of the pore fluids and their relation with the rock matrix.

Currently, an explicit account of the coupling between porous flows and deforming rock matrix during tidal flexing is not achievable using either the commonly assumed viscoelastic description of tidal heating, or the poroelastic description of fluid motion (Choblet et al., 2017; Vance et al., 2007). As we explain below, both these end-member approaches neglect the coupling between porous solid and fluid, which can result in enhanced heating in both phases. A model for Enceladus that can evaluate the production of heat in both the pore fluid and rock matrix simultaneously and consistently has not been proposed hitherto.

In this paper, we propose a method of evaluating heating of the rock and pore fluid in the core by assuming a poroviscoelastic rheology, which consistently accounts for the coupling between pore fluid flows and deformation of the rock matrix. Our approach is simplified in that we assume a homogeneous poroviscoelastic spherical body loaded cyclically with an axisymmetric degree-2 pattern. In section 2, and in Appendices A and B, we provide the analytical framework for solving this simplified problem. In section 3 we compare our results for heat production in both the solid and fluid components with the end-member viscoelastic solution in which the pore fluid is neglected; we also explore the spatial pattern of heating. In section 4 we explore how the evolution of the mechanical properties of the core may have changed the dominant heat-producing mechanism over time. Finally, we conclude with a discussion of our results and a sketch of how a more complete analytical description may be accomplished.
2. Methods

2.1. Overview of the Model

The core of Enceladus is modeled as a poroviscoelastic, spherical body with uniform porosity, permeability, elastic moduli, and viscosity. The core is assumed to be submerged in an overlying ocean and all pore spaces are filled with fluid with constant viscosity. During tidal deformation, the solid rocks in the core deform periodically, generating heat (i.e., tidal heating). The deformation of the rock matrix causes the pore spaces to compress/dilate, driving porous fluid flows and generating heat via fluid dissipation. As redistribution of pore pressure changes the local stress and strain rate in the rock matrix, the coupling between fluid and solid components in our model is intrinsically two way. To obtain the heating rates, the model seeks analytical solutions for quantities including strain, stress, and velocity based on the assumptions and constraints below.

The core is assumed to have properties of both a viscoelastic (i.e., viscous relaxation of the rock matrix) and poroelastic material (i.e., mechanical coupling between pore pressure and total stress), which is described by linear poroviscoelastic constitutive relations. The constitutive relations explicitly couple the pore pressure and fluid mass with the strain and stress of the rock matrix (see next section). The motion of the interstitial fluids, driven by spatial gradients of fluid pore pressure, satisfies Darcy’s law and mass conservation. The model assumes force balance while omitting the generation of seismic waves, leading to a quasi-static equilibrium condition for the total stress.

The quantitative framework is completed by boundary conditions and additional geometrical requirements of the solutions: At the surface of the core (i.e., seafloor), the fluid pressure and stress are continuous; at the poles of the core’s surface, the amplitude of strain is approximated by a function of the moon’s orbital motion and its degree-2 Love number determined by the rigidity and viscosity of the solid rock matrix (see next section); the model implicitly accounts for the gravitational potential caused by the moon’s orbital motion by imposing degree-2 spatial geometries and periodicity of the solutions and through the imposed boundary conditions. The model is simplified by assuming only axisymmetric degree-2 geometry, ignoring longitudinal variations of gravitational potential.

2.2. Model Setup and Solutions

In this work we focus entirely on the water-filled silicate core of Enceladus. We largely neglect the role of the overlying ocean and ice shell. While these components will modify the tidal deformation of the core, the changes expected are small while greatly complicating the analytical development. Parameter values assumed are tabulated in Table A1.

In a poroviscoelastic material, the total stress is shared between pressurization of the pore fluid and the deformation of the rock matrix. In comparison to the classical Maxwell viscoelastic description commonly assumed for tidal heating models, a poroviscoelastic description incorporates an additional contribution of pore fluid pressure to the bulk rheology. Its constitutive relation can be obtained by combining classical linear poroelasticity and a Maxwell viscoelastic model wherein viscous relaxation occurs in the shear components (Biot, 1941; Cheng, 2016; Rice & Cleary, 1976; Roberts & Nimmo, 2008; Tobie et al., 2005; Wu & Peltier, 1982).

\[
\frac{\partial \sigma}{\partial t} + \frac{\mu}{\eta_m} \sigma - \frac{1}{3} \frac{\mu}{\eta_m} \text{Tr}(\sigma) I = 2\frac{\partial \epsilon}{\partial t} + \left( K_m - \frac{2}{3} \mu \right) \frac{\partial \text{Tr}(\epsilon)}{\partial t} I - \alpha \frac{\partial P}{\partial t}. \tag{1}
\]

Here \(\sigma(r, \theta, t)\) and \(\epsilon(r, \theta, t)\) are the stress and strain tensors for the ensemble material, \(P(r, \theta, t)\) is the pore pressure, \(I\) is a unit tensor, and \(\mu\) and \(\eta_m\) are the rigidity (i.e., shear modulus) and viscosity of the rock matrix. The poroelastic coefficient \(\alpha\) (also known as the Biot constant) indicates the relative strength of the rock matrix in comparison with that of pure rock and has a value between 0 and 1, and \(K_m = (1 - \alpha)K_s\) is the bulk modulus of the rock matrix where \(K_s\) is the bulk modulus assumed for pure rock. In the current study we assume \(K_s = 10\ \text{GPa}\) or smaller, according to values assumed for Earth’s oceanic crust (Crone & Wilcock, 2005; Vance et al., 2007). We can verify that the Maxwell formulation in classical tidal heating studies can be obtained from Equation 1 when the pore pressure is decoupled from the matrix with \(\alpha = 0\) (Brusche & Sundermann, 1978; Peale & Cassen, 1978; Ross & Schubert, 1989; Segatz et al., 1988; Tobie et al., 2005;
The core of Enceladus is modeled as a spherical, homogeneous, poroviscoelasitic body with radius $R_0$. The body undergoes axisymmetric oscillation with a period of 33 hours. During periodic deformation, all quantities depend on radial position $r$ and angle $\theta$ (measured from vertical axis) and are independent of azimuthal angle $\varphi$. Lower left quarter shows a fluid-loading condition, where fluid pressure is continuous on the surface, and internal stress balances with loading pressure.

Under periodic loading, all the quantities in the system (denoted by $f$ here) are represented by complex amplitudes as $\bar{f} = Re (f(\theta)e^{i\omega t})$, where $f$ stands for strain, stress, pore pressure, or fluid velocity; $\bar{f}$ is the complex amplitude of $f$; and $\omega$ is the loading frequency. The constitutive relations, equilibrium condition, Darcy’s law and pore fluid continuity lead to analytical solutions for the displacement potential and the pore pressure (see Appendices A and B for derivation)

$$\Phi = -\frac{\alpha C_1}{k^2(K_m + 4\mu^*)} \left( \frac{3\sin kr}{k^2r^3} - \frac{3\cos kr}{k^2r^2} - \frac{\sin kr}{kr} \right) (3\cos 2\theta + 1)$$\hspace{1cm}(2)$$

$$+ C_2 r^2 (3\cos 2\theta + 1),$$

$$\hat{P} = C_1 (3\cos 2\theta + 1) \left( \frac{3\sin kr}{k^2r^3} - \frac{3\cos kr}{k^2r^2} - \frac{\sin kr}{kr} \right),$$

where $k$ is a complex wavenumber determined by material properties. In the present study, contributions from self-gravity and the tidal forcing from orbital eccentricity are considered in a simplified fashion and are implicitly reflected by the constant values of $C_1$ and $C_2 \propto C_1$, which are determined by the boundary conditions at the surface of the core (see Appendices A and B). The expressions for the displacement potential and pore pressure in Equation 2 further lead to solutions for the strain $\dot{\epsilon}$, stress $\sigma$, and Darcy’s velocity $\vec{q}$ (see Equation A9 in section A1). The volumetric heating rates averaged over one tidal period are therefore

$$h_{\text{side}}(r, \theta) = \frac{1}{T_{\text{sid}}} \int_{nT_{\text{sid}}}^{(n+1)T_{\text{sid}}} \frac{\partial \epsilon_{ij}}{\partial t} \left( \frac{\partial \epsilon_{ij}}{\partial t} \right) \, dt + \frac{1}{T_{\text{sid}}} \int_{nT_{\text{sid}}}^{(n+1)T_{\text{sid}}} \frac{\partial P}{\partial t} \, dt,$$

where $T_{\text{sid}} = 33$ hr is the orbital period for application to Enceladus. The volumetric heating rate $h_{\text{side}}$ is generated in the rock matrix and is the counterpart of tidal heating in a solid viscoelastic body. The last term in Equation 3 arises from the compression/dilation of the pore fluid ($\zeta$ is the relative amount of pore fluid entering the pore space) and typically has a negligible contribution. In Equation 4, $h_{\text{vis}}$ represents the viscous heat generated in the pore fluid. The global heating rates in solid or fluid are $H_{\text{side,vis}} = 2\pi \int_0^{R_0} h_{\text{side,vis}} r^2 \sin \theta \, dr$ and the total heating rate $H_{\text{total}} = H_{\text{side}} + H_{\text{vis}}$. Under the boundary conditions of force balance and fluid pressure continuity at the surface of the core (i.e., the fluid-loading boundary condition; see Appendices A and B), the ratio $H_{\text{vis}}/H_{\text{side}}$ is determined and represents the partitioning of total heat between the solid and fluid constituent in the core. To obtain the magnitude of the heating rates, an additional boundary condition is required to constrain $C_1$ and $C_2$. 

Wu & Peltier, 1982). We implicitly assume that the bulk viscosity of the rock matrix is infinitely large in comparison with the shear viscosity $\eta_m$; hence, the viscous relaxation process only occurs in the shear component, similar to previous studies.

The core of Enceladus is modeled as a spherical, homogeneous, poroviscoelasitic body under a fluid-loading boundary condition at its surface (see Figure 1). Following earlier works, we assume irrotational deformation with displacement $\vec{u} = V\Phi$ (Landau & Lifshitz, 1959; Love, 1927), where the displacement potential $\Phi(r, P_2(\theta))$ is axisymmetric and has a degree-2 pattern in zonal direction ($P_2$ is degree-2 Legendre polynomial with $m = 0$). As discussed below, this is a simplification of the actual tidal pattern, which includes both $P_2^m$ and $P_2^m$ components for eccentricity tides (e.g., Tobie et al., 2005).

Under periodic loading, all quantities in the system depend on radial position $r$ and angle $\theta$. The core of Enceladus is modeled as a spherical, homogeneous, poroviscoelasitic body with radius $R_0$. The body undergoes axisymmetric oscillation with a period of 33 hours. During periodic deformation, all quantities depend on radial position $r$ and angle $\theta$ (measured from vertical axis) and are independent of azimuthal angle $\varphi$. Lower left quarter shows a fluid-loading condition, where fluid pressure is continuous on the surface, and internal stress balances with loading pressure.
In practice, the deformation of the porous core is forced by tidal variations in the gravitational potential, with the surface of the core treated as an interface with displacement and stress coupled to the volumetric deformation and pressurization of the overlying ocean, which in turn is coupled to ice shell deformation. Such complexities are beyond the scope of the present study. Here we instead simplify the forcing and the boundary condition by assuming that the maximum strain at the poles is determined by the viscoelastic properties of the solid matrix in the core. This allows us to carry out a direct comparison between the heating in a standard viscoelastic body and the additional heating arising from porosity and fluid flow. For diurnal tides the maximum tidal strain $\epsilon_{\text{max}} \approx 3e M^5 m^3 R^3 a^3 h_2^2 (M\text{urray} & \text{Dermott, } 1999)$. Here $e$ is eccentricity of the orbit, $M$ and $m$ are the mass of the planet and moon, $R_0$ is the radius of the moon and $a$ the planet-satellite distance; we take $h_2 = \frac{5}{3} k_2$, which is the appropriate limiting behavior when rigidity dominates over gravity; $k_2$ is the tidal Love number. Following Kepler’s law the relation further becomes $\epsilon_{\text{max}} = \frac{9}{4\pi} e \frac{\omega^2}{\rho G} \frac{5}{3} k_2$, assuming a uniform body, where the complex rigidity $\mu^* = \mu \frac{\omega}{1 + 19\mu^*/2\rho g R}$ depends on the viscosity and rigidity of the matrix (e.g., Ross & Schubert, 1989). We assume that the amplitude of the radial strain at the polar surface is $\epsilon_0 = \epsilon_{\text{max}} = \frac{9}{4\pi} e \frac{\omega^2}{\rho G} \frac{5}{3} |k_2|$, which is used for constraining $C_1$ (Figure 2a and see the Appendices A and B).

2.3. Choice of Permeability and Viscosity

The material properties of the core are represented by four parameters: rock matrix rigidity $\mu$, viscosity $\eta_m$, permeability $\kappa$, and poroelastic coefficient $\alpha$, which by definition ranges from 0 to 1. We consider a range of high permeabilities and moderate to low rigidities and viscosities in our study. In earlier works, the permeability of Enceladus was assumed to be no more than $10^{-10} \text{m}^2$, given the typical value of the permeability of the oceanic crust on Earth (Choblet et al., 2017; Vance et al., 2007). Furthermore, Choblet et al. (2017) concluded that the permeability of the core of Enceladus needed to be smaller than $10^{-12} \text{m}^2$ for high-temperature hydrothermal circulation to occur, as suggested by the occurrence of silica nanoparticles.
In our work below we consider a wider range of possible permeability values based on the results of some recent studies on Earth’s seafloor, which suggest that the permeability is likely higher. In one study on the Cocos plate, numerical simulations suggested that permeabilities of $10^{-10}$ and $10^{-8} \text{ m}^2$ for recharge and discharge portions of one outcrop yield the best match between simulations and heat flow measurements (Lauer et al., 2018; Winslow et al., 2013). Borehole measurements of large permeabilities, up to $10^{-8} \text{ m}^2$, and porosities, up to 10%, have also been made at some locations (Lauer et al., 2018; Winslow et al., 2013). Although the structure of Earth’s oceanic crust (e.g., sediment layers) are different from that in the core of Enceladus, the characteristic permeability for the latter is anticipated to be of a similar order of magnitude or higher because of its high porosity. The density of the core inferred from gravity studies suggests a porosity of at least 20% (section 1), while some studies have assumed porosities of up to 50% extending into the center of the core (Choblet et al., 2017; Roberts, 2015; Vance et al., 2018). As a result, we postulate that $\kappa$ is likely higher than the typical value for Earth’s seafloor, and consider a range of permeability of $\kappa \in [10^{-12},10^{-7}] \text{ m}^2$ in our study.

In some early works, the rigidity and viscosity of the core of Enceladus were estimated based on the properties of solid rocks, with typical values of $\eta_m \sim 10^{20} \text{ Pa} \cdot \text{s}$ and $\mu \sim 100 \text{ GPa}$, leading to predictions of very small heating rates (e.g., Roberts & Nimmo, 2008). Other studies proposed lower rigidities and viscosities based on the presence of porosity, rock alteration (e.g., serpentinization), or weakening of the core, which result in higher heating rates (Choblet et al., 2017; Efroimsky, 2018; Roberts, 2015). In the recent work by Choblet et al. (2017), 10–30 GW of tidal heating results from a weak core, with an effective shear modulus $\mu_{eff} = |\mu'| \in [10^7,10^8] \text{ Pa}$ and dissipation function $Q_{\mu}^{-1} = \frac{1}{\omega} \frac{Im(\mu')}{|\mu'|} \in [0.2, 0.8]$, which, in the context of Maxwell viscoelastic rheology, corresponds to a low viscosity $\eta_m = \frac{1}{\omega Q_{\mu}} \mu_{eff}$, that is, less than $10^{13} \text{ Pa} \cdot \text{s}$, and a low rigidity $\mu = \frac{\mu_{eff}}{\sqrt{1 - (Q_{\mu}^{-1})^2}}$, that is, between 0.01 and 0.1 GPa. These values are interpreted as a consequence of weakening by cyclic loading, which has been observed in experiments of cyclic loading tests of cohesive soils (Choblet et al., 2017). However, solid viscosities $<10^{13} \text{ Pa} \cdot \text{s}$ resemble those of ice near its melting temperature, rather than those typical of near-solidus solid rock values (> $10^{18} \text{ Pa} \cdot \text{s}$). In our study, we assume a range of rigidities between 0.01 and 10 GPa, and a low- to moderate-viscosity range of $10^{12}$ to $10^{15} \text{ Pa} \cdot \text{s}$ to represent a moderately to severely weakened core.

3. Model Results

In this section we first compare our results with end-member viscoelastic tidal heating in which the effect of the pore fluid is ignored. We then investigate how the presence of pore fluid affects heat production in both the solid and fluid components. Finally, we investigate the spatial pattern of our tidal heating model.

3.1. Comparison With Classical Predictions for a Viscoelastic End-Member

When the pore fluid is decoupled from the deformation of the rock matrix ($\alpha = 0$ or $\kappa = 0$), the body becomes effectively viscoelastic and a heating rate $H_{\text{visco}}$ can be obtained (see Appendix A2). $H_{\text{visco}}$ corresponds to the viscoelastic end-member of the poroviscoelastic model, which can be benchmarked using the classical viscoelastic model. According to earlier studies, the theoretical heating rate predicted by the classical Maxwell model is $H_{\text{theory}} = -21\omega R_0^2 e^2 \text{Im}(k_2)/2G$ (Segatz et al., 1988; Tobie et al., 2005; Zschau, 1978). In our approach, $H_{\text{visco}}$ is determined by the loading frequency, rigidity, and viscosity (see Appendix A1). $H_{\text{visco}}$ is comparable to the theoretical prediction $H_{\text{visco}} \approx 57\% H_{\text{theory}}$ for a wide range of rigidity and viscosity values (Figure 2b). Due to the neglect of the additional degree-2 harmonic in the poroviscoelastic model, $H_{\text{visco}}$ is always smaller than $H_{\text{theory}}$. As our poroviscoelastic model can reproduce the main characteristics of the viscoelastic end-member, we consider our choice of geometry and boundary conditions to be justified for the purposes of this preliminary study.
3.2. Enhanced Heating and Heating Partition

During cyclic loading, the displacement in the poroviscoelastic body develops higher amplitudes at surface polar regions (Figure 3a). As in previous studies, phase lags exist between the stress and strain components, contributing to the generation of heat in the rock matrix (Figure 3b).

The coupling between the pore pressure and solid matrix allows for higher internal deformation relative to the viscoelastic end-member, causing the heating in the solid matrix to increase, even though the boundary strain remains small (determined by viscosity and rigidity). We find that the heating enhancement is especially prominent when the viscosity of the solid matrix is moderate or large: for example, at a rigidity of 1 GPa and viscosity of $10^{15}$ Pa·s, the total heating rate increases from 30 MW for the viscoelastic core to nearly 1 GW for the poroviscoelastic core when the poroelastic coefficient $\alpha = 0.9$, and to nearly 17 GW when $\alpha = 0.95$ and permeability $10^{-11}$ m$^2$ (Figure 4a). Since the total expected heat output of Enceladus is a few tens of GW (section 1), it is clear that poroviscoelastic dissipation in the core could supply a large fraction of Enceladus’s total heat budget.

According to the micromechanical relation, $K_m = (1 - \alpha)K_s$ ($K_m$ and $K_s$ are the bulk moduli of the rock matrix (frame) and the pure rock, respectively), a large $\alpha$ indicates a much more compressible porous rock matrix in comparison to a purely solid rock. When $\alpha > 0.9$, the compressible rock matrix leads to a second heating rate peak that emerges at moderate matrix viscosity (Figure 4a). We consider that large values of $\alpha (>0.9)$ are likely, given the large porosity expected in the core of Enceladus. In the study of tidal pumping in the shallow crust of Enceladus by Vance et al. (2007), a frame bulk modulus of 0.1 GPa and rock bulk modulus of 10 GPa were assumed, corresponding to $\alpha \sim 0.99$ in the study. Although a large $\alpha$ likely exists for the porous core, it is not required for enhanced heating and the emergence of a second heating peak: with a lower value for $K_s < 2$ GPa (e.g., caused by mineral alteration of water-rock reaction), even a moderate value of $\alpha < 0.8$ can lead to the same enhanced heating effect (see Figure B1a in Appendix B).

We further point out that the second heating peak (when it exists) occurs primarily in the solid component (see Figure B1b in Appendix B) and that the viscosity corresponding to the second peak varies primarily with the compressibility of the rock matrix (see Figures B1c and B1d in Appendix B). These observations suggest that the compressible rock matrix is the main contributor to the enhanced heating rate in the solid. As most tidal heating models assume an incompressible matrix, the effect of compressibility has not been extensively explored, although Hurford et al. (2006) explored the effects of varying material parameters including compressibility on the tidal response of the Earth. While Kaula (1964) found an increase in the heating rate of less than 3% when the compressibility of the lunar viscoelastic core was included, the enhanced heating rates, we observe here indicate that for a poroviscoelastic core, the compressible rock

Figure 3. (a) The maximum magnitude of radial displacement $|u_r|$ on a cross section of the spherical body. (b) The time series of tensile components of strain $\varepsilon_{\theta\theta}$ and stress $\sigma_{\theta\theta}$ as functions of time measured under the north pole ($\theta = 0^\circ$) at depth 5.6 km below surface. The example case has permeability $k = 1 \times 10^{-10}$ m$^2$, rock matrix rigidity $\mu = 1$ GPa, viscosity $\eta_m = 10^{17}$ Pa·s, Biot constant $\alpha = 0.2$. 

matrix and the subsequent coupling between the matrix with pore fluid pressure can influence the heating rate to a much larger extent than previously reported. Compressibility together with fluid permeability thus provides an alternative to ultralow solid viscosities as an explanation for high heat production rates in the core.

Besides enhanced heating in the solid component, the coupling between the rock and fluid phase causes fluid oscillation throughout the body, providing an additional heat source via viscous dissipation. The dissipative heating is especially prominent when the rock matrix viscosity and core permeability are moderate or high. At small (less than 10^{13} Pa·s) matrix viscosity, the poroviscoelastic core resembles the viscoelastic end-member, where viscoelastic deformation of the solid matrix dominates and the heating rates are consistent with the existing study by Choblet et al. (2017) (Figures 4a and 4b). Under the same boundary conditions, rigidity and rock viscosity, the total heating rate \( H_{\text{vis}} + H_{\text{tidal}} \) for poroviscoelastic body is higher than that of the viscoelastic counterpart \( H_{\text{visco}} \), with relative increase in heating rate \( \Delta H_{\text{total}} \%

\[ \Delta H_{\text{total}}\% = \frac{H_{\text{total}} - H_{\text{visco}}}{H_{\text{visco}}} \times 100 \]
Htidel + Hvis - Hvisco
Hvisco
ranging from 1% for small viscosities (ηm < 10^{14} \text{ Pa\cdot s}) to 1,000 times for large viscosities (see Figures 4c and 4d).

Both the heating rate of the solid rock \(H_{\text{tide}}\) and heating rate of the pore fluid \(H_{\text{vis}}\) vary with physical parameters including permeability, poroelastic coefficient, rigidity and viscosity (Figure 5). The dependence of \(H_{\text{tide}}\) on poroelastic parameters such as the permeability \(\kappa\) and poroelastic coefficient \(\alpha\) indicates that heating in poroviscoelastic bodies is not a straight superposition of viscoelastic and poroelastic end-members, because of the interaction between poroelastic diffusion and viscoelastic relaxation processes. The partition of heat between the fluid component and solid component in the core is represented by the ratio \(H_{\text{vis}}/H_{\text{tide}}\) and varies most sensitively with the viscosity of the matrix (see Appendix B Figure B2). Other parameters, including \(\alpha\), \(\kappa\), and \(\mu\), only affect the partition of heat to a moderate degree.

Overall, for a moderately weakened or un-weakened rock with viscosity \(\eta_m > 10^{16} \text{ Pa\cdot s}\), most of the heat is produced in the pore fluid. For some choices of \(\alpha\) and permeability, heat production rates can exceed 10 GW and thus contribute a significant fraction of Enceladus's global heat budget without requiring ultralow solid viscosities. For a severely weakened rock with viscosity \(\eta_m < 10^{15} \text{ Pa}\), most of the heat is produced in the solid matrix. For a system with severely weakened rock (i.e., low viscosity) with strong tidal heating, as proposed in Choblet et al. (2017), the contribution of porous flow is very small. As a result, we postulate that the heat generated via porous flows was most likely to play a role before the core became severely weakened, most likely in the early history of Enceladus (see section 4 below).

### 3.3. Spatial Pattern of Heating

The volumetric heating rate \(h_{\text{tide}}(r, \theta)\) and \(h_{\text{vis}}(r, \theta)\) (Figures 6a–6f) for the poroviscoelastic body develop spatial patterns in response to fluid diffusion accompanying the cyclic deformation of the solid matrix. We find that the heating in the fluid phase \(h_{\text{vis}}\) concentrates below the seafloor under the polar regions and decreases with depth, with high heating rates focusing in a shallow layer with depth similar to the “skin depth” determined by poroelastic parameters (Jupp & Schultz, 2004; Vance et al., 2007) (see Appendix B). The heating in the solid matrix \(h_{\text{tide}}\) is virtually constant, with a small decrease toward the surface over the skin depth. This lack of spatial variation does not capture all the features of the classical model (Beuthe, 2013; Tobie et al., 2005), due to the missing harmonic modes in the deformation potential; therefore, although the simplified geometry is capable of reproducing the total heating rate, an accurate description of spatial distribution requires the inclusion of more harmonic modes. The volumetric heating rates lead to a rough estimation of the maximum surface heat flux, which is an integration of \(h_{\text{tide}}\) or \(h_{\text{vis}}\) in the radial direction. Due to our assumption of axisymmetric geometry, the resulting heat flux varies only with latitude at the surface of the core (Figures 6g–6i). For heat generated in the solid matrix, the flux peaks at mid latitude but is almost constant; for heat generated in the fluid, the heat flux peaks in the polar regions and is smallest at midlatitudes. We note in particular that panels (h) and (i) show peak polar heat fluxes of roughly 280 and 100 mW m\(^{-2}\), enough to account for the measured polar flux (Howett et al., 2011).
4. Evolution of Heating Rate With Weakening of the Core

The enhanced heating rates we find may also have implications for the thermal evolution in the history of Enceladus. As postulated by Choblet et al. (2017), cyclic deformation leads to progressive weakening of the core, which causes a reduction in viscosity and rigidity. It is therefore possible that the core of Enceladus possessed higher viscosity and rigidity in its past, which, according to the Maxwell model, corresponds to a negligible heating rate. But incorporation of a second phase (i.e., fluid) changes this conclusion, as follows.

Direct measurements of how viscosity and rigidity evolve over time due to periodic loading are not available. However, existing experimental studies provide relations for both the damping ratio $D$ and effective modulus $G$, which are typical measurements in mechanical tests (Ishibashi & Zhang, 1993; Rollins et al., 1998; Seed et al., 1986). For Maxwell viscoelastic materials these quantities are determined by the
complex rigidity $\mu^*$ as $D = \text{Im}(\mu^*)/2|\mu^*|$ and $G = |\mu^*|$ (Choblet et al., 2017), which further lead to the relations $\eta_m = \frac{G}{2D\omega}$, $\mu = \frac{G}{\sqrt{1 - 4D^2}}$. Here we infer a weakening trajectory based on the experimental data in (Rollins et al., 1998) on cyclically deformed gravels, where $D$ and $G$ are measured as functions of cyclic shear strain. Fitting a linear relation between $D$ and $G$, we obtain from the experimental data the relation $D = -0.2086G/G_{\text{max}} + 0.1828$ and choose a maximum effective modulus of $G_{\text{max}} = 10$ GPa. During the weakening of the core, the effective modulus $G$ decreases over time, leading to an increase in damping ratio $D$, and reduction in matrix viscosity $\eta_m$ and rigidity $\mu$ (inset of Figure 7). According to viscoelastic tidal heating models, the heating rate increases as the core weakens (Choblet et al., 2017), and $10^{-3} – 10^{-2}$ GW of heat implies a severely weakened core with viscosity $\eta_m < 10^{13}$ Pa·s (Choblet et al., 2017). With the poroviscoelastic model, an enhanced heating rate is achievable at higher viscosity and rigidity (Figure 7), implying the possibility of intense heating in the early history of Enceladus, prior to severe weakening of its core. Prolonged heating in the deep past of Enceladus is certainly energetically possible, based on the rate at which energy can be supplied by Saturn (Nimmo et al., 2018). Furthermore, there are potentially interesting feedbacks, since early water-rock reactions could lead to alteration of the silicate mechanical properties (e.g., serpentinization) and a potential run-away effect.

5. Summary and Discussion

In this study we developed a poroviscoelastic description for the tidal heating process in the fluid-saturated core of an icy satellite and applied it to Enceladus. The poroviscoelastic rheology incorporates the coupling between the deformation of the solid rock matrix and fluid motion and requires reevaluation of the heat generated during tidal flexing in both the solid and fluid component of the core. Despite simplifications in loading geometry and boundary conditions, the new model is reasonably consistent with the classical viscoelastic models at its viscoelastic end-member state, reproducing the bulk part (57%) of the predicted total heating rate. When fluid-solid coupling effects are incorporated, the poroviscoelastic model predicts high heating rates, which can reach $10^{-3} – 10^{-2}$ GW without requiring the ultralow solid viscosities invoked by previous
work (Choblet et al., 2017; Efroimsky, 2018; Roberts, 2015). Based on the characteristics of these heating rates, we postulate that the heating rate enhancement results from both the compressibility of the solid matrix, and viscous dissipation of the fluid flows. The model also suggests that Enceladus may have undergone intense tidal heating early in its history, before the core was weakened. The volumetric fluid heating rate is maximized at shallow depths at the poles, and the surface heat flux from the fluid component peaks at the poles. Although our model provides a simplified description of the tidal deformation process, the enhanced heating rates suggest the importance of considering tidal heating problems in a two-phase framework where the fluid and solid components are kinematically coupled. To obtain more precise heating rates and distributions, future analytical work will need to incorporate an improved treatment of boundary conditions and the tidal forcing.

Our poroviscoelastic model also yields some results that are consistent with observations. The model predicts that the fluid heating (both volumetric heating rate and heat flux) likely concentrates in the polar regions, which could have possible implications for thickness variations of the ice shell of Enceladus and the high polar heat flux (Beuthe et al., 2016; Čadek et al., 2016; Hemingway & Mittal, 2019; Howett et al., 2011). The thinning of polar ice used to be primarily attributed to increased heating at the base of the ice shell near its melting point, while the effect of core heating was considered to be too small to be relevant (Běhouková et al., 2012; Tobie et al., 2008). Our model suggests that the local heating in the pore fluid and the resulting heat flux peak at polar surface regions. These high heating rates and heat flux could contribute to the thinning of polar ice shell.

There are several obvious ways in which this pilot study could be extended. One is to include a full treatment of the degree-2 tidal potential, rather than the axisymmetric potential assumed here. Doing so would allow a full exploration of the tidal forcing, and in particular help to provide a detailed constraint on the maximum strain in the polar regions. An intermediate step would be to focus on the effect of librations: the librational potential consists of a single degree-2 harmonic (e.g., Richard & Rambaux, 2014), and librations may have played an important role in Enceladus’s evolution (Wisdom, 2004; Wilson & Kerswell, 2018). A second obvious improvement would be to use the heating derived to calculate the resulting thermal structure. This is not straightforward, however, because particularly in the high-permeability cases, the effects of fluid convection will need to be treated (cf. Choblet et al., 2017), likely in a parameterized fashion. But doing these calculations is important because it is not clear whether the permeabilities and heating rates assumed here are consistent with the high-temperature, water-rock reactions inferred by Hsu et al. (2015). Likewise, the periodic introduction of warm fluid into the base of the overlying ocean is likely to have important consequences for the ocean circulation above (cf. Vance et al., 2007) and the evolution of the ice shell thickness.

It is also of some interest to explore where else this analysis may be applied. Larger bodies with oceans and rocky cores, such as Europa, are likely to have a thinner permeable layer owing to the higher pressures. But the midsized Saturnian satellites, in particular Dione (which may have an ocean; Zannoni et al., 2020) are likely candidates for heating generated by a poroviscoelastic response to tidal forcing. A less obvious candidate is Io, which likely has a subsurface, partially molten permeable region in which tidal forcing is important (de Kleer et al., 2019). We conclude, in common with Choblet et al. (2017), that the core of Enceladus is likely a region of high heat production and fluid motion. Future work that studies the forces driving this heating and fluid motion in more detail will increase our capacity to evaluate the thermal evolution, the locations and likelihood of geochemistry and biochemistry below the sea floor of Enceladus.

**Appendix A: Quantitative Details**

**A1. Solutions for Periodically Loaded Poroviscoelastic Body**

During periodic deformation, interstitial pore water can flow through the porous and permeable solid matrix, driven by the gradient of pore pressure $P(r, \theta, t)$. Assuming a periodic form for all quantities $f(r, \theta, t) = Re \left[ f(r, \theta)e^{i\omega t} \right]$, Darcy’s law and the continuity equation may be written as...
where the undrained modulus \( K_u \) is the bulk modulus of the rock-fluid ensemble (see Table A1). Since we simplify the geometry of the problem in the model, we do not explicitly account for the tidally varying gravitational potential (which involves all three degree-2 geometries) in the forcing equation. Instead, we simply assume a quasi-static equilibrium condition \( \boldsymbol{\nabla} \cdot \mathbf{\ddot{\sigma}} = 0 \) here and enforce the tidal forcing through the surface condition as described below. By choosing solutions with axisymmetric degree-2 pattern and periodicity, the axisymmetric part of the gravitational potential is implicitly accounted for (see below). The constitutive relations (A1, A2-1, and A5) and the quasi-equilibrium condition lead to

\[ \nabla^2 \ddot{P} + k^2 \ddot{P} = 0, \quad (A6) \]

with a complex wavenumber

\[ k = \frac{1 - i}{\sqrt{2}} \sqrt{\frac{\omega}{c} \frac{K_m^0 + i \omega \tau}{K_m^0 + i \omega \tau}} \]

where material constants \( K_m^0, K_u^0 \) and poroelastic diffusivity \( c \) are
When the rock matrix is deformable but incompressible, the pore fluid does not share the load of the system \((\alpha = 0, K_m = \infty)\), and the evolution of the pore fluid pressure reduces to the Navier equation for porous flows in an incompressible matrix, which is typically assumed in hydrothermal circulation studies, for example (Choblet et al., 2017; Fisher et al., 2003). The general form of solution for Equation A6 in spherical coordinates, with an implicit boundary condition that the solution is finite at \(r = 0\), is

\[
P = \sum_{l} \sum_{m} C_{l} m_{j}(kr)P_{lm}^{m}(\cos\theta)e^{j\mu_{l}}.
\]

where \(j_{l}(kr)\) is an degree-\(l\) spherical Bessel function, \(P_{lm}^{m}\) is degree-\(l\) associated Legendre polynomial, and \(C_{l} m_{j}\) are coefficients to be determined by initial and boundary conditions. With the axisymmetric condition \((m = 0)\) and imposed degree-2 symmetry \((l = 2)\), the solution reduces to

\[
P = C_{1}(3 \cos 2\theta + 1) \left( \frac{3 \sin kr}{kr} - \frac{3 \cos kr}{kr^2} - \frac{\sin kr}{kr} \right), \tag{A7}
\]

Substituting Equation A7 into the equilibrium condition \(\nabla^{2}\Phi = \frac{\alpha}{(K_m + \frac{4}{3}\mu)} \hat{P}\) leads to the solution for the displacement potential

\[
\Phi = -\frac{\alpha C_{1}}{k^2(K_m + \frac{4}{3} \mu)} j_{2}(kr)(3 \cos 2\theta + 1) + C_{2}r^{2}(3 \cos 2\theta + 1), \tag{A8}
\]

where the second term is a general solution to the Laplace equation in viscoelastic heating problem (Kaula, 1964; Segatz et al., 1988; Tobie et al., 2005; Wu & Peltier, 1982). Under the constitutive relations, Equations A7 and A8 lead to solutions for other quantities,

\[
\begin{align*}
\dot{\varepsilon}_{rr} &= \frac{\partial^{2}\Phi}{\partial r^{2}}, \\
\dot{\varepsilon}_{\theta\theta} &= \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \Phi}{\partial r} \right) + \frac{1}{r^{2}} \frac{\partial^{2} \Phi}{\partial \theta^{2}}, \\
\dot{\varepsilon}_{rr} &= \frac{1}{r^{2}} \frac{\partial}{\partial \theta} \left( \frac{1}{r} \frac{\partial \Phi}{\partial \theta} \right) - \frac{1}{r^{2}} \frac{\partial^{2} \Phi}{\partial r \partial \theta}, \\
\dot{q}_{r} &= -\frac{k C_{1}}{\eta_{f}} (3 \cos 2\theta + 1) \frac{dj_{2}(kr)}{dr}, \\
\dot{q}_{\theta} &= \frac{6 k C_{1}}{\eta_{f}} \sin 2\theta \frac{dj_{2}(kr)}{dr}, \\
\dot{\zeta} &= C_{1} \frac{\alpha^{2} \left( K_u + \frac{4}{3} \mu \right)}{\left( K_m + \frac{4}{3} \mu \right) \left( K_u - K_m \right)} (3 \cos 2\theta + 1) j_{2}(kr),
\end{align*}
\]

where \(q_{r}\) and \(q_{\theta}\) are complex amplitudes for the radial and tangential components of the Darcy's velocity. The solutions (A7–A9) contain two complex constants \(C_{1}, C_{2}\), which are constrained by the following
boundary conditions. We specify continuity of fluid pressure and force balance at the surface of the core (Liao et al., 2018)
\[ \sigma_{rr}(R_0, \theta) = -P_{\text{ocean}}(\theta), \quad P(R_0, \theta) = P_{\text{ocean}}(\theta), \]
where \( \sigma_{rr}(R_0, \theta) \) is the radial component of the total stress in the core at the surface. \( P_{\text{ocean}}(\theta) \) is the fluid pressure applied by the overlying ocean at the seafloor. Without specifying the ocean’s pressure, we use the above two relationships to derive the fluid-loading condition \( P(R_0, \theta) + \sigma_{rr}(R_0, \theta) = 0 \), which leads to
\[ C_2 = C_1 \left[ \frac{1}{2} k^2 \left( K_m + \frac{4}{3} \mu^* \right) \frac{d^2 j_2}{dr^2} (kR_0) + \frac{1}{2} \frac{\alpha}{K_m + \frac{4}{3} \mu^*} j_2(kR_0) \right]. \]

To constrain \( C_1 \), an additional boundary condition on the maximum strain at the poles is assumed. For diurnal tides the maximum tidal strain \( \varepsilon_{\text{max}} \approx 3 \frac{M R_0^3}{m a^3} h_2 \), where \( e \) is eccentricity of the orbit, \( M \) and \( m \) are the mass of the planet and moon, \( R_0 \) is the radius of the moon, and \( a \) the planet-satellite distance; \( h_2 = \frac{5}{3} k_2 \) is the tidal rising Love number (Murray & Dermott, 1999). Following Kepler’s law the relationship may be written as \( \varepsilon_{\text{max}} = \frac{9}{4\pi} \frac{\omega^2}{\rho G} \frac{5}{3} k_2 \), \( k_2 = \frac{3}{5} \frac{1 + 3 \mu/2 \rho gR}{1 - 2 \mu/\rho gR} \), We assume that at the pole of the core the magnitude reaches the maximum value \( \varepsilon_0 = \frac{9}{4\pi} \frac{\omega^2}{\rho G} \frac{5}{3} k_2 \). For each set of \( \eta_m \) and \( \mu \), the boundary condition \( \varepsilon_{rr}(R_0, 0^\circ) = \varepsilon_0 \) is used for constraining \( C_1 \) via equation A8.

The volumetric heating rates \( h_{\text{tide}} \) and \( h_{\text{vis}} \) are obtained by (Al-Hadhrami et al., 2003; Cheng, 2016; Segatz et al., 1988; Tobie et al., 2005)
\[
\begin{align*}
\frac{h_{\text{tide}}}{\Omega} &= \sum_{n=1}^{\infty} \frac{\left( r^{(n+1)T_u} \frac{\delta \epsilon_{qy}}{\delta t} + r^{(n+1)T_u} P_r \frac{\delta \epsilon_{qy}}{\delta t} \right)}{n T_u} dt, \\
\frac{h_{\text{vis}}}{\Omega} &= \frac{1}{\Omega} \int_0^{(n+1)T_u} \frac{\eta_f}{k} \delta q \cdot \delta q dt.
\end{align*}
\]
(A10)
where \( \Sigma_{ij} \) indicates summation of all the indices. Substituting complex formulations into Equation A10 leads to
\[
\begin{align*}
\frac{h_{\text{tide}}}{\Omega} &= \frac{\omega}{2} \sum_{n=1}^{\infty} \left[ \Re(\sigma_q) Im(\epsilon_q) - Im(\sigma_q) Re(\epsilon_q) \right] - \frac{\omega}{2} \left[ \Re(\sigma_q^*) Im(\epsilon_q^*) - Im(\sigma_q^*) Re(\epsilon_q^*) \right], \\
\frac{h_{\text{vis}}}{\Omega} &= \frac{1}{2} \sum_{n=1}^{\infty} \left( |\epsilon_q|^2 + |\epsilon_q^*|^2 \right).
\end{align*}
\]
(A11)
and the total heating rate is calculated by volume integration of each heating rate
\[
\begin{align*}
H_{\text{tide, vis}} &= 2\pi \int_0^{R_0} r \int_0^\pi h_{\text{tide, vis}}(r, \theta) r^2 \sin \theta \, d\theta \, dr.
\end{align*}
\]

A2. End-Member Case: Tidal Heating of an Effectively Viscoelastic Body

The poroviscoelastic body behaves effectively as a purely viscoelastic one if the pore pressure is decoupled from the loading of the solid matrix (\( \alpha = 0 \)), or when the pore fluid is unable to flow (e.g., \( \kappa = 0 \)). In these cases, the strain-stress relation becomes identical to that of a viscoelastic body (Roberts & Nimmo, 2008; Ross & Schubert, 1989; Tobie et al., 2005)
\[
\dot{\sigma}_{\text{visco}} = \left( K - \frac{2}{3} \mu^* \right) \nabla^2 \Phi_{\text{visco}} + 2\mu^* \dot{\epsilon}_{\text{visco}},
\]
(A12)
where \( K \) is the bulk modulus (\( K = K_m \) if the system is drained or \( K = K_0 \) if the system is undrained). The quasi-equilibrium condition \( \nabla \cdot \sigma = 0 \) leads to the Laplace equation for potential \( \Phi_{\text{visco}} \) and a solution with degree-2 symmetry,
\[ \nabla^2 \phi_{\text{visco}} = 0 \quad \Rightarrow \quad \phi_{\text{visco}} = C_3 r^2 (3\cos 2\delta + 1), \]  
(A13)

which reduces the strain-stress relation to
\[ \sigma_{\text{visco}} = 2\mu^* \epsilon_{\text{visco}}, \]

and leads to heating rate
\[ h_{\text{visco}} = \frac{1}{T_{\omega}} \int_{n}^{(n+1)T_{\omega}} dt \text{Re}(\sigma_{ij}) \text{Re} \left( \frac{\partial \epsilon_{ij}}{\partial t} \right) - \frac{\omega R}{2} \text{Im}(2\mu^*) \Sigma |\epsilon_{ij}|^2 = 96C_2^2 \omega \text{Im}(\mu^*). \]  
(A14)

where \( C_3 \) is to be determined by the same maximum strain boundary condition. The heating rate \( h_{\text{visco}} \) is uniform in space because of the simplified form of Equation A13 arising from the implicit condition \( \Phi_{\text{visco}}|_{r \rightarrow 0} \neq \infty \).

Appendix B: Additional Results

Figures B1–B4 show the volumetric heating rates as functions of viscosity \( \eta_m \), the ratio between viscous dissipation in fluid and tidal heating in solid as a function of the viscosity \( \eta_m \), Biot constant \( \alpha \) and permeability \( \kappa \), the maximal tidal fluid responses, and the tidal and viscous heating in a shallow crust, respectively.

**Figure B1.** (a) Total heating rate for different solid bulk modulus \( K_s \). A second heating peak appears when \( K_s \) is small, indicating that the enhanced heating is linked to increased compressibility of the rock matrix. (b) Total heating rate, solid heating rate, and fluid heating rate as functions of solid viscosity for one set of parameters. (c) Total heating rates for different \( \alpha \) values and (d) total heating rates for different permeability values. Panels (c) and (d) indicate that the second heating rate peak migrates to higher viscosity for larger \( \alpha \) but does not depend on permeability.
Figure B3. Maximum pore pressure $|\vec{P}|$, radial component $|\vec{q}_r|$, and tangential component $|\vec{q}_\theta|$ of Darcy’s velocity. The solutions are obtained based on maximum strain condition, viscosity $\eta_m = 10^{16}$ Pa·s, rigidity $\mu = 1$ GPa, $\alpha = 0.1$, $\kappa = 10^{-8}$ m$^2$, $\phi_o = 20\%$. Globally averaged radial fluid velocity is $<|\vec{q}_r|/\phi_o> = 3.79 \times 10^{-6}$ m/s and tangential velocity is $<|\vec{q}_\theta|/\phi_o> = 2.20 \times 10^{-6}$ m/s.

Figure B4. Examples of volumetric heating rate and contours in shallow crust in a hemisphere for two different parameter combinations $\kappa$ and $\alpha$ combination. For both cases $\eta_m = 10^{17}$ Pa·s, $\mu = 1$ GPa, and $\phi_o = 0.2$. White dash lines indicate the skin depth $\delta = \sqrt{c/\omega}$ for oscillating poroelastic material (Jupp & Schultz, 2004). The horizontal axis shows colatitude $\theta$, where $\theta = 0$ indicates polar region.

Figure B2. Contours of ratio between heating in pore fluid and solid matrix $H_{\text{vis}}/H_{\text{tide}}$ varying with pairs of parameters. For panel (a), permeability $\kappa = 1 \times 10^{-10}$ m$^2$; for panel (b), poroelastic coefficient $\alpha = 0.2$. The dependence of $H_{\text{vis}}/H_{\text{tide}}$ on rigidity $\mu$ is negligible and hence is not shown.
Data Availability Statement

Matlab codes for realizing the analytical solutions are available at Code Ocean (Liao, 2020).

References


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