Reflections on “Parametric acoustic array,” source of virtual-array sonars

Kenneth G. Foote

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REFLECTIONS

The *Reflections* series takes a look back on historical articles from *The Journal of the Acoustical Society of America* that have had a significant impact on the science and practice of acoustics.

Reflections on “Parametric acoustic array,” source of virtual-array sonars

Article: Parametric acoustic array
Author: Peter J. Westervelt
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OVERVIEW

“Parametric acoustic array” is the title of Peter J. Westervelt’s seminal paper[1] on the subject, with antecedents.[2,3] It describes the effect of nonlinearity on simultaneous, overlapping acoustic waves propagating in the same direction in a fluid medium. For two harmonic primary waves, the result is the generation of two new, scattered waves, at the sum- and difference-frequencies. If the primary waves are similar in frequency, the difference-frequency will be low compared with both the primary- and sum-frequencies. Its absorption, consequently, will be relatively low, and the difference-frequency wave will survive to propagate beyond the distance at which absorption of the other waves renders them negligible. The medium thus acts as a low-pass filter. The primary beams are assumed to be planar and collimated. The difference-frequency beam is remarkably directional, notwithstanding its low frequency. Moreover, it has no sidelobes. In his paper, Westervelt gave analytic expressions for the scattered pressure field, intensity, total radiated power, and beam width. In conclusion, he noted the applicability of the concept to both transmission and reception. As a physical theory, the parametric acoustic array is exemplary, consisting of a set of mathematical propositions, deduced from a small number of principles, that aim to represent as simply, as completely, and as exactly as possible a set of experimental laws.[4]

$$\Box^2 p_s = -\rho_0(\partial q/\partial t)$$

$$q = {\rho_0}^{-2}{C_0}^{-4}\left[1 + {}^{1}\!/\!{}_{2}\rho_0{C_0}^{-2}\left(d^2p/d\rho^2\right)_{\rho=\rho_0}\right](\partial/\partial t){p_i}^2$$

Equations (7) and (8) in Westervelt’s paper (Ref. 1). This describes the scattered pressure field p_s due to source q through the nonlinear interaction of the incident pressure field p_i. $\Box^2$ is the d’Alembertian operator, ρ_0 and C_0 are the respective mean mass density of and sound speed in the fluid medium, and the bracketed expression is the coefficient of nonlinearity. Equations from Westervelt, “Parametric Acoustic Array,” J. Acoust. Soc. Am. 35, 535–537 (1963). Copyright 1963 Acoustical Society of America (Ref. 1).

Terminology: The eponymous array was named “parametric” for the parameter of nonlinearity in the fundamental equation, where it appears in a combination with a convection term. The array was characterized as “end-fire,” as the new beams are formed in the direction of the virtual sources in the medium. The process of generation is also called “parametric conversion.” In common parlance, “parametric sonar” refers to a realization of the parametric acoustic array as a device to transmit sound, with accompanying reception almost always accomplished by ordinary linear processes. The name is tantamount to the “virtual-array sonars” of the title.

IMPACT OF THE ARTICLE

Westervelt described several approximations in the opening section of his paper, with evaluation for a particular incident pressure field. Relaxing or removing these, as well as generalizing to practicable realizations, stimulated much subsequent work. This included, for example, determination of effects of finite primary-wave aperture, signal demodulation, cylindrical and spherical spreading, identification of different operating regimes, treatment of effects of diffraction, medium inhomogeneities, and noise. Early experimental demonstrations included measurements in water and air[3,5,6] as well as proof that the parametric conversion does occur in the medium.[7] Discussions on the scattering of sound by sound were also stimulated. Development and applications of parametric sonar have been aided by the Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation, derived via Burgers’ equation, with diffraction, absorption, and nonlinearity included.[8]

Notwithstanding the general weakness, or inefficiency, of parametric conversion, many special or niche applications of the parametric acoustic array have been recognized, especially to exploit its accessible, formidable bandwidth and exceptional directionality at low frequencies. Underwater examples include, among others, communication in shallow water, sub-bottom profiling, ensonification for fish swimbladder-resonance spectroscopy, naval sonar, and measurements in confined volumes when low frequencies are required. For applications in air, the parametric array loudspeaker is prominent. Patents have followed, in abundance, for both transmitting and receiving applications.

BREVITY

Few readers of "Parametric acoustic array" have not noticed its brevity. Some profound works of science, indeed of the human mind,[9] have been exceptionally brief.[10–12]

ORIGIN

The story of the parametric acoustic array began in London about 1951, when Westervelt was asked to deliver a steak-and-kidney pie to the illustrious Captain H. J. Round. In Round's garret-laboratory, an underwater 18-kHz transducer was sitting on a bench, pinging. Westervelt noticed a very powerful low-frequency, but directional, sound. He wondered, was it being generated at the transducer, in his ears or brain, or in between?

By KENNETH G. FOOTE
Woods Hole Oceanographic Institution, Woods Hole, Massachusetts 02543, USA

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