Supporting Information for "High-frequency internal waves and thick bottom mixed layers observed by gliders in the Gulf Stream"

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Contents of this file

1. Text S1.

2. Figure S1.

Introduction

In this supporting information, we present the derivation of ΔPE, the change in potential energy due to formation of a bottom mixed layer from an initial state of linear stratification.

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Text S1.

Consider a water column with an initially linear potential density profile (σ_0) and constant vertical density gradient $\frac{\partial \sigma_\theta}{\partial z}$ (Fig. S1, blue). Complete mixing of the lower portion of this water column with zero mixing in the upper portion of the water column would result in a potential density profile (σ_{mixed}) as indicated by the red profile in Fig. S1. A bottom mixed layer of thickness Δz_{ml} would result, and the resulting density profile (σ_{mixed}) would have a step at depth $-H + \Delta z_{\mathrm{ml}}$.

To conserve mass in this one-dimensional problem, the potential density in the resulting bottom mixed layer must be equal to the average density of the original profile within Δz_{ml} of the seafloor. We write the initial potential density profile as $\sigma_0 = \sigma_{\mathrm{surf}} + \frac{\partial \sigma_\theta}{\partial z} z$, where σ_{surf} is the surface density. It follows that the density within the bottom mixed layer is $\sigma_{\mathrm{mixed}} = \sigma_{\mathrm{surf}} + \frac{1}{2}\frac{\partial \sigma_\theta}{\partial z}\left(-2H + \Delta z_{\mathrm{ml}}\right)$.

The change in potential energy due to formation of such a bottom mixed layer is $\Delta PE = \int_{-H}^{0} \Delta\sigma_\theta g z \, dz$, where the $\Delta\sigma_\theta = \sigma_{\mathrm{mixed}} - \sigma_0$ is the change in potential density due to mixing, g is gravity, and H is the bottom depth. For the profiles defined above, the change in potential density is then

$$\Delta\sigma_\theta = \begin{cases} 0, & z > -H + \Delta z_{\mathrm{ml}} \\ \frac{\partial \sigma_\theta}{\partial z}\left(-H + \frac{\Delta z_{\mathrm{ml}}}{2} - z\right), & -H < z \leq -H + \Delta z_{\mathrm{ml}} \end{cases} . \tag{1}$$

Substituting into the definition of potential energy change and evaluating, we have

$$\Delta PE = \int_{-H}^{-H+\Delta z_{\mathrm{ml}}} \frac{\partial \sigma_\theta}{\partial z}\left(-H + \frac{\Delta z_{\mathrm{ml}}}{2} - z\right) g z \, dz \tag{2}$$

$$= g\frac{\partial \sigma_\theta}{\partial z}\int_{-H}^{-H+\Delta z_{\mathrm{ml}}}\left(-Hz + \frac{\Delta z_{\mathrm{ml}}}{2} z - z^2\right) dz \tag{3}$$

$$= g\frac{\partial \sigma_\theta}{\partial z}\left[-\frac{H}{2}z^2 + \frac{\Delta z_{\mathrm{ml}}}{4}z^2 - \frac{z^3}{3}\right]_{-H}^{-H+\Delta z_{\mathrm{ml}}} \tag{4}$$

$$= g\frac{\partial \sigma_\theta}{\partial z}\left[-\frac{H}{2}\left(H^2 - 2H\Delta z_{\text{ml}} + \Delta z_{\text{ml}}^2 - H^2\right)\cdots\right. \quad (5)$$

$$+ \frac{\Delta z_{\text{ml}}}{4}\left(H^2 - 2H\Delta z_{\text{ml}} + \Delta z_{\text{ml}}^2 - H^2\right)\cdots \quad (6)$$

$$\left. - \frac{1}{3}\left(-H^3 + 3H^2\Delta z_{\text{ml}} - 3H\Delta z_{\text{ml}}^2 + \Delta z_{\text{ml}}^3 + H^3\right)\right] \quad (7)$$

$$\Delta PE = -\frac{g}{12}\frac{\partial \sigma_\theta}{\partial z}\Delta z_{\text{ml}}^3. \quad (8)$$

The change in potential energy due to formation of a bottom mixed layer from an initial state of linear stratification is proportional to the cube of the mixed layer thickness and directly proportional to the initial stratification (8).

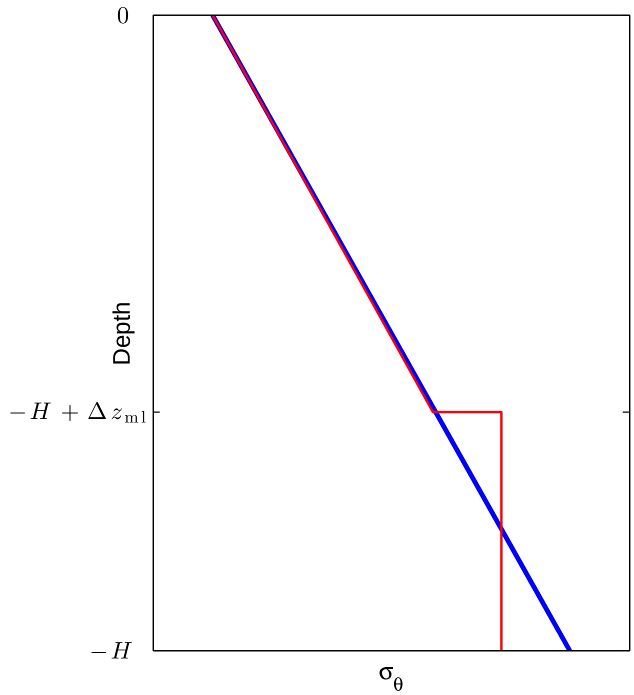

Figure S1. Profiles of potential density σ_θ. The blue initial profile (σ_0) is linearly stratified. The red profile (σ_{mixed}) results from complete mixing of σ_0 between a depth of $-H + \Delta z_{\mathrm{ml}}$ and the seafloor at depth $-H$.