Abstract: The ocean basins have almost exactly the correct surface area and average depth to hold Earth’s water. This study asserts that three processes are responsible for this. First, the crust is thickened by lateral compression from mountain formation. Second, Earth’s continental crust is leveled by erosion. Third, due to the efficiency of erosion, the average elevation is a few hundred meters above sea level. A theoretical fluid model, suggested partly by laboratory experiments, includes an ocean of specified depth. The resulting continents are tabular (that is, their elevation view is rectangular). The surface lies above sea level, contributing to a well-known double maximum in Earth’s elevation corresponding to continents and ocean basins. Next, a simple hydrostatic balance between continent and ocean gives average depth and area of present oceans and continents within 33%. Further calculations with a suitable correction to fit present Earth cover a wide range of possible crust volumes for earlier Earth. With the present water volume, ocean area always exceeds 25% of the globe. For all possible water volumes, average continental crust thickness always exceeds 23.4 km. This may explain why cratons have thicknesses comparable to younger crust so that they are found on Earth’s surface today. Therefore, mountain building, and erosion have enabled water to carve its own cistern in the form of the accumulated ocean basins. The wide range of areas and depths of oceans and continents found here can constrain models of early earth. Similar calculations can be done for earthlike planets as well.

Keywords: freeboard, continental crust depth, ocean depth, continent area, ocean area, continental crust and water volumes
1. Introduction

It is well known that Earth’s ocean basins average approximately 3800 m (2.5 miles) depth covering approximately 70% of the surface and containing 97% of the world’s water (e.g. Sverdrup et al. 1942). The deep ocean basins contain even closer to 100% of Earth’s liquid water during glacial periods. Therefore, continents and oceans have come to a balance so that the ocean basins hold almost all of the water. This balance possibly has some effect on planetary climate, although there is at least one suggestion that this might only apply to second order (Kuhn et al 1989).

Geochemists know that orogeny, volcanism, and sedimentation, contribute to the chemical evolution and building of continental material (e.g. Dewey and Windley 1981, McLennan and Taylor 1983, Cogley 1984, Taylor and McLennan 1995, Saal et al. 1998, Clift and Vannuchi 2004, Hawkesworth et al. 2010, Walther 2005, O’Neill et al. 2015). The mechanical consequences of orogeny, volcanism and sedimentation seem to be studied less. How did the continents and ocean basins mechanically evolve so that the oceans have exactly the correct area and average depth to hold Earth’s water? However, the overall dimensions of continents and oceans have their origin in the balances between mountain building and erosion rate (Harrison 1994, Zhang 2005, et al., Roberts 2015).

Here, the freeboard concept (Wise 1972, 1974) is added as in Whitehead and Clift 2009, which leads to mean continent elevation being only a few hundred meters above sea level.

Mountain building is a direct consequence of the great rigid “tectonic” plates moving over Earth’s surface. It is well known that the ocean floor on each plate is produced and moves away from mid-ocean ridges at divergent margins, and the floor is consumed by sinking (subducting) into the interior of the earth (the mantle) in subduction zones at convergent margins. Therefore, any fixed thing within a plate, including a continent, migrates from divergent to convergent margins. Once a continent arrives at a subduction zone, compression and orogeny occurs and mountain belts are produced (Press 2003, Johnson and Harley 2012). Over 80 orogenic events exist in the geological record over roughly four billion years (Kearey et al. 2009, Johnson and Harley 2012) pointing to the possibility of plate tectonics existing over that period of time. Although each event has a special story (e.g. Royden 1993a, Kaufman and Royden 1994, Huerta et al. 1996, 1998, 1999, Thatcher et al. 1999, Yin and Harrison 2000), during each event,
the continental crust is thickened. Orogeny produces a thickness increase of 2.5% in the past 65 my (Clift and Vannuchi 2004, Whitehead and Clift 2009). The thickening decreases the total continent area by 1.7% and consequentially increases ocean area.

This persistent crustal thickening is balanced to first order by redistribution of continental material by erosion (Harrison 1994). The erosion rates are sufficiently great to erode the continent surface down to an elevation close to sea level. Estimates of time scales for erosion of mountain belts are a few hundred million years, [Veizer and Jansen 1985; Harrison 1994, Zhang 2005, Clift et al. 2009]. The average surface elevation is presently only 835 meters above sea level, so earth has tabular continents that lead to the well-known double maximum in elevation distribution (Sverdrup et al. 1942, Harrison 1988, 1990 1998, Harrison et al 1981, 1983, 1985). This distribution is not observed for other rocky planets and moons (Uchupi and Emery 1993).

The eroded continental material deposited on the seafloor is ultimately buried at subduction zones (Clift et al., 2009). This material feeds volcanism that produces new continent material through andesitic lavas at continent margins. (Gazel et al. 2015). The cycle of water into and out of the mantle is also involved in generating new continental crust (Höning and Spohn 2016). Another contributor to continent area increase is internal deformation, which is best documented as strongly localized divergence within mountainous regions [Kaufman and Royden, 1994], [Huerta et al., 1998, 1999], [Royden, 1993b, Huerta et al., 1996], [Thatcher, et al., 1999], [Bird, 1979, Kay and Kay, 1993] [Conrad and Molnar, 1997, Houseman and Molnar, 1997]. For example, calculations based on measurements of Himalayan spreading and uplift have produced estimates of the viscosity of the continental material [England and Molnar, 1997, Flesch et al., 2000]. There is no present estimate of continent divergence globally.

Finally, the freeboard effect notes that continent edges are almost exactly at sea level as a consequence of erosion, which is more effective under air than under the ocean surface (Wise 1972, 1974, Harrison, 1998, 1999, Hynes, 2001, McElroy and Wilkinson, 2005). This effect, plus the strength of erosion (loc. cit.) limits the average elevation of the continents to under 1 km above the sea surface.
The balance between mountain building; continent extension; erosion; sedimentation; the burial of sediments in the mantle; and the eventual return of some of the sediment material and water to the continents is summarized in Figure 1. The purpose of this paper is to present a simple fluid mechanical model of this and to report some consequences of ensuing calculations. In Section 2, some fluid experiments show how thickening of a floating fluid from convergence of an underlying fluid leads to elevation of the surface. In Section 3, a fluid theory produces the tabular nature of continents when the floating fluid is in the presence of an ocean. In Section 4, a simple calculation explores a basic consequence of the constraint that continent elevation has a fixed value above sea level. This simple isostatic calculation quantifies one of the fundamental features of earth and oceans, yet it appears to be absent from textbooks and summarizing articles about Earth (e.g. Schubert and Sandwell, 1989, Turcotte and Schubert 2002, Press 2003, Grotzinger and Jordan 2010, King 2015, Condie 2016). Then, additional calculations use other possible volumes of crust and water for an earlier Earth constraint.
2. Suggestive laboratory experiments

Laboratory experiments were constructed to quantify the balance of crustal thickening and spreading. The first experiment has a layer of high viscosity silicon oil floating above corn syrup in a rectangular container (Figure 2). The roller on the right rotates counterclockwise, and the one on the left rotates clockwise at the same rate. Convergence at the oil-syrup interface driven by the two rollers thickens the center of the layer of floating oil and elevates the surface (Figure 2b). For sufficient roller speed, the oil separates from the walls (Figure 2c). Vertical thickening along the center of the tank increases with roller speed (Figure 3). This agrees with a linear theory developed in the Appendix. Using densities and viscosities of the laboratory materials, the maximum elevation of the top surface $e$, (see the table in supplementary material for a list of symbols) as a function of roller angular rotation rate $\Omega$, is given by the formula $e = 1.36 \times 10^{-3} \Omega$ and the depression of the interface $d_{in}$ is given by $d_{in} = 3.31 \times 10^{-3} \Omega$. Therefore, elevation is proportional to driving rate $\Omega$.

Figure 2. (a) The experimental apparatus with a flow driven by 2 rollers. This produces two overturning cells in the bottom fluid that exert stress to the upper fluid. (b) The top layer thickens at the center with upward deflection of the center for slow roller speed. (c) At higher roller speed, the surface layer is swept into an isolated body.
Figure 3. Elevation of the top and depression of the interface as a function of roller rotation rate $\Omega$. The dashed lines are from equations (A20) and (A21) in.

The second experiment shows that the elevated surface is not dependent on Newtonian viscosity. It has a top layer of 600 polypropylene balls of 0.64 cm diameter combined with 100 balls of 0.32 cm diameter floating in silicon oil with kinematic viscosity 10 cm$^2$ s$^{-1}$ lying under air. The combination of sizes of balls produces a mushy matrix. They occupy half of the interior volume of a transparent circular cylinder whose rotational axis is tangential to gravity. A circulation cell in the oil is driven by steady slow counterclockwise rotation, sweeping the floating balls toward the descending region on the left (Figure 4). There, the balls collect in a deep patch. The surface of the balls in the patch lies above the level of the fluid, even though they are located above the descending oil.
Figure 4. Photograph of a cluster of floating spheres in a counterclockwise slowly rotating cylinder 13.3 cm in diameter half full of silicon oil. The camera looks along both the axis of the cylinder and along the rotation axis. Gravity is directed downward.

All experiments show that the top surface of the thickened patch extends upward. The elevation of the top layer in Figure 3 depends on the speed of the driving, so this experiment does not include the two different rates of erosion under air and under water that are vital to Wise’s explanation of freeboard. Therefore, a simplified model and its theory is developed that will incorporate the two different rates of flattening for continent surfaces below and above water consistent with the freeboard concept.

3. A mechanical model of a continent with the ocean present.

This model has floating viscous fluid that is thickened by convergence from a velocity imposed at the base of the continent material. To incorporate gravitational spreading that balances this convergence, we take note of a popular parameterization for erosion that flattens the continent surface in accordance with the relation

\[
\frac{\partial h}{\partial t} = -Kh
\]  

Typically, the erosion rate \( K \) is proportional to an erosion coefficient determined by the land size, and inversely proportional to an erosion boundary layer depth and an isostatic factor (e.g. Zhang 2005). In addition, \( K \) typically is a nonlinear function of slope.

However, for simplicity our model makes \( K \) a constant and we assign a viscosity to the continent material, making it a Newtonian fluid.

Our model is a simplification of the dynamics of the continental crust (Figure 5a). Flow at the base of the crust imposed by mantle flow pushes the deep continental material toward the right (Figure 5a,b). The surface tilts upward toward the right to provide a hydrostatic pressure gradient to drive the return flow. This is the exact balance discussed by Zhang (2005), who parameterized erosion more fully than is to be done here. Although earth has more than one continent, this model represents an average of all the continents.
Figure 5. Elevation views of the simplified equilibrium state of the continents and oceans. Vertical direction is stretched greatly. (a) A cartoon of the dynamics. Surface flow corresponding to erosion is generated by viscous flow within the continent, driven by bottom velocity. (b) Geometry of the theory. The surface flow shown by dashed arrows represents downhill movement of the surface through erosion.

In the mathematical model (Figure 5b), three fluids representing ocean, continent and mantle have densities $\rho_o$, $\rho_c$, and $\rho_m$. They all lie in a field of gravity with acceleration $g$ downward. Air lies over everything but it has negligible density and viscosity. The ocean has depth $d_o$, and the continent has kinematic viscosity $\nu$. The continent has three regions: 1. The first is an old continent region under the ocean surface. 2. This is an old continent region lying under air. 3. Finally there is a region of mountains.

In all three regions, motion is imposed at the base of the continent by specifying velocities. Regions 1 and 2 are characterized by different surface restoring forces as models of erosion under water and under air, and they have a constant positive normal
speed $U_1 = U_2$. Region 3 has a greater normal speed $U_3$. This faster flow rate under mountains represents convergence of the mantle in subduction zones that produce mountain belts. The return flows are shown in Figure 5b by dashed arrows. Return flows are meant to represent and to simulate erosion and not actual creep of the continent material. Slope is largest in Region 1 from a smaller restoring force that represents small erosion rate. Regions 2 and 3 are characterized with two different bottom speeds. Therefore, the mountains have a greater return flow in a negative direction than the return flow of old continent under air. This is represented by the dashed arrow under $L_3$.

Surface slope is smallest in region 2.

The continent thickness and surface slope are calculated as a function of the ocean depth, velocities, and physical properties of the materials. It will be assumed that the aspect ratio $d_0/L_2$ is small. Although the speeds are free to vary over wide ranges of possible relevance to Earth, Stokes flow is anticipated. This limit is defined using the speed under region 2 as a typical speed so that $U_2 d_0 / \nu \ll 1$. To model the weak erosion rate of the continent surface under water compared to erosion rate of the continent surface under the atmosphere, we specify $\rho_2 - \rho_1 \ll \rho_2$. In other words, this model uses a small surface restoring force to represent small erosion under the ocean. Thus, the density of ordinary water is not used but instead a density close to continent density is given for the ocean. This small density difference is an artifice to insure that the surface restoring force of the continent under water is much less than the surface restoring force to the continent under air.

In this model, the flow within the continent is the only unknown flow. The continent fluid obeys continuity and Stokes flow.

$$\nabla \cdot \vec{u}_i = 0$$

$$\nabla p_i = \mu \nabla^2 \vec{u}_i - g \rho c \vec{k}$$

with $\vec{u}_i$ the velocity vector and $p_i$ the pressure. The ocean and atmosphere are low in viscosity compared to the continent, producing negligible stresses on the continent except for hydrostatics. In addition, atmospheric pressure is zero at sea level and along the
continent surface. Third, the flow is strictly two dimensional in the vertical and lateral plane.

The vertical origin is located at the ocean floor (Figure 5), and the lateral origin is located at the continent-sea surface point of intersection. Taking the limit $d_0^2/L_2^2 \ll 1$, (with $L_2$ a typical lateral length), the common lubrication approximation exists with negligible vertical velocities and lateral derivatives. The hydrostatic equation governs vertical forces for the continents.

We first develop the equations in the regions under air, (Regions 2 and 3) The pressure obeys $p_2=0$ at $z = h_i(x)$ ($i=2,3$), where $h_i$ is the surface elevation above the ocean floor. The hydrostatic approximation is

$$p_i = g \rho_c (h_i - z)$$

(4)

Therefore, the lubrication approximation is valid in the lateral direction, and since it has only vertical derivatives, the equation in each region is

$$\frac{1}{\rho_c} \frac{\partial p_i}{\partial x} = g \frac{\partial h_i}{\partial x} = -\nu \frac{\partial^2 u_i}{\partial z^2}$$

(5)

with solution

$$u_i = \frac{g}{\nu} \frac{\partial h_i}{\partial x} \frac{z^2}{2} + A_i z + B_i$$

(6)

This approach resembles simple lubrication theory as applied for example for asthenospheric flow and return flow (Turcotte and Schubert 2002). The depth of the base of the continent is determined by setting pressure below it to the hydrostatic value using the hydrostatic approximation,

$$z = \delta h_i(x)$$

(7)

where $\delta = \rho_i/(\rho_m - \rho_c)$. We set $u_i = U_i$ there to get

$$B_i = U_i - \frac{g}{\nu} \frac{\partial h_i}{\partial x} \frac{z^2}{2} + A_i \delta h_i.$$ 

(8)

In addition, the lateral shear at the continent surface is set to zero giving
\[ A_i = -\frac{g}{\nu} \frac{\partial h_i}{\partial x} h_i. \]  

(9)

Inserting these in (6),

\[ u_i = \frac{g}{\nu} \frac{\partial h_i}{\partial x} \left( \frac{1}{2} (z^2 - \delta^2 h_i^2) - h_i (z + \delta h_i) \right) + U_i. \]  

(10)

Lateral volume flux \( F_i \) is

\[ F_i = \int_{-\delta h_i}^{\delta h_i} u_i \, dz = -\frac{1}{3} \frac{g}{\nu} \frac{\partial h_i}{\partial x} (1 + \delta)^3 h_i + U_i(1 + \delta) h_i. \]  

(11)

Steady State

For a continent that is steady and fully developed, \( F_i = 0 \) so that

\[ \frac{\partial h_i}{\partial x} = \frac{3 \nu U_i}{g h_i^2 (1 + \delta)^2}. \]  

(12)

This integrates to

\[ h_i^3 = \frac{9 \nu U_i x}{g (1 + \delta)^2} + C_i. \]  

(13)

In region 2, \( h = 0 \) at \( x = 0 \) so \( C_2 = d_0^3 \) and the elevation is

\[ h_2^3 = d_0^3 + \frac{9 \nu U_2 x}{g (1 + \delta)^2}. \]  

(14)

In region 3, the integration starts from the point where the surfaces are matched at \( x = L_2 \) so the elevation is

\[ h_3^3 = d_0^3 + \frac{9 \nu U_3 x}{g (1 + \delta)^2} + \frac{(U_2 - U_3) \nu L_2}{g (1 + \delta)^2}. \]  

(15)

Now we consider regions 1 under water. To calculate the surface, we use (13) with \( g \) replaced by reduced gravity \( g' = g(\rho_c - \rho_o)/\rho_c \) so that

\[ h_1^3 = d_0^3 + \frac{9 \nu U_1 x}{g'(1 + \delta)^2}. \]  

(16)

The lateral length for region 1 is found setting \( h = 0 \), at \( x = -L_1 \) so

\[ L_1 = \frac{g'(1 + \delta)^2 d_0^3}{9 U_1 \nu}. \]  

(17)
Since we specify $g' \ll g$, for the purpose of determining a volume of the crust in this region, we approximate $L_1=0$.

For use in the following section, it is necessary to find the volume of the continent. Assuming that this 2 dimensional model has a typical width $w$ in the third direction, and noting that the total depth of the continent at each point is given by $(1+\delta)h_i(x)$, the volume of each region is

$$V_1 = 0$$  

$$V_2 = w \int_0^{L_2} (1 + \delta) \left( \frac{9U_\alpha V_x}{g} \right)^{\frac{1}{2}} dL_1$$  

$$V_3 = \frac{w}{3} \left( \frac{9U_\alpha V}{g} \right)^{\frac{1}{2}}$$

$$V_3 = \frac{w}{3} \left( \frac{9U_\alpha V}{g} \right)^{\frac{1}{2}}$$

$$\left\{ L_2 + L_3 + \frac{g(1+\delta)d_0^3}{9U_3} + \frac{(U_2-U_3)L_2}{9U_3} \right\}^{\frac{1}{2}} - \left\{ L_2 + \frac{g(1+\delta)d_0^3}{9U_3} + \frac{(U_2-U_3)L_2}{9U_3} \right\}^{\frac{1}{2}}$$

$$V_1 = 0.$$  

In regions 2 and 3, the elevation is expanded as a Taylor series about the left hand point that starts each region so that

$$V_2 = wd_0 (1 + \delta) L_2 \left\{ 1 + \frac{vU_2 L_2}{6g(1+\delta)^2 d_0^3} \right\}$$  

$$V_3 = wd_0 (1 + \delta) L_3 \left\{ 1 + \frac{vU_2 L_2}{3g(1+\delta)^2 d_0^3} \right\} \left( 1 + \frac{vU_3 L_3}{3g(1+\delta)^2 d_0^3} \right)$$

To make realistic tabular continents, the term on the right within the brackets in (22) must be small so that the old continent has roughly constant thickness. This is true if

$$\frac{vU_2 L_2}{6g(1+\delta)^2 d_0^3} < 1.$$  

Using present Earth values in mks units of $g=10 \text{ ms}^{-2}$, $d_0=3800 \text{ m}$, $\rho_2 = 2800 \text{ kg/m}^3$, $\rho_3 = 3300 \text{ kg/m}^3$, and $L_2=10^7 \text{ m}$, this requires that $vU_2 < 2.5 \times 10^3$. If one uses a speed associated with 0.1 times a value for mantle convection of $3 \times 10^{-9} \text{ ms}^{-1}$ the
continent kinematic viscosity in this model has to be significantly smaller than $22.5 \times 10^{12}$ m$^2$s$^{-1}$. This is a value lower than commonly used mantle viscosity values that are approximately $10^{17}$ m$^2$s$^{-1}$ (Turcotte and Schubert 2002), but such a small value is appropriate here since it quantifies the effects of erosion. It is similar in magnitude to values of erosion by Zhang (2005) with timescales of $10^8$ years, and who used an empirical value for uplift rate of similar magnitude that fits the present continents. The smallness of this viscosity compared to mantle viscosity gives a measure of the strength of erosion needed for Earth to have tabular continents (also discussed by Zhang loc.cit.). This low value suggests that the early experiments in Section 2 and the theory in Appendix 1 might be revisited using lower continent viscosity. However, a layer corresponding to the ocean should also be added. In region 3, one would specify a lateral length perhaps $L_3=10^6$ m with bottom speed 20 times greater.

The important result for this model of the continent is that the body is not tabular because of its strength, but because the rate for erosion flattening the continent is greater than the rate of mountain generation. This result is also pointed out by Zhang (loc.cit.). However, we also have added the fact that the ocean imposes a freeboard constraint. Although it can be argued that the freeboard-orogeny-erosion balance applies to earth, supporting this through analysis from data about orogeny in addition to the rates of erosion already quantified by Zhang (loc.cit.) is beyond the scope of this paper.

Transient adjustment

The surface evolution is driven by conservation of volume flux

$$\left(1+\delta\right) \frac{\partial h}{\partial t} = \frac{\partial F}{\partial x},$$  \hspace{1cm} (24)

which can be advanced numerically. The timescale is $v/gd_0^2$, the speed scale is $U_2v/gd_0^2$ and the dimensionless area flux scale is $gd_0^3/v$. Figure 6 shows a calculation in which there is only one region corresponding to Region 2 with no Region 3. The surface $h$ gradually approaches a slope with a value of $3 \times 10^{-5}$. Note that an initial bump is substantially smoothed out after one time unit.
Figure 6. Transient response of surface elevation and flux driven by a moving bottom. (a) Uniform speed $Uzv/gd_c^2=10^{-5}$. A bump that is initially placed in the middle relaxes with time. Elevation values are multiplied by five to show details. Time is normalized by dividing by timescale $v/gd_c$. Time intervals are 0.1 units. A value of $10^{-3}$ is subtracted from flux to offset it from zero. (b) There are different driving speeds in two regions. On the right, the speed is ten times greater. Time intervals are 0.01 units before $t=0.1$, then intervals are 0.1 up to $t=1$, then intervals are 1 (blue/green) up to $t=4$. Flux $F$ is normalized by dividing by $gd_c^3/v$ and a value of $0.15x10^{-3}$ is subtracted to offset it from zero.

4. Areas and thicknesses of ocean and continent

In section 2, the experiments essentially had no ocean at all, so $d_0=0$. In section 3, the ocean depth $d_0$ is specified. Here, the value of $d_0$ is determined using simple hydrostatic equations. In the previous section, the pressure under the ocean was not a factor in the model. The pressure under the continents was hydrostatic and consistent with lubrication theory. In this section, the ocean lies over Earth’s mantle and contributes to pressure in the mantle. The mantle, continent and ocean are not moving and a hydrostatic balance exists between ocean and continent. The cartoon in Figure 7a (taken from Figure 1) shows the factors contributing to the thickness of the continent. The cartoon in Figure 7b shows the hydrostatic model analyzed here. The continents are aggregated together as one solid body that is floating on a liquid mantle. It lies next to a motionless liquid ocean.
Figure 7. (a) Sketch of the first-order factors modifying the thickness of the continental crust. (b) The hydrostatic balance of ocean basins and continents. (Simplified from Figure 5 in Whitehead and Clift, 2009).

Pressures within the mantle under ocean and continent are calculated using hydrostatics and set equal to each other. Mantle pressure under the oceans equals the weight per unit area from the accumulation of ocean water of depth $d_o$ and mantle of thickness $d_m$. This is set equal to the weight per unit area under the continents of thickness $d_c$ due to the continental crust. Hydrostatic pressure is calculated using the densities of ocean ($\rho_o = 1030 \text{ kg m}^{-3}$), continent ($\rho_c = 2800 \text{ kg m}^{-3}$), and mantle ($\rho_m =3300 \text{ kg m}^{-3}$). The equality of pressure under ocean and continent obeys the formula

$$2800gd_c=1030gd_o+3300gd_m+3300gD \tag{25}$$

with $g$ the acceleration of gravity. The constant $D$ is added to the equation to correct for everything left out of this model. The list of possible causes of corrections is long and includes many layers in the ocean floor, within the continents and in the upper mantle as well as features of mantle convection and even the density of air.

Defining $E$ as the elevation of the continent above sea level, and using the relation $E+d_o+d_m=d_c$ to eliminate $d_m$ from (25),
Next, the known values of the volumes of continental crust and water are incorporated. Using volume $V$, area $A$ and the subscript $o$ for ocean, and $c$ for continent, where

\begin{equation}
d_c = \frac{V_c}{A_c} \tag{27}
\end{equation}
and

\begin{equation}
d_o = \frac{V_o}{A_o} \tag{28}
\end{equation}
Using the fact that the two areas add up to the surface area of Earth

\begin{equation}
A_o + A_c = 5.1 \times 10^8 \text{ km}^2 \tag{29}
\end{equation}
\begin{equation}
A_o \text{ is eliminated using (28) and (29) and (26) becomes}
\end{equation}

\begin{equation}
E = \frac{500}{3300} d_c - \frac{2270 V_o d_c}{3300 \left(5.1 \times 10^8 d_c - V_c\right)} \tag{30}
\end{equation}
This reduces to the quadratic equation for crust thickness

\begin{equation}
2.55 \times 10^{\text{11}} d_c^2 - \left(500 V_c + 2270 V_o - 1.683 \times 10^{\text{12}} (D - E)\right) d_c - 3300 V_c (D - E) = 0 \tag{31}
\end{equation}
with solution

\begin{equation}
d_c = F \pm \sqrt{F^2 + 1.294 \times 10^{-8} V_c (D - E)} \tag{32}
\end{equation}
where $F = \left(500 V_c + 2270 V_o - 1.683 \times 10^{\text{12}} (D - E)\right)/5.1 \times 10^{\text{11}}$.

Does (32) produce a reasonable prediction of depth of continental crust on Earth as a function of water and crust volumes? First, we ignore all deviations between this model and Earth by setting $D=0$ and using the values $V_c = 7.679 \times 10^9 \text{ km}^3$ and $V_o = 1.178 \times 10^9$ [Whitehead and Clift 2009]. Using the present value of continent elevation above the sea surface $E=0.835 \text{ km}$ [Turcotte and Schubert 2002] in equation (32), we get $d_c = 28.1 \text{ km}$. This depth is 73% of the present average crust thickness $d_c = 38.4 \text{ km}$, [Whitehead and Clift 2009]. Continent area using this thickness and the present volume of crust is $2.67 \times 10^8 \text{ km}^2$, which is 133% of the actual area of $2 \times 10^8 \text{ km}^2$. Consequent ocean area is $2.53 \times 10^8 \text{ km}^2$, which is 81% of the actual area of $3.10 \times 10^8 \text{ km}^2$. The fact that all these values are within 34% of earth’s values indicates that this extremely crude model is a good first approximation.

It is clear that secular changes in the value of $D$ might arise, for example from secular changes in heat flow (Schubert and Reymer 1985). It is not the object here to
determine a better value of the present value of $D$ from direct geophysical measurements (e.g. Gossler and Kind 1996), although that would make a useful future study. Instead, we use the present crust thickness $d_c = 38.4$ to calculate the value of $D$ for present Earth and it is $D=-2.369$ km. Naturally, using (26) with this value of $D$ gives the present value of ocean depth $d_o = 3.8$ km [Whitehead and Clift 2009] as well as the correct ocean and continent areas.

It is useful to insert $D=-2.369$ km in (32) and find possible thicknesses and areas for earlier earth by considering plausible ranges of volumes of early continental crust and water. Here Earth’s present area is used. In addition, since early erosion is not easily quantified, a number of values of $E$ [Müller et al. 2008] are considered. Figure 8a shows the continental crust thickness as a function of $V_c$ for assorted values of $E$ using the present day value of $D$ and the present ocean volume (a value consistent with the present ocean area $A_o = 310 \times 10^6$ km$^2$, and ocean depth $d_o = 3.8$ [Whitehead and Clift 2009]. Values of ocean depth (Figure 8b) can also be found since it is linearly related to crust thickness by (26). The curves are the same as those in 8a but with different offset.

Figure 8 shows a number of things: First, $d_c$ extends from approximately 25 to 66 km in thickness. Second, the spread of the curves for different values of $E$ means that
continent thickness is sensitive in detail to continent elevation and hence, presumably to erosion rate. Third, since $E$ and $D$ are subtracted in equation (32), thickness is also sensitive to the exact value of $D$. Therefore, if early continent or mantle structures differ significantly from their present structures, the effect would be similar to the effects of different levels of erosion. Fourth, ocean depth varies considerably because $d_o$ ranges from just over 2 km up to almost 11 km. Since (26) shows that $d_o$ is linearly proportional to $d_c$, the curves in (a) and (b) have the same shape even though relative offsets are different. Fifth, and most importantly, even with this wide range of $E$ (hence $D$), continental crust thickness never is less than 25 km. The result that crustal thickness exceeds a certain value is consistent with the fact that the crust of cratons has thicknesses of the order of present crust (King 2005), resulting in cratons covering a significant part of the present continents (Figure 9).

![Figure 9](http://earthquake.usgs.gov/data/crust/maps.php)  (a) Map of geological provinces of the world. The shields are cratons extending to the surface and the platforms are cratons covered with sediment. (From public domain by USGS - [http://earthquake.usgs.gov/data/crust/maps.php](http://earthquake.usgs.gov/data/crust/maps.php)). (b) Continental crust thickness (km). (From public domain by USGS - [http://quake.wr.usgs.gov/research/structure/CrustalStructure/](http://quake.wr.usgs.gov/research/structure/CrustalStructure/)).

The crust thickness $d_c$ is related to areas of continent and ocean in (27)-(29). Figure 10 shows results for the same parameters as used to make Figure 8 plotted against areas. First, we see again $d_c<25$ km. Second, continent area extends up to approximately 80% of the total area of earth in the limit of three times the present crust volume. Third, ocean area correspondingly is as small as 20% in the same limit. Fourth, very small values of $V_c$ result in vanishing continent area so that Earth is largely covered by water. Fifth, present Earth (stars) is in a region sensitive to changes so relative thickness and areas change with different volumes of materials and erosion rates. Summarizing the
results shown in Figures 8 and 10, continent crustal thickness is likely to range between
the extremes of 25 to 70 km, but for all crust volumes, continent area is less than 80% of
earth’s area and the ocean area is more than 20%.

Figure 10. (a) Continent thickness as a function of the areas of the continents and oceans. Solid
curves are for 5 different values of mean continent elevation above sea level \( E \) and the
sixth dashed curve is for present elevation. (b) Ocean depth versus areas of the continents
and oceans collapses to a single curve for all \( E \). Water volume is fixed at the present
value \( V_o=1.178\times10^9 \text{ km}^3 \) and \( D=-2.369 \text{ km} \).

Next, we investigate sensitivity of continent crust thickness and ocean depth to
the volume of planetary water \( V_o \). Since past values are poorly constrained, a wide range
of values of water volume from 0.01 to 10 times the present volume is used. Mean
continent elevation is set to the present value \( E=0.835 \text{ km} \). Figure 11 shows crust
thickness as a function of crust volume and areas. Crust thickness extends from 23.4 km
to almost 180 k. Figure 12 shows ocean depth. It ranges from almost zero to 35 km.
Figure 11. Continental crust thickness for 8 values of the ratio of water volume to the present value as a function of (a) continental crust volume and (b) areas of oceans and continents (the dashed lines are offset from the solid lines and they use the scale shown on the right). The stars show present Earth. \( E = 0.835 \) km and \( D = -2.369 \) km.

Figure 12. As in Figure 11 but for ocean depth instead of crust thickness.

In summary, the analysis in this section shows that continental crust is greater than about 22 km (Figures 8, 10, 11, and 12). With present water volume on Earth, continents occupy less than about 75% of Earth’s area, and oceans occupy more than 25% of the surface area. These results assume first that there are circumstances where continents have evolved long enough for the dynamic balance described here to have become established on earth. Second, it assumes Earth has similar material properties and weathering rates so that erosion is strong enough to limits the elevation of the continent material. Third, it assumes similar values of \( D \).

5. Summary

Floating oil thickens when subjected to convergence by an underlying fluid, but it thickens with convergence speed and thus has shortcomings as a model of continents on earth. A theoretical fluid model using an ocean layer of given thickness and Newtonian viscosity produces a tabular continent. Using the hydrostatic pressure balance under the continents and ocean basins and using the fact that the average continent surface elevation is less than 1 km above the ocean surface, calculations show that the average thicknesses and the areas of Earth’s continents and oceans are readily estimated as a
function of volumes of continental crust and water. For application to earlier earth, and
for almost all possible volumes, the continental crust exceeds 22 km in thickness and
total continent area is less than 75% of Earth’s surface. This constraint may be useful to
those studying the evolution of Earth. If suitable changes in surface area are made,
calculations may also be useful for other earthlike planets or moons.

Admittedly, the kinetic details and consequences from erosion are crudely
represented in the simplified model in section 3. This does not seem to matter because if
erosion is strong enough to level mountains in a fraction of a planet’s lifetime, the tabular
nature of continents is not sensitive to erosion rates. Our model poorly represents other
aspects of erosion, too. Material is not swept off the top of the continents, moved to the
oceans or to low regions of a continent and swept into the mantle at subduction zones
only to be partially returned by volcanism in this model. However, even though this
model does not produce a realistic cycle of mantle material from surface to mantle and
back, a statement of conservation of the total continent material at any one time plus the
conservation of water on the planet is used to determine areas and average thickness of
oceans and continents.

The theory in Section 3 shows that that tabular continents arise from the simple
balance between orogeny, erosion and freeboard. This balance is likely to exist over wide
ranges of governing parameters with predictable thicknesses and areas of ocean and
continental crust linked to volumes of continental crust and water. In the process, water
on Earth, through the act of erosion on land, (and water on any other earthlike planet) has
carved its own cistern. This cistern, the ocean, holds most all of the water and causes
orogeny and erosion on the continents to form tabular continents. Note also that the	
tabular nature of continents certainly has effects on the collision and breaking apart of the
individual continents during Wilson cycles. Therefore, Earth’s water exerts a first order
effect on the areas of continents and oceans and presumably thereby it affects the pattern
of mantle circulation throughout all the depth of the mantle possibly all the way to the
core.

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Appendix  The deflection of a crustal layer by mantle circulation

This two-layer calculation gives the amount of thickening and thinning of floating fluid from circulation of fluid below in the limit of large viscosity (Whitehead 2003). A layer of viscous fluid representing continental crust lies above a circulating mantle. The top layer (Figure A1) has viscosity, $\mu_1$ density $\rho_1$ and average thickness $d$. It lies above the “mantle” fluid with viscosity $\mu_2$ density $\rho_2$, and average depth $D_m$ in a field of gravity $g$. Simplifications are: 1. Only steady flows are considered. 2. The matching conditions between layers are applied at the level mean interfaces so that the equations are linear. 3. The flows exist for small Reynolds number so that inertial forces are much smaller than viscous forces. 4. The flow is two-dimensional.

Figure A1. The two layer model driven by velocity at the lower boundary.

Accordingly, the equations of two-dimensional viscous flow are used

$$0 = -\nabla p_i + \mu_i \nabla^2 \vec{u}_i$$ (A1)
where \( i = 1, 2 \) denote the top and bottom layers, respectively, with \( \hat{u}_i = u_i \hat{i} + w_i \hat{j} \) and the \( \hat{i} \) and \( \hat{j} \) unit vectors in the horizontal and vertical directions. The continuity equation is

\[
\frac{\partial u_i}{\partial x} + \frac{\partial w_i}{\partial z} = 0 \quad \text{(A2)}
\]

The boundary conditions at the interface between the two layers (which is taken to be at \( z=0 \)) are the matching of lateral velocity \( u_1 = u_2 \), vertical velocity \( w_1 = w_2 \), and lateral stress

\[
\mu_1 \left( \frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x} \right) = \mu_2 \left( \frac{\partial u_2}{\partial z} + \frac{\partial w_2}{\partial x} \right). \quad \text{(A3)}
\]

In addition, the condition of vertical stress determines a value for interface deflection \( \eta_z \).

\[
2\mu_1 \frac{\partial u_1}{\partial x} + P_1 - 2\mu_2 \frac{\partial u_2}{\partial x} - P_2 = g (\rho_2 - \rho_1) \eta_z \quad \text{(A3)}
\]

Boundary conditions on the top of the upper layer are applied at \( z=d \). These conditions are zero vertical velocity, zero lateral stress (often called free-slip), and zero vertical stress that will produce a value for \( \eta_1 \). The final two boundary conditions are imposed at the bottom of the deep layer at \( z=-D_m \). They are that the vertical velocity is equal to zero, that \( w_2 = 0 \), and that the imposed velocity \( u_2 = U \sin kx \).

The solutions are of the form

\[
w_i = \{ A_i \sinh k\bar{z} + B_i \sinh k\bar{z} + C_i \cosh k\bar{z} \} \cos kx. \quad \text{(A4)}
\]

The boundary and interface conditions can be expressed as the matrix equation

\[
\begin{bmatrix}
\text{kD} & -sD - kD_s cD & cD + kD_m sD \\
-sD_m & D_m sD & -D_m cD \\
k & 0 & 1 & -k & 0 & 0 & -\mu_i \\
0 & \mu_2 & 0 & 0 & -\mu_i & 0 & 0 \\
0 & 0 & 0 & sd & dsd & dcd \\
0 & 0 & 0 & 2k^2 sd & 2kcd + 2k^2 dsd & 2kds + 2k^2 dcd
\end{bmatrix}
\times
\begin{bmatrix}
A_2 \\
B_2 \\
C_2 \\
A_1 \\
B_1 \\
C_1
\end{bmatrix}
= \begin{bmatrix}
Uk \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}. \quad \text{(A5)}
\]
where \(cD = \cosh kD_m, cd = \cosh kd, sD = \sinh kD_m\) and \(sd = \sinh kd\). The solutions for the constants are

3. \(A_1 = 2Uk^2 \mu_2 \left[ sD - kD_m cd \right] / \text{Det} \)

4. \(B_1 = 2Uk^2 \mu_2 \left[ sD - kD_m cD \right] (sd)^2 / \text{Det} \)

5. \(C_1 = -2Uk^2 \mu_2 \left[ sD - kD_m cd \right] sdc \left( \text{sdcd} \right) / \text{Det} \)

6. \(A_2 = 2Uk^2 \left[ \mu_1 D_m sD (sd)^2 + \mu_2 D_m cD (-kd + \text{sdcd}) \right] / \text{Det} \)

7. \(B_2 = 2Uk^2 \left[ \mu_1 (sD - kD_m cD) (sd)^2 \right] / \text{Det} \)

8. \(C_2 = 2Uk^2 \left[ -\mu_1 kD_m sD (sd)^2 + \mu_2 sD (kd - \text{sdcd}) \right] / \text{Det} \)

where

10. \(\text{Det} = -2k \mu_1 \left[ (sD)^2 - k^2 D_m^2 \right] (sd)^2 + 2k \mu_2 \left[ sD cD - kD_m \right] \left[ kd - \text{sdcd} \right] \).

The solution for \(\eta_1\) gives values of the elevation of the surface corresponding to continent elevation

13. \(\eta_1 = \eta_{01} \frac{kd \cosh kD_m - \sinh kD_m}{\sinh^2 kD_m - k^2 D_m^2} \left[ 2k \cosh kd - 3 \sinh kd \right]
\left[ \sinh^2 kd + \mu_0 \left[ \sinh kD_m \cosh kD_m - kD_m \right] \left[ \sinh kd \cosh kd - kd \right] \right] \)

14. \(\) (A6)

where \(\eta_{01} = \frac{2Uk^2 \mu_2}{g \rho_1}\) and \(\mu_0 = \mu_2 / \mu_1\). The solution for \(\eta_2\) gives the depth of the deflection of the interface between the two fluids corresponding to “continental roots”

17. \(\eta_2 = \eta_{02} \frac{kd \cosh kD_m - \sinh kD_m + \lambda \sinh kD_m \sinh^2 kd + \mu_0 \lambda \cosh kD_m \left[ \sinh kd \cosh kd - kd \right]}{\sinh^2 kD_m - k^2 D_m^2 \left[ \sinh^2 kd + \mu_0 \left[ \sinh kD_m \cosh kD_m - kD_m \right] \left[ \sinh kd \cosh kd - kd \right] \right]} \)

18. \(\) (A7)

where \(\eta_{02} = \frac{2Uk^2 \mu_2}{g \left( \rho_2 - \rho_1 \right)}\) and \(\lambda = D_m / d\).
Since continental crust is very thin compared to the depth of the mantle we take $kd$ small in (A6) and (A7) and also take the crust to be more viscous than the mantle (England and Molner 1997), so $\mu_0 = \mu_2 / \mu_1 \ll 1$. Then, (A6) (A7) simplify to

$$\eta_1 = \frac{-2U \mu_2 k [kD_m \cosh kD_m - \sinh kD_m]}{g \rho_1 d [\sinh^2 kD_m - k^2 D_m^2]} \quad \text{(A8)}$$

$$\eta_2 = \frac{-\rho_1 \eta_1}{\rho_2 - \rho_1} \quad \text{(A9)}$$

An important result in these limits is that the viscosity of the top layer has dropped out and is not important in determining the deflections. This arises because the surface and the interface develop values that allow stress from the mantle circulation to balance buoyant restoring forces produced by topography. Therefore, this simplified model of the production of continent by mantle convergence does not depend on the viscosity of the continents, or if continent viscosity is a proxy for a very small value of erosion, it does not depend on the strength of erosion.

Figure A2 shows values of deflection divided by layer depth for Equations A6 and A7 and for equations (A8) and (A9). For this calculation, the velocity $U = 3 \times 10^{-9}$ ms$^{-1}$ was used, which is the spreading rate of the Pacific plate. This model therefore has the same magnitude for the flow speeds in the deep and top shallow mantle, which is appropriate for uniform viscosity mantle convection. The other parameters used are the acceleration of gravity $g=9.8$ ms$^{-2}$, density difference between mantle and continent with the typical value $\rho_2 - \rho_1 = 600$ kg/m$^3$, and upper layer thickness $d=15$ km. This is the value if all continental crust were spread evenly over the globe. This calculation is an estimate of what happens in two cases, first, before the continent material is segregated into lumps and second, what happens in the experiments in Figure 2. Panel (a) has the
results for whole mantle convection with $D_m=2880$ km. The viscosity used is the well-known value for deep mantle convection $10^{21}$ pa-s. The ratio of viscosity between the lower and upper layer are $\mu_0 = 0.01$ and 100 in equations (A6) for the surface and (A7) for the interface (long and short dashes, respectively). In addition, curves from equations (A8) and (A9) are plotted as solid curve and they lie very close to the curve for $\mu_0 = 0.01$, so that excellent agreement exists between the general formulas and the approximations. Generally speaking, the value of the viscosity of the upper layer is not important as long as viscosity is more than ten times greater than mantle viscosity.

For parameters with the deflection of order one, the layer would break up and the calculation is invalid. Panel (a) indicates that cells with wavelength greater than about 6000 km produce interface deflections indicating break up (thick grey line). Therefore, in this model, continental crust could not cover earth but would instead be broken up into lumps of individual continent. Panel (a) also shows that wavelengths shorter than about 5000 km would not form proto-continent, but longer wavelengths produce a bottom interface amplitude of order one.

The size of deflection for parameters associated with shallow mantle convection (mantle layer 600 km deep and viscosity of $10^{19}$ pa s) are shown in figure A2b. The break-up criterion is not found since the amplitude is not close to one even for the longest wavelength mantle motion.
Figure A2 Elevation of the surface of the top layer and the interface between the two layers as a function of the driving wavelength (a) Results for a deep mantle flow. The long dashed lines are for $\mu_0 = 10^2$ the short dashed lines are for $\mu_0 = 10^{-2}$ and the solid lines are for equations (A1) and (A2). The thick grey line indicates the order one amplitude limit of the calculation. (b). Results for shallow lower layer. The symbols are from the approximations in equations (A1) and (A2) and the solid curves from equations (A6) and (A7).

A close view of the velocity field using parameters for both the crust and mantle with $\mu_0 = 10$ is shown in Figure A3. The upper panel has stretched vertical coordinates and a different velocity axis scale than the lower panel. The predominant stress exerted by the lower layer onto the upper layer is from shear $\mu_2 \left[ \frac{\partial u_2}{\partial z} \right]$ (the boundary condition $w = 0$ imposes the stress component $\mu_2 \left[ \frac{\partial w_2}{\partial x} \right] = 0$). The shear imposed by the lower layer drives a flow in the upper layer that has a return flow near the surface driven by the surface elevation slope. The overturning cell in the top layer is opposite in sense from the overturning cell in the lower layer. The shear exerted by the bottom circulation is almost completely balanced by the surface elevation slope so that the surface elevation slope is not a function of the viscosity of the upper layer. The balance between surface elevation and stress at the base of the surface layer might also be true for mountain belts over a convecting mantle and thus used to estimate shear stress that the mantle exerts on the bottom of mountain belt. The stress induced by surface slope has been used in
conjunction with Global positioning data of divergence to estimate an equivalent viscosity for the Himalayas (England and Molnar 1997).

Finally this is applied to the roller experiment shown in Fig. 3 using (A8) and (A9) and simplifying near the origin

\[ \eta_t = \frac{3U\mu}{2\rho_gD_m} \]  

(A20)

and for the surface between oil and syrup
\[ \eta_2 = \frac{3U\mu}{2(\rho_2 - \rho_1)gd^2k} \]  
\hspace{10cm} (A21)

We use velocity given by \( U = 2\pi\Omega \), and the following physical values: syrup density \( \rho_2 = 1439 \, \text{kg/m}^3 \), silicon oil density \( \rho_1 = 1020 \, \text{kg/m}^3 \), syrup viscosity 32.3 Pa\text{-}s, with roller radius \( r = 0.018 \, \text{m} \), depth between rollers and the oil/syrup interface \( d = 0.04 \, \text{m} \), and wavenumber \( k = 39 \, \text{m}^{-1} \) to get, in meters

\[ \eta_1 = 1.36 \times 10^{-3} \Omega \]  
\hspace{10cm} (A22)

and

\[ \eta_2 = 3.31 \times 10^{-3} \Omega . \]  
\hspace{10cm} (A23)

These are shown as dashed curves in Figure 3.

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