POLYMODE NEWS

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In this issue:

TEMPERATURE FINESTRUCTURE OF THE MODE-I EDDY

by Terry Joyce

Recent observations of temperature/salinity finestructure in the main thermocline of the northwest Atlantic (Joyce, 1976) show that large-scale T/S variability increases linearly northward by a factor of 2 on vertical scales of 2-50 m. This is associated with either internal waves or vertical mixing processes as the Gulf Stream is approached from the Sargasso Sea. The temperature finestructure of the MODE-I eddy was examined to discover whether any finestructure exists on the meso-scale. For this purpose, the data from an interval of dense CTD sampling (days 129-140, 9 May-20 May, 1973) was examined.

Dynamic heights between 500-1500 dbars from the MODE-I Draft Synoptic Atlas (1974) for the chosen time interval are shown in Figure 1. Also shown are CTD stations selected for the analysis of finestructure. The analysis consists of: (a) breaking the temperature profile between 600 and 800 dbars into four pieces, (b) first-differencing and Fourier transforming, (c) ensemble-averaging to obtain a sample temperature gradient spectrum, and (d) integrating over wavenumber bands to obtain variability on scales of 10-50 m and 2-10 m.

In Table I, the summary statistics for these stations appear. To obtain a non-dimensional estimate of the variability, the

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NOTES ON BISPECTRAL CALCULATIONS ON INTERNAL WAVES

by Mel Briscoe

In a companion article, McComas points out that bispectra can provide evidence of certain nonlinear interactions. I have been making a series of preliminary bispectral calculations from various kinds of internal wave data, and the observational picture is muddy. This is a brief status report. (Note that not only are internal waves a potential sink of eddy kinetic energy [Müller and Olbers, 1975; Frankignoul and Joyce, personal communication], but bispectra are simply the first step in the higher-order spectral analysis that yields trispectra, quadspectra, etc.; Tukey calls these "polyspectra," so the relationship to POLYMODE is at least nominal.)

Strictly speaking, a bispectrum is the double Fourier transform (i.e., over two frequencies, ω_1 and ω_2) of the triple correlation product (i.e., $R(\tau_1, \tau_2) = \langle x(t)x(t+\tau_1)x(t+\tau_2)\rangle$).

The bispectrum is usually calculated in the frequency domain after using the Fast Fourier Transform to get the frequency coefficients. I use

$$\beta(\omega_1, \omega_2) = \langle dZ_1(\omega_1)\, dZ_2(\omega_2)\, dZ_3^*(\omega_1 + \omega_2)\rangle$$

where the dZ are the Fourier coefficients, and the brackets indicate averaging over a number of segments of the time series being analyzed.

I like to think of the energy spectrum as a frequency-dependent description of the variance, and of the bispectrum as a bi-fre-

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NOTES from the Editor

We are pleased to note that Carl Wunsch of the Massachusetts Institute of Technology has been awarded the Texas Instruments Foundation's 1975 Founder's Prize. His accomplishments in applying modern data-processing techniques combined with hydrodynamic theory to oceanographic problems were cited. Also cited were his roles in international oceanographic programs (POLYMODE?), education, and academic affairs.

This award carries a monetary value of $35,000.

-- F. W.

CONTRIBUTORS

Dr. Melbourne G. Briscoe
Woods Hole Oceanographic Institution
Woods Hole MA 02543
(617) 548-1400, ext. 524

Dr. Terrence M. Joyce
Woods Hole Oceanographic Institution
Woods Hole MA 02543
(617) 548-1400, ext. 530

Dr. C. Henry McComas
Institute of Geophysics and Planetary Physics
Scripps Institution of Oceanography
La Jolla CA 92093
(714) 452-2072

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POLYMODE OFFICE NOTES

A meeting of the POLYMODE Theoretical Panel was held at Harvard University on 1-2 June to consider theoretical proposals to be reviewed at the National Science Foundation (NSF) Panel Review in mid-August.

Proposals which do not require review by the POLYMODE Organizing Committee, such as some renewal proposals, should be sent to the office of the International Decade of Ocean Exploration (IDOE) before 28 June for inclusion in the mid-August panel. POLYMODE IDOE proposal deadlines for the next calendar year are listed in POLYMODE News No. 4.

A NSF panel meeting for IDOE proposals will be held on 7 July.

The POLYMODE* News is produced at the Woods Hole Oceanographic Institution. It is edited by Ferris Webster and Leigh Stoecker.

If you have material of interest for this newsletter, please get in touch with either of the above at the Woods Hole Oceanographic Institution, Woods Hole, MA, 02543, Telephone (617) 548-1400.

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SUMMARY OF NONLINEAR INTERNAL WAVE MEETING

reported by Hank McComas

On 6 and 7 April, 1976, a meeting was held at the Institute of Geophysics and Planetary Physics (IGPP) in La Jolla to discuss nonlinear internal wave dynamics, compare recent work, and suggest future directions for theoretical and observational research in this field.*

In recent years, observations of oceanic internal waves have shown that the Garrett and Munk (GM) model is surprisingly good (Briscoe, 1975). Although this model contains no dynamics, it describes the shape and general energy level of the internal wave spectrum in regions of the ocean where one might expect a different internal wave field due to the proximity of strong currents, continental slopes, rough bottom, and other possible sources.

Beyond the basic description of the wave field is the important question of where the energy comes from, where that energy goes, and how it gets redistributed. Because we are unable to do the complete radiation problem, investigating all the sources, sinks, and the transfer among the waves, we must attack smaller pieces of the problem in simplified situations. The following will indicate the present battle plan and suggest new campaigns for understanding the internal wave regime.

One successful theoretical approach in this regard is the energy transfer within the wave field by weak nonlinear resonant interactions. McComas (1975) has shown that there are certain rapid interactions occurring which could account for some of the features of the spectrum as well as transfer energy to the inertial band. In an attempt to confirm or deny this finding, he is comparing theoretical predictions to observational evidence (bispectra) of such interaction. The theoretical ideas seem to be confirmed by observations taken from Arctic Ice Island T3 (Neshyba and Sobey, 1975) but not, as yet, from Briscoe's analysis of the IWEX and MISERY open ocean data (see accompanying article). A data exchange will be made, and other cross-checking of the computations will be carried out in an attempt to understand the apparent disagreement. More calculations will be made by Briscoe, but, at present, the usefulness of bispectra as a tool in investigating nonlinear interaction in the open ocean is unresolved.

Another technique that may be used to look at nonlinear interactions is to investigate perturbations to the universal GM model and develop time scales and "mean free paths" for the ensuing energy transfer. One particular problem that will be investigated by McComas is the return of a perturbed spectrum to almost equal intensities of upward and downward propagating waves, an observed feature of the wave field. The time and length scales for the process may indicate why vertical propagation of energy has been so difficult to observe. Ken Watson (University of California) pointed out that the problem of input from surface waves could readily be investigated in this manner, as both the model to assess the space-time scales and energy levels of the appropriate perturbation and the model for subsequent redistribution are currently available. He described his work with Bruce West and Bruce Cohen (1976) on surface-surface internal wave interactions in a realistically stratified ocean. Unlike the constant-buoyancy-ocean case, there is a strong coupling that seems to indicate that the surface wave field alone is a sufficient source for the high-frequency portion of the internal wave spectrum.

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*The meeting was attended by Walter Munk, Hank McComas, Jim Cairns, Blyth Hughes, and Peter Worcester of IGPP; Mel Briscoe of W.H.O.I.; Ed Sobey of Oregon State; Myrl Hendershott of Scripps; Rob Pinkel of Marine Physical Laboratory at Scripps; Tom Powell of U. of C., Davis; and Ken Watson of U. of C., Berkley.

NOTES ON BISPECTRAL CALCULATIONS ON INTERNAL WAVES (continued)

quency-dependent description of the skewness. In the same way that only the frequencies ω_1 and $-\omega_1$ contribute to the energy spectrum at each frequency, only frequencies ω_1, ω_2, and $\pm\omega_1 \pm\omega_2$ in the bispectrum contribute at each bi-frequency (ω_1, ω_2). Call $\pm\omega_1 \pm\omega_2 = \omega_3$: then the bispectrum has contributions only **when three Fourier components related by $\omega_1 \pm \omega_2 \pm \omega_3 = 0$ are phase-locked. In other words, if there are two frequencies in a time** series that are interacting to produce sum and difference components (beat notes), the bispectrum will show a non-zero value at that bi-frequency. For a linear Gaussian process, the bispectrum vanishes.

Bispectra may provide information which is otherwise difficult to obtain. For example, if frequencies 8 and 12 are interacting to give products at frequencies 4 and 20, a traditional energy spectrum might show peaks at 4, 8, 12, 20 without indicating whether or not the peaks are coupled. The bispectrum would be non-zero at (8,12), which tells you what you want to know.

Figure 3a is an example calculated from artifically-generated data. The display actually shows the bi-coherence, which is the modulus of the bispectrum normalized by the autospectral energy at ω_1, ω_2, and ω_3. It is called bi-coherence because of the similarity of its formulation to that of the traditional coherence. The display shows quadrant number 1 of McComas' Figure 5.

The artifical signal was

$$s = g + a(\sin 16t + \sin 32t + \dots + \sin 96t)$$

where g is Gaussian noise with zero mean and unity variance. In Figure 3a, a = 10. The signal contains phase-locked harmonics of frequency = 16, such that many different sum and difference triplets are present for example, 16 + 16 = 32, 32 + 16 = 48, 48 + 16 = 64, ... , 80 + 16 = 96. Hence, in the bi-coherence, one sees peaks at $(\omega_1, \omega_2) = (16,16), (32,16), \dots , (80,16)$. These produce sum "notes" of the "beating," and show up along the line $\omega_2 = 16$. There are also peaks along $\omega_2 = 32$, because there $(\omega_1, \omega_2) = (32,32), (48,32), \dots , (64,32)$, etc. The final peak occurs at (48,48).

Figure 3b shows the bi-coherence of a different test signal, namely

$$s = n + 10g \sin 16t - 50 \cos 32t$$

where n is non-Gaussian noise (actually, $n = g^2$). Since n contains all frequency components, one possible triplet for "beating" is x + y = 32, where x and y are any two frequencies imbedded in the noise n; this shows up in Figure 3b as the ridge running from (16,16) to (32,0), i.e., $\omega_1 + \omega_2 = 32$. Also, we can have 32 + x = y, which shows up as the ridge along either $\omega_1 = 32$ or $\omega_2 = 32$. Note that the sin 16t term cancels out in the averaged triple products because it is multiplied by g = Gaussian noise.

The point of these two examples is that different kinds of phase-locking and nonlinearities can give different patterns, and it is the *patterns* that we are looking for.

McComas' theoretical work suggests three different limiting self-interactions: "induced diffusion," which appears along a ridge at $\omega_2 = f$ = inertial frequency, and $\omega_1 = f$ to buoyancy frequency (= N); "elastic scattering," which appears along the same ridge as induced diffusion; and "parametric subharmonic instability," which appears along the line $\omega_1 = \omega_2$.

(One differentiates between induced diffusion and elastic scattering by computing

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NOTES ON BISPECTRAL CALCULATIONS ON INTERNAL WAVES (continued)

cross-bispectra between vertically-separated instruments: the vertical scales of the two interactions are different; hence, maintainance of a significant cross-bispectrum at large vertical separations suggests that elastic scattering, not induced diffusion, is taking place. The importance of which interaction is operative arises from the direction of energy transfer being from low to high frequencies for induced diffusion, but vice-versa for the elastic scattering.)

I have taken a first look at auto-bispectra of temperature and current records from IWEX (Sargasso Sea, main thermocline) and MISERY (Pacific Ocean, between Hawaii and California) and the patterns suggested by McComas are not evident, nor are any other patterns. The only mooring record I have discovered that shows a semi-clear pattern in the bispectrum is from a vector-averaging current meter at 2988 m depth near the crest of the Muir Seamount. Figure 4 shows the auto-bi-coherence for the north component of current. The apparent ridge of interactions along ω_2 = f to 2f is curious; since the local buoyancy frequency is around 0.6 c.p.h., most of the significant points must correspond to non-internal wave fluctuations!

This work is all preliminary, but has the potential of indicating which frequencies are involved in non-linear transfers, and how active those transfers are. One would hope that fluctuations in non-linear activity and character could be matched to, say, fluctuations in eddy kinetic energy and Cox number of the finestructure.

Reference

Müller, Peter and Dirk J. Olbers (1975) On the dynamics of internal waves. J. Geophys. Res., *80*, 3848-3860.

SUMMARY OF NONLINEAR INTERNAL WAVE MEETING (continued)

In regard to vertical propagation of energy, McComas pointed out that the sign of a quadrature of vertical displacement and vertical velocity could unambiguously decide the direction of net energy flux. Jim Cairns (Scripps) noted that such simultaneous records would shortly be available. He is working with data from a yo-yoing capsule which measures vertical displacement of an isotherm directly. The vertical velocity can be inferred from the known motion of the capsule itself.

Cairns also suggested that there may be a lot to learn by observing the growth of the internal wave field from completely calm water, similar to watching the growth of the surface wave field as the wind begins to blow. He noted that the conditions at Lake Tahoe provide a unique opportunity to do this. After the predictable winds hit the lake, Powell has observed that a well-developed, GM-like internal wave spectrum arises in the previously quiescent water. We should be able to observe the development, and possible decay, of the spectrum. The dynamics here are simpler than in the open ocean, and so is the experiment. Thus, it provides a good, inexpensive testing ground for a future open-ocean experiment, as well as providing a useful "radiation balance" experiment. If an energy balance experiment such as this cannot be successfully conducted in this controlled environment, there would seem little point in attempting it in the open ocean.

The group also discussed conventions for the display of (auto-) bispectra and cross-bispectra. Similar to spectra, where only half the frequency range is unique, i.e., $E(\omega) = E(-\omega)$, these higher-order spectra have certain symmetries in the bi-

(continued page 6, right column)

TEMPERATURE FINESTRUCTURE OF THE MODE-I EDDY (continued)

temperature gradient variance is divided by the square of the mean temperature gradient. This number, the Cox number, C_T, is not meant to parameterize all of the variance, but to serve as index of the finestructure.

The variation of C_T across the eddy for each of the wavenumber bands is shown in Figure 2. On scales of 2-10 m, there is a significant variation in C_T across the eddy; C_T is large in regions of high vertical shear. No significant variation of C_T on 10-50 m scales is evident. The percent variation of C_T is small, amounting to about 40 percent.

Results suggest that regions of high vertical shear of low-frequency currents could apparently be "sources" of temperature finestructure variability.

References

Joyce, T. M. (1976) Large-scale variations in temperature/salinity finestructure in the main thermocline of the northwest Atlantic. (under revision, to appear in Deep-Sea Res.)

Draft Synoptic Atlas--MODE-I (1974) (Unpublished manuscript) The MODE Executive Office, 54-1417 M.I.T., Cambridge MA 02139.

SUMMARY OF NONLINEAR INTERNAL WAVE MEETING (continued)

frequency plane (ω_1, ω_2) of Figure 5. It is recommended that auto-bispectra be displayed in octant 1, cross-bispectra with ω_1 and ω_2 from the same record in the unique octants 1 and 8, and cross-bispectra of three different records in the half plane $\omega_1 \geq 0$. The third frequency ω_3 is defined by the rule $\omega_1 + \omega_2 + \omega_3 = 0$.

References

Briscoe, Melbourne G. (1975) Preliminary results from the trimoored Internal Wave Experiment (IWEX). J. Geophys. Res., *80*, 3872-3884.

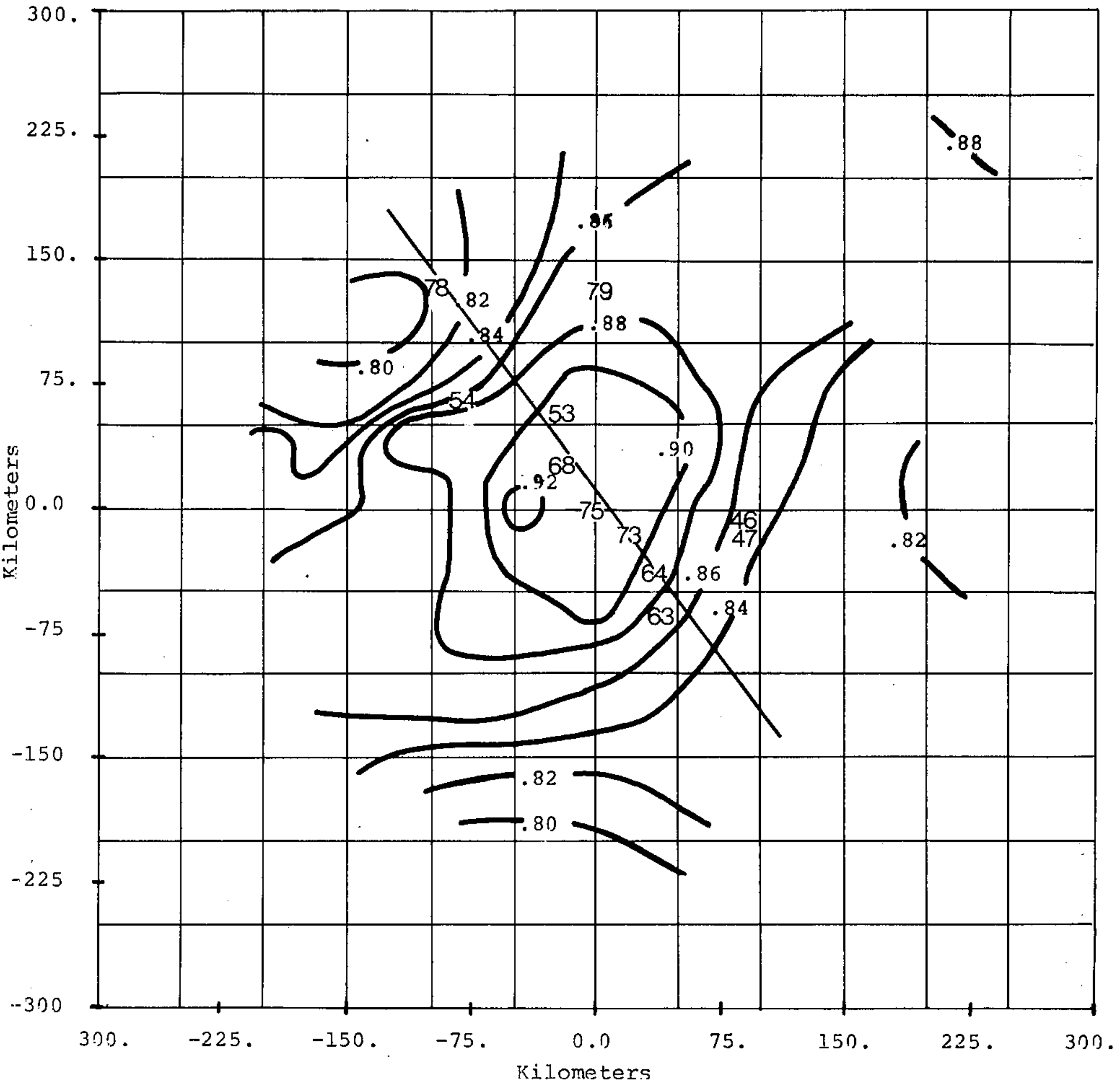
McComas, C. Henry (1975) Nonlinear interactions between internal waves. Ph.D. Thesis. The Johns Hopkins University.

Neshyba, S. and E.J.C. Sobey (1975) Vertical cross-coherence and cross-bispectra between internal waves measured in a multiple-layered ocean. J. Geophys. Res., *80*, 1152-1162.

Watson, Kenneth, Bruce West, and Bruce Cohen (1976) Coupling of surface and gravity waves: a mode coupling model. J. Fluid Mech. (in press).

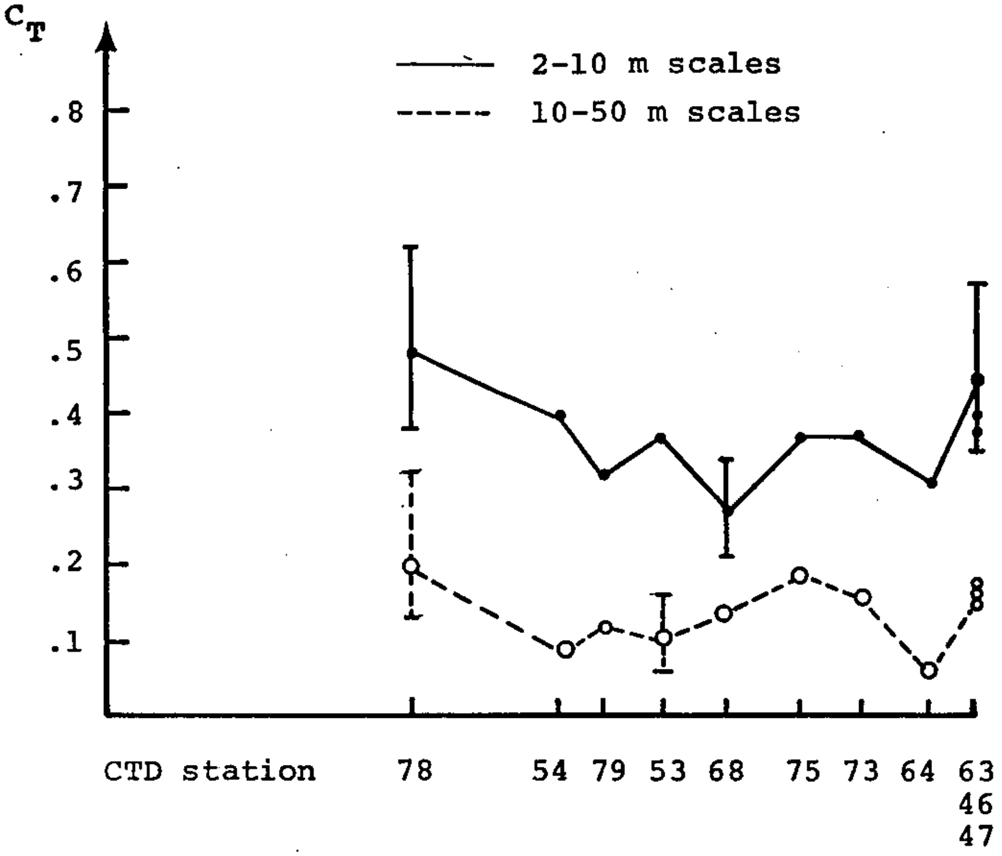
Chain 112 Station	Var (T_z) 10-50 m ($°C/m)^2$	Var (T_z) 2-10 m ($°C/m)^2$	$\bar{T}_z$ °C/m	(C_T) 10-50	(C_T) 2-10
46	$.705 \times 10^{-4}$	$.218 \times 10^{-3}$	.0227	.136	.421
47	$.915 \times 10^{-4}$	$.208 \times 10^{-3}$	.0237	.169	.372
53	$.581 \times 10^{-4}$	$.210 \times 10^{-3}$	.0238	.102	.369
54	$.591 \times 10^{-4}$	$.272 \times 10^{-3}$	.0262	.086	.395
63	$.741 \times 10^{-4}$	$.222 \times 10^{-3}$	.0224	.147	.442
64	$.378 \times 10^{-4}$	$.190 \times 10^{-3}$	.0246	.062	.313
68	$.641 \times 10^{-4}$	$.128 \times 10^{-3}$	.0217	.136	.272
73	$.788 \times 10^{-4}$	$.186 \times 10^{-3}$	.0224	.156	.369
75	$.926 \times 10^{-4}$	$.188 \times 10^{-3}$	.0226	.181	.366
78	1.093×10^{-4}	$.261 \times 10^{-3}$	.0233	.202	.481
79	$.652 \times 10^{-4}$	$.182 \times 10^{-3}$	.0237	.116	.324

Table I
(Joyce)



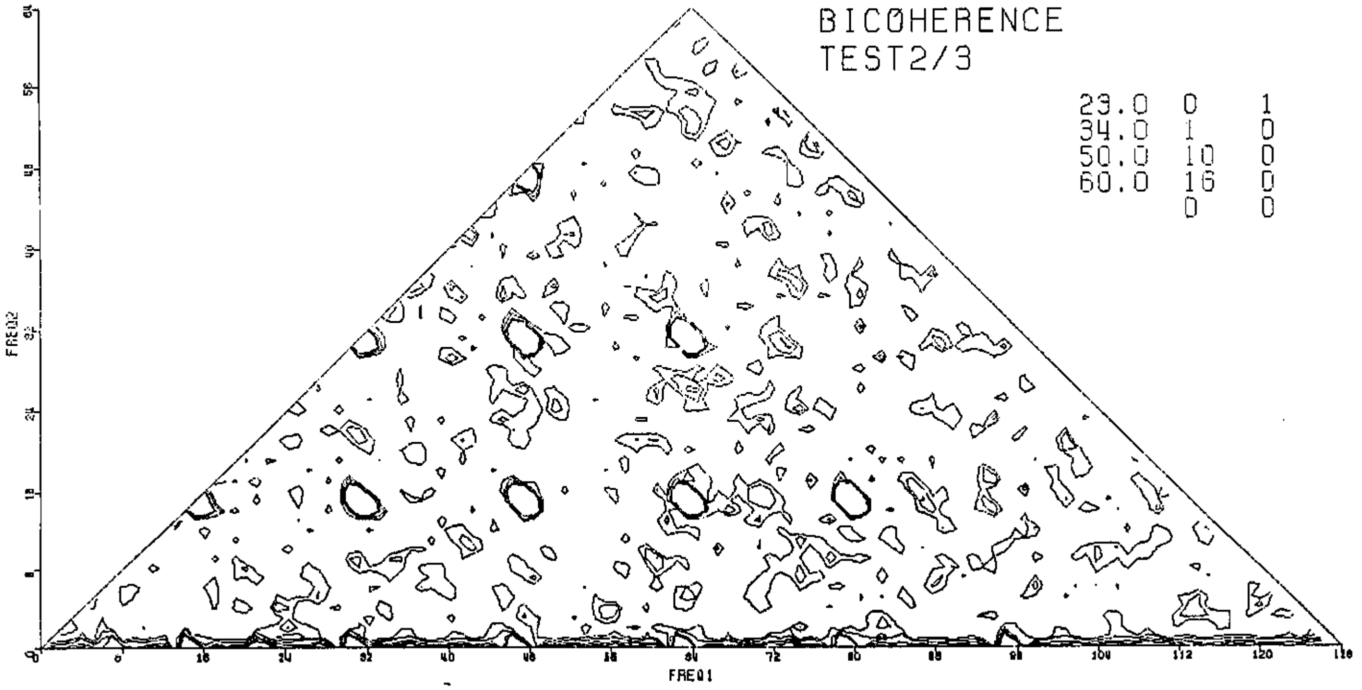
Dynamic heights between 500-1500 dbars at the MODE central mooring (28°N, 69°40'W) for days 129-140 (9-20 May, 1973) from the MODE-I atlas. The bold numbers are the CTD stations selected for fine-structure analysis. The contour interval is .02 dynamic meters.

Figure 1 (Joyce)



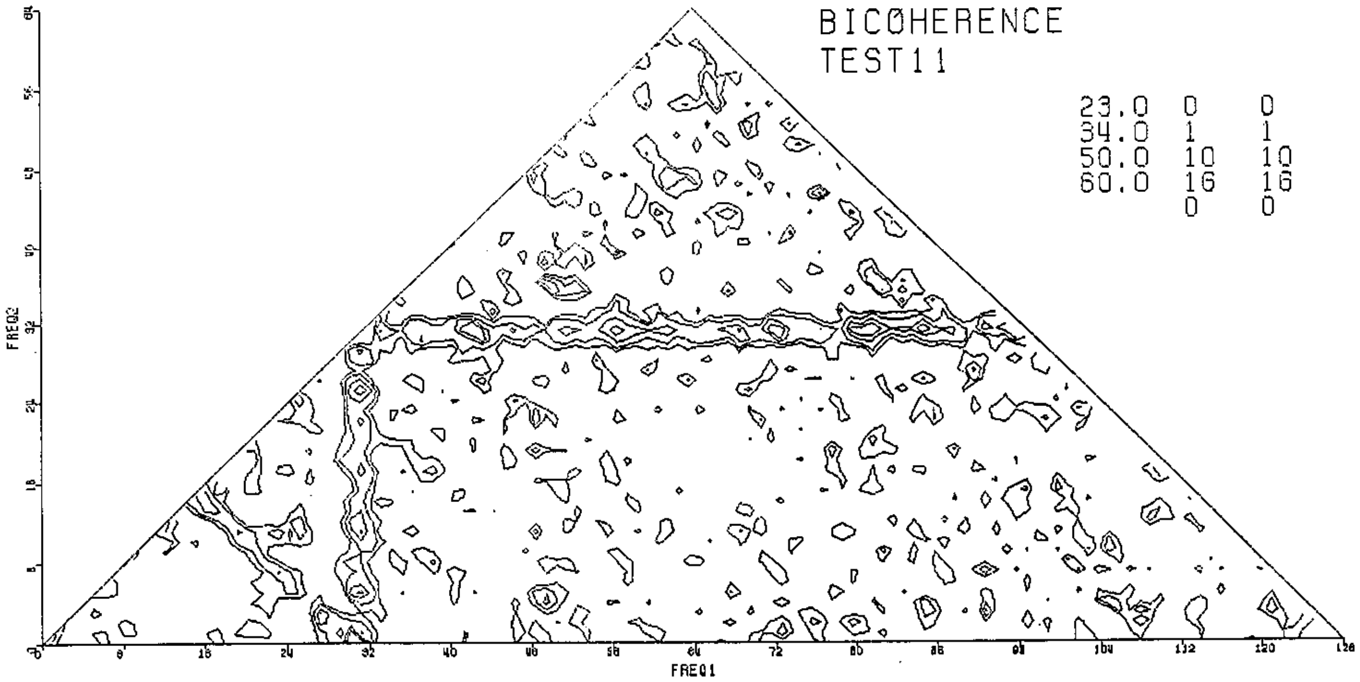
The variation of C_T across the MODE eddy along the line shown in Figure 1 for each wavenumber band. Error bars are 95 percent confidence limits.

Figure 2 (Joyce)



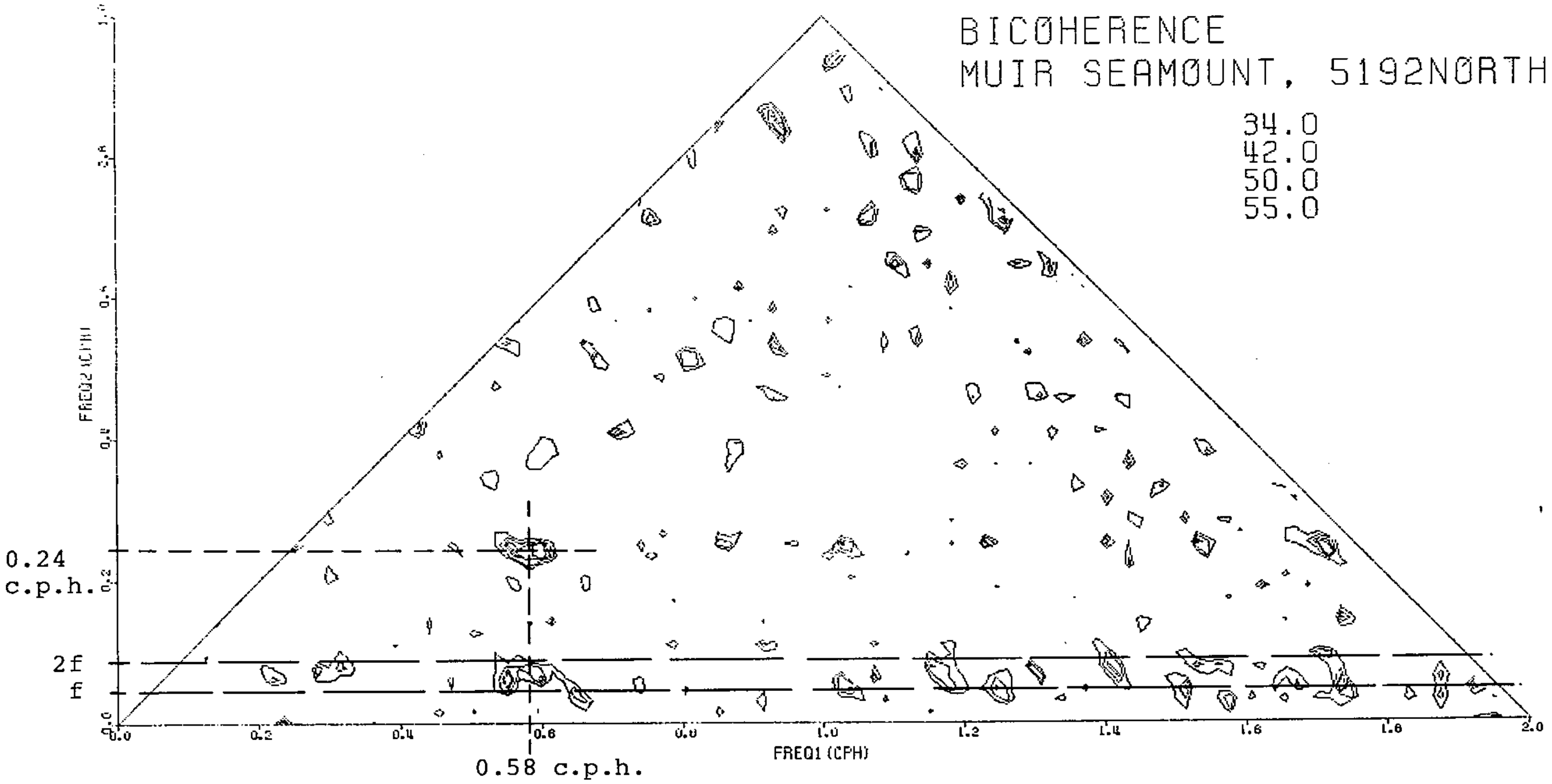
Bi-coherence (normalized bispectral modulus; see text) of a test signal composed of phase-locked sums of sinusoids of harmonically-related frequencies. Signal contains Gaussian noise which produces the "dirty bispectral window" through which the sine harmonics are seen. Contours are numerically obtained confidence levels: 95, 99, 99.9, ~100 percent.

Figure 3a



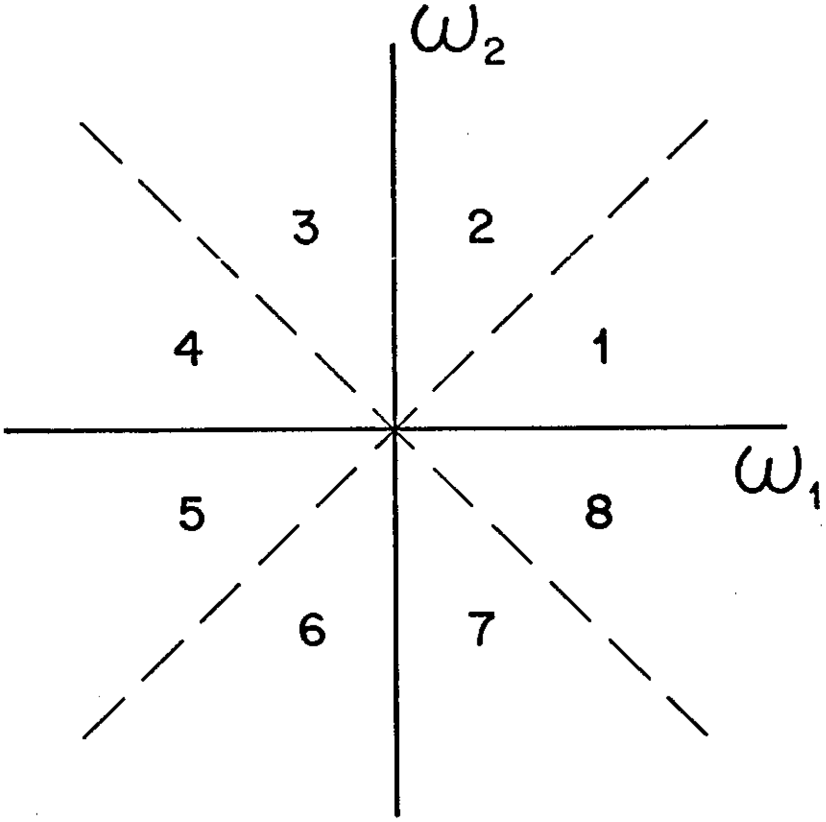
Bi-coherence of a test signal composed of non-Gaussian white noise plus a sinusoid. Contours as in Figure 3a.

Figure 3b
(Briscoe)



Bicoherence of north component of current from VACM 5192 above Muir Seamount. Record was broken into 20 64-hour segments, overlapped 50 percent with 19 additional segments. Full cosine (Hanning) tapers were applied to each segment; the bispectrum was estimated by averaging over the 39 segments. Contour levels shown correspond to 99, ∿99.4, 99.9, and ∿100 percent confidence that the hypothesis of zero bicoherence is rejected; numerical simulations gave the levels. The figure should be interpreted as showing: (1) a ridge of interactions between near-inertial frequencies (f-2f) and pairs of high frequencies; (2) an interaction involving 0.58 and 0.24 c.p.h. motions. (1) corresponds to a derived theoretical result; (2) is of unknown origin, although 0.58 c.p.h. is approximately the minimum buoyancy frequency in the water column.

Figure 4 (Briscoe)



The bifrequency plane for bispectral representation. Conventions for the display of auto-bispectra and cross-bispectra on this plane are discussed in the text. The eight octants are numbered according to this convention.

Figure 5 (McComas)