Supporting Information for "On the Southern Ocean CO_2 uptake and the role of the biological carbon pump in the 21st century": Box model equations

J. Hauck[1], C. Völker[1], D.A. Wolf-Gladrow[1], C. Laufkötter[2], M. Vogt[2], O. Aumont[3], L. Bopp[4], E. Buitenhuis[5], S. C. Doney[6], J. Dunne[7], N. Gruber[2], T. Hashioka[8], J. John[7], C. Le Quéré[5], I. D. Lima[6], H. Nakano[9], R. Séférian[10], I. Totterdell[11]

Contents of this file

1. Box model equations

Introduction This file gives equations for the box model that is set up to represent three surface boxes of the Southern Ocean with the southern border defined by the Antarctic continent and the northern boundary of the boxes as 58°S, 44°S and 30°S.

Physical transport: The Ekman transport across the box boundaries is calculated as

$$ekman = \frac{-\tau}{\rho \cdot f} \cdot L \quad (1)$$

where ρ is the density of sea water, f is the Coriolis parameter, L is the length of the (northern) box boundary and the wind stress τ is given as

$$\tau = u^2 \cdot dc \cdot \rho_{air} \quad (2)$$

with the zonal wind speed u, the drag coefficient dc and the density of air ρ_{air}.
The total outflow of water for a given box is then defined as the difference of the Ekman transports:

$$mtr_{box1} = -ekman_{box1} \quad (3)$$

$$mtr_{box2} = ekman_{box1} - ekman_{box2} \quad (4)$$

$$mtr_{box3} = ekman_{box2} - ekman_{box3} \quad (5)$$

where the southernmost box (box 1, Eq. 3) experiences no inflow from the south (Antarctic continent). The surface outflow is balanced by upwelling:

$$upw = -mtr \quad (6)$$

To close the mass balance for prescribed Antarctic Bottom Water (AABW) formation (see Eq. 14), this amount has to be added to the upwelling flux:

$$upw_{box1} = upw_{box1} + aabw \quad (7)$$

Tracer transport: We consider horizontal ($advh$) and vertical ($advv$) advection of the tracers T^i with i=$\{DIC, ALK\}$:

$$adv_{T^i} = advh_{T^i} + advv_{T^i} \quad (8)$$

with the horizontal advection calculated from Ekman transport:

$$advh_{T^i\ box1} = -ekman_{box1} \cdot T^i_{box1} \quad (9)$$

$$advh_{T^i\ box2} = ekman_{box1} \cdot T^i_{box1} - ekman_{box2} \cdot T^i_{box2} \quad (10)$$

$$advh_{T^i\ box3} = ekman_{box2} \cdot T^i_{box2} - ekman_{box3} \cdot T^i_{box3} \quad (11)$$

and vertical advection of the tracer T^i calculated from upwelling and AABW formation:

$$advv_{T^i} = upw \cdot T^i_{vt} \quad (12)$$

where the tracer concentration used to calculate vertical transport T^i_{vt} is the deep tracer concentration (T^i_{100m}) when upwelling is positive and the surface tracer concentration (T^i) is used when upwelling is negative, i.e. downwelling occurs:

$$T^i_{vt} = \begin{cases} T^i_{100m} & \text{if} \quad upw > 0 \\ T^i & \text{if} \quad upw > 0 \end{cases} \quad (13)$$

In the first (southern) box, additional vertical tracer transport due to Antarctic Bottom Water formation is prescribed:

$$advv_{T^i\ box1} = advv_{T^i\ box1} - aabw \cdot T^i_{box1} \quad (14)$$

The tracer fluxes are converted to concentration changes by taking into account the volume of the box V and the time step dt:

$$cadv_{T^i} = adv_{T^i}\ \frac{1}{V}\ dt \quad (15)$$

Source minus sink terms: The only source or sink for alkalinity are vertical and horizontal advection:

$$SMS(T^{ALK}) = cadv_{T^{ALK}} \quad (16)$$

[1]Alfred-Wegener-Institut, Helmholtz-Zentrum für Polar- und Meeresforschung, Postfach 12 01 61, 27515 Bremerhaven, Germany
[2]Environmental Physics, Institute of Biogeochemistry and Pollutant Dynamics, ETH Zürich, Zürich, Switzerland
[3]Laboratoire de Physique des Oceans, Centre IRD de Bretagne, Plouzane, France
[4]Laboratoire des sciences du climat et de l'Environnement (LSCE), IPSL, CEA-UVSQ-CNRS, UMR8212, Gif-sur-Yvette, France
[5]Tyndall Centre for Climate Change Research, University of East Anglia, Norwich Research Park, Norwich NR47TJ, UK
[6]Woods Hole Oceanographic Institution, Department of Marine Chemistry & Geochemistry, Woods Hole MA, USA
[7]NOAA Geophysical Fluid Dynamics Laboratory, Princeton, New Jersey, USA
[8]Core Research for Evolutional Science and Technology, Japan Science and Technology Agency, Tokyo, Japan
[9]Meteorological Research Institute, Tsukuba, Ibaraki, Japan
[10]CNRM-GAME, Centre National de Recherche Météorologique-Groupe d'Etude de l'Atmosphère Météorologique, Météo-France/CNRS, 42 Avenue Gaspard Coriolis, 31100 Toulouse, France
[11]Met Office, Exeter, UK

0886-6236/15/$12.00

Source and sink terms for DIC include horizontal and vertical advection, as well as air-sea CO_2 flux (F_{CO_2}) and a term for biological carbon fluxes that considers gross primary production (GPP) and a loss term ($LOSS$) that includes respiration and remineralization:

$$SMS(T^{DIC}) = adv_{T^{DIC}} - GPP + LOSS + F_{CO_2} \quad (17)$$

Export production is considered implicitly as the residual of $GPP - LOSS$. GPP and $LOSS$ are converted from rates to concentration change per time step by:

$$GPP = gpp\frac{dt}{d2s} \quad (18)$$

$$LOSS = (resp + remin)\frac{dt}{d2s} \quad (19)$$

The CO_2 flux (F_{CO_2}) is calculated as

$$F_{CO_2} = k \cdot dCO_2^{star} \quad (20)$$

with k being the piston velocity calculated following *Wanninkhof* [1992, Eq. 8] including chemical enhancement and

$$dCO_2^{star} = pCO_2^{air} \cdot ff - CO_2^{oce} \quad (21)$$

where pCO_2^{air} is prescribed, ff is a correction term for the non-ideality of the gas [*Weiss and Price*, 1980, Eq. 13] and CO_2^{oce} is derived from DIC and ALK by iteratively solving the equation for H^+ using the coefficients K_1, K_2, K_B and K_W from *Millero* [1995, Eq., 35, 36, 52, 63] . The CO_2 flux is then scaled by the amount of ice-free area (1-f_{ice}) in the box:

$$F_{CO_2} = F_{CO_2} \cdot (1 - f_{ice}) \quad (22)$$

References

Millero, F. J. (1995), Thermodynamics of the carbon dioxide system in the oceans, *Geochimica et Cosmochimica Acta*, *59*(4), 661–677, doi:10.1016/0016-7037(94)00354-O.

Wanninkhof, R. (1992), Relationship between wind speed and gas exchange over the ocean, *Journal of Geophysical Research*, *97*, 7373–7382, doi:10.1029/92JC00188.

Weiss, R., and B. Price (1980), Nitrous oxide solubility in water and seawater, *Marine Chemistry*, *8*(4), 347 – 359, doi: 10.1016/0304-4203(80)90024-9.

J. Hauck, Alfred-Wegener-Institut, Helmholtz-Zentrum für Polar- und Meeresforschung, Postfach 12 01 61, 27515 Bremerhaven, Germany(judith.hauck@awi.de)

Table S1. Variables in the box model

Short name	unit	description
$ekman$	$m^2\ s^{-1}$	Ekman transport
τ	$kg\ m^{-1}\ s^{-2}$	wind stress
u^2	$m^2\ s^{-2}$	zonal wind speed squared
mtr	$m^3\ s^{-1}$	volumetric Ekman transport
upw	$m^3\ s^{-1}$	upwelling
$advhT^i$	$mmol\ s^{-1}$	horizontal tracer advection
T^i	$mmol\ m^{-3}$	tracer concentration in surface box
$cadv_{T^i}$	$mmol\ m^{-3}$	conc. change by tracer advection
GPP	$mmol\ m^{-3}$	conc. change by gross primary production
$LOSS$	$mmol\ m^{-3}$	conc. change by biological loss processes
gpp	$mmol\ m^{-3}\ d^{-1}$	gross primary production rate
$resp$	$mmol\ m^{-3}\ d^{-1}$	respiration rate
$remin$	$mmol\ m^{-3}\ d^{-1}$	remineralisation rate

Table S2. Parameters in the box model

Short name	unit	value	description
ρ	$kg\ m^{-3}$	1025	density of sea water
ρ_{air}	$kg\ m^{-3}$	1.2	density of air
dc	-	$1.3\ 10^{-3}$	drag coefficient
$aabw$	$m^3\ s^{-1}$	$8\ 10^6$	Antarctic Bottom Water formation rate
dt	s	2592000	time step
$d2s$	s	86400	convert from day to second
f	s^{-1}	$-1.24\ 10^{-4}$	Coriolis parameter [a]
		$-1.01\ 10^{-4}$	
		$-7.29\ 10^{-5}$	
V	m^3	$2.471\ 10^{15}$	volume of box[a]
		$3.809\ 10^{15}$	
		$4.412\ 10^{15}$	
L	m	$2.120\ 10^7$	Length of northern box boundary[a]
		$2.878\ 10^7$	
		$3.464\ 10^7$	

[a] From south to north