Modeled alongshore circulation and force balances onshore of a submarine canyon

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Abstract Alongshore force balances, including the role of nonlinear advection, in the shoaling and surf zones onshore of a submarine canyon are investigated using a numerical modeling system (Delft3D/SWAN). The model is calibrated with waves and alongshore flows recorded over a period of 1.5 months at 26 sites along the 1.0, 2.5, and 5.0 m depth contours spanning about 2 km of coast. Field observation-based estimates of the alongshore pressure and radiation-stress gradients are reproduced well by the model. Model simulations suggest that the alongshore momentum balance is between the sum of the pressure and radiation-stress gradients and the sum of the nonlinear advective terms and bottom stress, with the remaining terms (e.g., wind stress and turbulent mixing) being negligible. The simulations also indicate that unexplained residuals in previous field-based estimates of the momentum balance may be owing to the neglect of the nonlinear advective terms, which are similar in magnitude to the sum of the forcing (pressure and radiation stress gradients) and to the bottom stress.

1. Introduction

Refraction of incident waves over shelf bathymetry, including canyons, deltas, and reefs, can result in a spatially variable wave field that can lead to alongshore variable surf zone forcing and circulation [Long and Ozkan-Haller, 2005; Thomson et al., 2007; Apotsos et al., 2008; Gorrell et al., 2011; Shi et al., 2011]. As a result, bathymetric features well outside the surf zone can result in alongshore-variable surf zone forcing and flows, even along relatively uniform shorelines. The wave-averaged depth-integrated alongshore momentum balance in the surf zone can be given as [Feddersen et al., 1998]:

$$\rho(\eta+h) \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\rho g(\eta+h) \frac{\partial \eta}{\partial y} - \frac{\partial S_{xy}}{\partial x} - \frac{\partial S_{yy}}{\partial y} - \tau_b - \tau_{wad} - v p(\eta+h) \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

(1)

where $\rho$ is water density (assumed 1025 kg/m$^3$), $\eta$ is the deviation of the mean water surface from the still water depth $h$, $u$ and $v$ are the depth and time-averaged current velocities in the cross shore ($x$) and alongshore ($y$) directions, respectively, and $t$ is time. The left-hand side of the equation is the total acceleration ($\frac{dv}{dt}$), which is the sum of local acceleration and the advective acceleration terms contributing to the alongshore balance. The acceleration is balanced by the alongshore pressure gradient ($\rho g(\eta+h) \frac{\partial \eta}{\partial y}$), where $g$ is the gravitational acceleration), the gradients of the diagonal and alongshore components of the radiation-stress tensor $S_{xy}$ and $S_{yy}$ [Longuet-Higgins and Stewart, 1964], the alongshore components of the bottom- and wind-stress vectors $\tau_b$ and $\tau_{wad}$ and the turbulent momentum flux owing to horizontal mixing $v p(\eta+h) \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$, where $v$ is the horizontal eddy viscosity. For alongshore-uniform waves, alongshore flows are driven primarily by the radiation-stress forcing term ($\frac{\partial S_{xy}}{\partial x}$) that results from the breaking-induced cross-shore dissipation of obliquely incident waves [Longuet-Higgins, 1970]. If an alongshore wave height gradient is present, a corresponding gradient in the wave setup (increase in mean sea level toward shore resulting from breaking waves [Longuet-Higgins and Stewart, 1964; Lentz and Raubenheimer, 1999; Raubenheimer et al., 2001]) is introduced, leading to an alongshore pressure gradient. Alongshore pressure gradients have been shown to drive alongshore flows at O(100 m) scales onshore of a submarine canyon [Apotsos et al., 2008; the same field site used here] and an ebb-tidal delta [Shi et al., 2011; Hansen et al., 2014]. Alongshore pressure gradients resulting from smaller-scale (O(10 m)) bathymetric features, such as a
gap in a sand bar, also can be important for driving surf zone flows [Putrevu et al., 1995; Slinn et al., 2000; Haller et al., 2002; Chen et al., 2003; Haas et al., 2003; Schmidt et al., 2005]. Prior studies have suggested that
the forcing of the alongshore currents by breaking obliquely incident waves and alongshore pressure gra-
dients is balanced primarily by bottom stress [Longuet-Higgins, 1970; Feddersen et al., 1998; Feddersen and
Guza, 2003; Apotsos et al., 2008]. However, modeling studies have suggested that nonlinear advection
resulting from alongshore variations in forcing and bathymetry also may be a significant contribution to the
momentum balance [Long and Özkan-Haller, 2005; Kumar et al. 2011, 2012; Hansen et al., 2013; Wilson et al.,
2013].

The third-generation wave model Simulating WAves Nearshore (SWAN) has been used to model waves in
coastal regions with gradual bathymetric variations [Booij et al., 1999; Ris et al., 1999; Zubier et al., 2003], to
examine the effects of offshore islands [Rogers et al., 1999; Zalite et al., 2003], and to model
the waves onshore of a steep submarine canyon [Magne et al., 2007; Gorrell et al., 2011]. Numerical wave-
current coupled models based on the nonlinear shallow water equations have been used to simulate surf
zone flows onshore of alongshore-variable bathymetry [Long and Özkan-Haller, 2005; Benedet and List, 2008;
Shi et al., 2011; Hansen et al., 2013; Wilson et al., 2013]. However, there have been few model-data compari-
sions of alongshore surf zone flows or momentum balances.

Here, alongshore flows and forcing from the shoreline to 6 m water depth onshore of a submarine canyon
are investigated using a combination of observations and numerical model simulations (SWAN and
Delft3D). In particular, model predictions of waves and flows and terms in the momentum balance are com-
pared with observations. The model is used to examine the details of the alongshore force balance in areas
that were not instrumented and to determine the contribution of terms neglected in field-based evalua-
tions of the momentum balance.
2. Observations

Observations were collected for 48 days in October and November 2003 at Black’s Beach onshore of the Scripps submarine canyon near La Jolla, CA (Figure 1). Collocated acoustic Doppler velocimeters (ADV) and pressure sensors were deployed at 26 sites along the 1.0, 2.5, and 5.0 m depth contours, forming nine cross-shore transects (Figure 1b) [Apotsos et al., 2008]. The ADVs were sampled at 2 or 16 Hz for 3072 s (51.2 min) every hour. Sample volumes were located approximately 0.3, 0.3, and 0.9 m above sand level for sensors located in 1.0, 2.5, and 5.0 m depths, respectively. The pressure sensors were sampled at 2 Hz and were located approximately 0.5 m above the bed in 5.0 m depth and buried approximately 0.5 m below the bed in 1.0 and 2.5 m depth to avoid pressure disturbances owing to flow over the instrument [Raubenheimer et al., 2001].

Bathymetry and topography were surveyed over the instrumented area approximately weekly. Bathymetry was surveyed to 6–8 m depth using a personal watercraft equipped with a single beam echo sounder and a Differential GPS (DGPS) system. Topographic surveys extending from the bluff base to the shoreline were conducted during low tide using a DGPS receiver mounted on an all-terrain vehicle or dolly. Both the bathymetric and topographic surveys consist of cross-shore transects with approximately 25–50 m alongshore spacing. Moreover, hourly estimates of the sand levels at the instrument locations in 1.0 and 2.5 m depth were obtained with the downward-looking ADVs. Additional bathymetry and topography were obtained from a U.S. Geological Survey seamless digital elevation model (DEM) compiled using several data sources, including LiDAR and multibeam bathymetry [Barnard and Hoover, 2010]. Bathymetry of the continental shelf (offshore of the DEM) is from NOAA National Ocean Service (NOS) surveys.

Water levels, water depths, wave frequencies, wave heights, and mean velocities were estimated for six 512 s (~8.5 min) periods each hour, and wave directions were estimated hourly [see Apotsos et al., 2008]. Mean water levels were estimated assuming the pressure signal was hydrostatic. Setup was defined as the increase in the mean water level relative to that measured on the 3.5 m isobath at \( y = 1300 \) m (sensor not shown in Figure 1) [Apotsos et al., 2008]. Mean water depths were estimated from the water levels and the

Figure 2. (a) Wave heights (\( H_{\text{rms}} \)), (b) peak period, and (c) peak direction observed at the CDIP Outer Torrey Pines buoy (depth ~ 550 m) versus date in 2003. The gray area indicates 27 October discussed in the text.
bathymetry (based on the surveys and the ADV measurements). Centroidal (energy weighted) incident wave frequencies and root mean square wave heights ($H_{rms}$) ($2\sqrt{2}$ times the standard deviation of the water surface fluctuations) were estimated from the pressure fluctuations between 0.05 and 0.30 Hz assuming linear wave theory and exponential decay of wave fluctuations through the bed [Raubenheimer et al., 1998]. Mean wave angles were estimated using the colocated velocity and pressure observations [Kuik et al., 1988; Herbers and Guza, 1990; Herbers et al., 1999]. The diagonal component of the radiation-stress tensor ($S_{xy}$) was calculated at the 2.5 and 1.0 m depth sensors with linear wave theory using the estimates of wave height, direction, and frequency, and water depth [Apostos et al., 2008]. The alongshore component of the radiation-stress tensor ($S_{yy}$), as well as alongshore gradients ($\frac{\partial S_{yy}}{\partial y}$) were negligible relative to the remaining forcing terms, and thus are neglected.

Offshore wave conditions were obtained from the Coastal Data Information Program (CDIP) Outer Torrey Pines waverider buoy in 550 m water depth approximately 15 km NW of the experiment area. Frequency-directional (two-dimensional) wave spectra were estimated from the buoy observations every 30 min using the maximum entropy method [Lygre and Krogstad, 1986]. Wind speeds and directions were recorded by a meteorological station on the end of Scripps Pier at the southern end of the experiment area (Figure 1a). Offshore $H_{rms}$ wave heights ranged from 0.3 to 1.3 m, peak periods ranged from 5 to 22 s, and peak incident wave directions (direction from) ranged from about 180° (south) to 290° (WNW) (Figure 2). Wind speeds ranged from calm to 16 m/s, but usually were light (mean 1.8 m/s). Maximum hourly averaged wave heights were 1.4 m at the 5 m depth sensors, and alongshore currents reached 0.8 m/s at the 1 m depth sensors. Tides were mixed semi-diurnal with a maximum range of 2.1 m recorded at the NOAA Scripps Pier tide gauge.

3. Numerical Model
The observations from 1 October to 8 November were simulated with the numerical hydrodynamic model Delft3D [Lesser et al., 2004, version 6.01.01.2703] coupled with the phase-averaged wave model SWAN [Booij
et al., 1999, version 40.72]. Delft3D solves the time-varying nonlinear shallow water equations on a staggered Arakawa-C grid using an alternating-direction-implicit solver [Lesser et al., 2004]. The circulation model consists of three domains: a large regional tide model (Figure 3a) forced with spatially variable, satellite-derived tidal constituents [Egbert and Erofeeva, 2002], and two higher-resolution curvilinear, shoreline-following “surf zone” domains (Figure 3b) that are two-way coupled and run simultaneously. The tide model is run without wave forcing and provides barotropic tidal forcing to the boundary of the outer surf zone domain via a combination of Neumann (depths \( \leq 5 \text{ m} \)) and Riemann (depths \( \geq 5 \text{ m} \)) boundary conditions. In this application, these boundary conditions minimized boundary effects and allowed the nested surf zone domains to be smaller and more computationally efficient. Resolution of the surf zone domains in shallow water adjacent to the shoreline is \( \approx 4 \text{ m} \) in the cross shore and \( \approx 8 \text{ m} \) in the alongshore. Given the spatial resolution, a 1.5 s time step was used to minimize numerical error. The model was run in depth-averaged (2DH) mode to minimize run times. Although the vertical flow structure is not resolved, this approach reproduces the alongshore dynamics, which has weak vertical structure in shallow water [Reniers et al., 2004].

The bathymetry and topography data independently were organized into triangulated irregular networks that were used to interpolate elevation values onto the numerical domain nodes. In areas where multiple data sets overlapped, preference was given to the weekly surveys first and the NOS surveys last. Elevation data sets were merged smoothly when creating the bathymetry used in the model to avoid spurious flows resulting from artificial discontinuities in bed level. The bathymetry from the survey date closest to the time period of interest was used in the model.

The model bottom stress includes both a current-induced stress \( \tau_c \) and a wave-generated stress \( \tau_w \), with both terms parameterized using quadratic drag laws [Soulsby et al., 1993],

\[
\tau_c = \frac{\rho g \mathbf{U} \cdot \mathbf{U}}{C_z}, \tag{2}
\]

and

\[
|\tau_w| = \frac{1}{2} \rho f_w u_{orb}^2 \tag{3}
\]

where \( \mathbf{U} \) is the total Generalized Lagrangian Mean (GLM) velocity vector (the sum of time-averaged Eulerian and Stokes’ drift components), \( f_w \) is a friction coefficient \([0,0.1]\) for oscillatory flow [Swart, 1974], and \( u_{orb} \) is the wave orbital velocity estimated from the wave height, frequency, and wavelength using linear theory. A spatially uniform Chezy \((C_z)\) roughness of 70 m\(^{0.5}\)/s (equivalent to a drag coefficient, \( C_d \), of 0.002) was used in both the cross shore and alongshore directions in all hydrodynamic domains. The total bottom stress, \( \tau_b \), including current and wave components, is converted to an Eulerian reference frame by correcting for the Stokes’ drift component of the GLM velocity. The total horizontal eddy viscosity \((\nu \text{ in equation (1)})\) in each grid cell is the sum of the background \( \nu_0 \) and turbulent \( \nu_t \) components. The background horizontal eddy viscosity was calibrated to 0.5 m\(^2\)/s. The turbulent component (typically about 0.25–1.00 m\(^2\)/s) that results from wave breaking inside the surf zone is estimated as [Battjes, 1975]:

\[
\nu_t = h_r \left( \frac{D_r}{\rho} \right)^{\frac{1}{2}}, \tag{4}
\]

where \( D_r \) is the wave roller dissipation [Nairn et al., 1990; Stive and de Vriend, 1994; Reniers and Battjes, 1997] and \( h_r \) is the total water depth \((h + \eta)\).

Wave propagation and evolution are simulated on three nested SWAN domains, with the largest extending seaward of the continental shelf break. The two largest domains are refined versions of those described previously [Gorrell et al., 2011], with the smallest domain the same as the combined surf zone hydrodynamic domains (Figure 3b). The largest wave domain is forced uniformly along the open boundaries with the 2-dimensional spectra derived from the offshore buoy observations. Winds were assumed to be spatially uniform, and white-capping dissipation was included [van der Westhuysen et al., 2007]. Although nonlinear triad interactions are not included, quadruplet interactions are. Within each domain, the spectral wave action balance [Booij et al., 1999] is solved using 88 directional bins \((\approx 3.5^\circ/\text{bin})\) and 37 frequency bins logarithmically.
direction, and frequency interpolated to the 8.5 min averaging times of the observations) to ensure
ulated in the same manner as the field-based estimates (using the modeled water level and wave height,
the model outputs the computed momentum terms at each grid cell, the modeled forcing terms are calcu-
approximate depth of the “1.0 m depth” sensors [Haselmann et al., 1973]. Stationary SWAN simulations
were averaged, producing an estimate at the “1.0 m depth” sensor. Curves of setup versus depth were gen-
dient between the 1.0 m depth sensor and the shoreline, where \( S_{xy} \) was estimated as the gradient between the 2.5 and 1.0 m depth sensors, and the gra-
reaching the coast from the SSW (Figure 2). Results also are analyzed for 10 October 2003 (the
bimodal spectra or in areas of strong refraction (e.g., landward of the canyon head), this assumption is not optimal [Feddersen, 2004],
and likely contributes to model errors. However, the conclusions are insensitive to the method used to pro-
the resulting peak in alongshore flow is shoreward of the wave breakpoint, consistent with laboratory and field observations [Reniers and Battjes, 1997]. A second
advantage of the roller module is the inclusion of enhanced eddy viscosity owing to wave breaking (equa-
tion (4)). The primary disadvantage of the roller module is that only a single frequency and direction are
used to compute the radiation-stress gradients. For relatively narrow-banded frequency and directional
spectra the use of a single frequency and direction is sufficient. In contrast, for bimodal spectra or in areas
of strong refraction (e.g., landward of the canyon head), this assumption is not optimal [Feddersen, 2004],
and likely contributes to model errors. However, the conclusions are insensitive to the method used to pro-
vide the radiation-stress gradients
Instantaneous output from the flow model, including water level (tide plus setup), depth-averaged velocity,
and all terms of the cross shore and alongshore momentum equations at every grid cell, were saved every 10 min. For comparison with the observations, output from the grid cells closest to the instrument sites was extracted, and all vector quantities were rotated into the cross shore and alongshore coordinate system defined during the experiment (based on compass bearings at each transect). Model output and data sampling methods differ (instantaneous versus time-averaged). However, saving additional model output to enable averaging resulted in overly large files for each week run. Furthermore, wave forcing within the flow model is derived from stationary solutions to the phase-averaged action balance equation [Booij et al., 1999]. Model-data agreement was better when the model output and observations were averaged over 1 h periods than when the model output was interpolated to the 8.5 min times of the observational estimates. The model skill over the 1.5 month experiment is discussed in Appendix A.

4. Simulated Alongshore Momentum Balance
Analysis of the alongshore momentum balance is focused on 27 October 2003 (the case study described by Aputos et al. [2008]) when near-temporally constant narrow-banded, low-frequency (18 s), ~0.75 m high waves approached the coast from the SSW (Figure 2). Results also are analyzed for 10 October 2003 (the case study examined by Long and Ozkan-Haller [2005]) when a large rip current was simulated and observed visually at \( y = 1600 \) m. The alongshore variability of wave heights (Figures 4a–4c) and flows (Figure 4d) predicted on 27 October (Figure 4) and October 10 (not shown) are consistent with the observations (Figures 4c and 4d).

4.1. Model Validation With the Field-Estimated Momentum Balance
The model skill at reproducing the alongshore forcing is evaluated by comparing the pressure and radiation-stress gradients estimated from the model with those estimated from observations. The cross-shore gradient of \( S_{xy} \) was estimated as the gradient between the 2.5 and 1.0 m depth sensors, and the gradient between the 1.0 m depth sensor and the shoreline, where \( S_{xy} \) was assumed zero. These two gradients were averaged, producing an estimate at the “1.0 m depth” sensor. Curves of setup versus depth were generated for 24 h at each transect so that setup could be estimated along the bathymetric contour at the approximate depth of the “1.0 m depth” sensors [Aputos et al., 2008]. Alongshore pressure gradients were estimated as the gradient between the adjacent upcoast and downcoast sites (central difference). Although the model outputs the computed momentum terms at each grid cell, the modeled forcing terms are calculated in the same manner as the field-based estimates (using the modeled water level and wave height, direction, and frequency interpolated to the 8.5 min averaging times of the observations) to ensure
comparisons of like quantities (e.g., pressure gradients were estimated using the water level difference between the sensor locations).

The modeled flows and forcing are overlaid on the field estimates (as presented in Figure 7 of Apotsos et al. [2008]) to examine the momentum balance (Figure 5, compare red with green symbols, and blue with orange symbols). Far from the canyon ($y < 2300$ m), the observed and predicted radiation-stress gradients were northerly directed (waves were from the south, positive gradient) and were larger than the opposing alongshore pressure gradients (Figure 5, compare red and green circles with squares), resulting in northerly directed alongshore flows (Figure 5, $y = 2300$ m velocities are positive). Closer to the canyon head ($y = 1450$ m), refraction of the incident waves over the canyon resulted in large alongshore wave height gradients and correspondingly large setup and pressure gradients toward the south (negative forcing in Figure 5) that were larger than the opposing radiation-stress gradients (Figure 5, compare blue and orange pluses with crosses), resulting in southerly directed alongshore flows (Figures 4d and 5, $y = 1450$ m velocities are negative). Near the canyon the model underestimates the observed alongshore pressure gradient by 10–20% (Figure 5, compare blue with orange crosses). Far from the canyon, the model reproduces the magnitude of the forcing, but underestimates the velocity (Figure 5, compare red with green symbols). Despite the errors between the observed and modeled alongshore forcing and velocity (see Appendix A), the model reproduces the relative importance of these terms at these locations (Figure 5), with pressure-gradient-dominated southerly flow near the canyon and radiation-stress-gradient-dominated northerly flow far from the canyon, suggesting the model can be used to examine the momentum balance further.

### 4.2. Modeled Momentum Balances

The model is used to investigate the alongshore force balance, retaining the terms previously assumed small, for the entire field site including the region with the largest alongshore variability ($1450 < y < 1900$ m) where instruments could not be deployed because of its popularity as a surfing location (spatial gap in the sensor array, Figure 1). The momentum terms from the 10 min instantaneous output were cross-shore integrated [Hansen et al., 2013] from the shoreline to 6 m still water depth, then time averaged over 24 h on 27 October (gray region in Figure 2). The shoreline is defined as 0.25 m depth because the model does not resolve the physical processes in shallower depths owing to the grid resolution. The
The offshore extent of the surf zone was chosen as 6 m depth, where the surf zone forcing decays to near zero. Results and conclusions do not change if the offshore integration depth is increased, but do change if the depth is decreased below ~5 m where surf zone flows are present. Although the computational domain is approximately shoreline following, all momentum terms were rotated into the local cross-shore and alongshore coordinate system defined by the orientation of the 0.25 m depth contour. Results are not qualitatively dependent on the cross-shore integration, with the force balances along individual bathymetric contours similar to each other. However, the cross-shore integration reduces the importance of the horizontal mixing terms in (1), while providing a more representative estimate of surf zone forcing compared with estimates on individual depth contours. Wind stresses and \( \frac{\partial S_{xy}}{\partial y} \) are small (and neglected in the discussion below). Thus, the dominant terms in the time-averaged cross-shore integrated momentum balance are the total acceleration, the alongshore-pressure gradient, the diagonal radiation-stress gradient, and the bottom stress.

Similar to previous results [Apostos et al., 2008], the modeled time-averaged cross-shore integrated alongshore force balance for 27 October (Figures 6b–6d) is spatially variable, with flows diverging at \( y \sim 1600 \) m (Figures 4d and 6a), consistent with the observations (Figure 4d). The model acceleration term \( \frac{DV}{Dt} \), while small relative to the magnitudes of the pressure and radiation-stress gradients, is of the same magnitude as both the bottom stress (Figure 6b) and the sum of forcing by pressure and radiation-stress gradients (Figure 6d). Northward radiation-stress gradients dominate the forcing north of the canyon (1600 \( \leq y \leq 2000 \)) and southward alongshore-pressure gradients dominate for 1400 \( \leq y \leq 1600 \) (Figure 6d). The strongest northward flows are at about 1800 \( \leq y \leq 2200 \) (consistent with the observations, Figure 4d), which is north of the maximum total forcing \( y = 1875 \) m in Figure 6d. The model suggests this offset is owing primarily to the advective terms because the difference between the forcing and bottom stress is accounted for by acceleration (Figure 7a, compare black (forcing) and blue (bottom stress) curves with near perfect agreement between black and red-dashed (sum of bottom stress and acceleration) curves). For example, at \( y \sim 2050 \) m the flows remain about 0.5 m/s although the net forcing is decreasing (Figures 6a and 6d), consistent with a contribution of \( \frac{DV}{Dt} \) to the force balance (Figure 7a). The strongest modeled southward flows are at \( y \sim 1550 \) m (Figure 6a), where the net southward forcing is largest (Figure 6d). The contribution to \( \frac{DV}{Dt} \) of the local acceleration is small, whereas the contributions of the two advective terms are spatially variable, but of similar overall magnitudes (Figure 7b).

Field-based results for the 50 1 h periods that met the criteria of (1) having large wave height gradients near the canyon head and (2) the sensors along the 1.0 m isobaths were in the surf zone [from Apostos et al., 2008] suggest that the sum of the radiation-stress and pressure forcing is balanced by the quadratic bottom stress, estimated as \( p \lVert \mathbf{U} \rVert^2 \) (computed using the ADV measured velocities) multiplied by a drag.
coefficient estimated as the least-squares slope between the forcing and bottom stress (with $R^2 = 0.71$ and 0.75 near and far from the canyon, respectively). In contrast, the numerical model predicts that the bottom stress (see equations (2) and (3)) and total acceleration, which is dominated by the nonlinear advection ($\nabla u \cdot \nabla v$, Figure 7b), are of similar magnitude (Figures 6b and 7a). The near perfect model agreement between the sum of the radiation-stress and pressure forcing terms and the sum of the total acceleration and bottom stress (Figure 7a, compare the black with the dashed red curve) indicates that the horizontal mixing and wind stress terms are negligible in the time-averaged cross-shore-integrated momentum balance.

The predicted force balance, and in particular the importance of the nonlinear advective terms, is similar for a range of wave conditions. For example, although the forcing terms were larger for the 1 m high, near-normally incident waves on 10 October (Figure 2, and the case study described in Long and Özkan-Haller [2005]), the simulated alongshore flows were similar in magnitude to those on 27 October, owing to a partial balance between the northward radiation-stress forcing and the southward pressure-gradient forcing (not shown). Similar to 27 October, the model suggests that both nonlinear advection and bottom stress are necessary to balance the net forcing. However, the locations of the flow convergences and divergences are dependent on the offshore wave conditions. For example, in contrast to 27 October, the simulated and observed 10 October alongshore flows converged north of the canyon head at $y \sim 1600$ m, and the model predicts offshore directed flows extending beyond the 10 m depth contour, similar to a rip-current. A strong rip current at roughly this location has been predicted previously [Long and Özkan-Haller, 2005] and is evident in images from 10 October (not shown).

5. Discussion

In the model, the sum of the pressure and radiation stress gradients (the forcing) is almost perfectly balanced (Figure 7a) by the sum of the bottom stress and total acceleration (which is dominated by nonlinear advection, Figure 7b). When nonlinear advection is neglected, the modeled forcing is correlated with a mean-current-induced bottom stress normalized by a drag coefficient (0.002) based on the Chezy
roughness (equation (2)), but there is significant scatter (Figure 8a). The mean current-induced bottom stress roughly corresponds to the field-estimated stress $q_v U$, because the effects on the stress of the near normally incident waves (orthogonal to the mean current) are small [Soulsby et al., 1993]. North of the canyon at $y = 2300$ m, where the modeled $Dv/Dt$ is small (Figure 7a), the least squares slope of the balance is 0.0026, similar to the model drag coefficient of 0.0020 and there is limited scatter ($R^2 = 0.73$, black circles in Figure 8a). Adjacent to the canyon at $y = 1450$ m, where the model predicts large $Dv/Dt$, there is considerably more scatter ($R^2 = 0.26$, red squares in Figure 8a) and the least squares slope, 0.0035, is about 50% larger than the model drag coefficient. Including $Dv/Dt$ in the balance removes nearly all scatter ($R^2 = 0.98$ and 0.89 for $y = 2300$ and 1450 m, respectively) and the drag coefficient (slope) is within 20% of that set in the model (Figure 8b). As suggested previously [Apotsos et al., 2008], these results indicate that neglecting nonlinear advection could be the cause of the scatter in field-based force balances. For example, the mean residuals from the least squares fits to the observations [Apotsos et al., 2008] were $-0.54$ and 0.35 N/m$^2$ for $y = 1450$ and 2300 m, respectively, similar to the model predicted $Dv/Dt$ along the 1 m contour (approximate depth of field-estimated forcing). In addition, the quadratic dependence of both the bottom stress and advective terms suggests nonlinear advection could be accounted for in the field momentum balances by altering (increasing or decreasing, depending on the signs of the terms) the estimated bottom stress. Thus, neglecting the advective terms may have biased the field-estimated drag coefficients (0.0024 and 0.0025 at $y = 2300$ and 1450 m, respectively) [Apotsos et al., 2008].

Similar to previous results [Long and Ozkan-Haller, 2005], these new numerical results indicate that the acceleration term, principally nonlinear advection, can be as important as the sum of forcing (pressure and radiation-stress gradients) and as the bottom stress. Although the inclusion of nonlinear advection in the momentum balance does not change the underlying forcing (the direction of alongshore currents primarily is controlled by the sum of the pressure and radiation-stress gradients), the simulations suggest the
nonlinear advective terms are important to the spatial lags between the peaks in forcing and alongshore currents or bottom stresses, and alter the locations of flow convergences and divergences (Figures 6 and 7). Thus, nonlinear advection may be important to surf zone flow convergences (and rip locations) and morphologic change [Wu and Liu, 1984; de Vriend, 1987; Long and Özkan-Haller, 2005].

A scaling parameter, referred to as the shallow water Reynolds number \( R_w \) that approximates the alongshore length scales over which nonlinear advection likely is important is computed as [Wilson et al., 2013]:

\[
R_w = \frac{h_0 v_0 k}{\mu},
\]

where \( v_0 \) represents the background, alongshore-independent, velocity in water depth \( h_0 \), \( k = \frac{2\pi}{L} \) with \( L \) an alongshore length scale, and \( \mu \) is a linear friction coefficient set to 0.002 m/s (following that used by Wilson et al. [2013]). The Reynolds number \( R_w \) is computed from the 24 h averaged numerical results on the 27 October along the 1 m still water depth contour \( (h_0 = 1 \text{ m}) \) over length scales ranging from 50 to 500 m, with \( v_0 \) given by the mean alongshore velocity magnitude over the respective alongshore length scale (Figure 9). Large \( R_w \) \(( \approx 10 \text{ for } 0.5 \text{ m/s flows})\) occurs at short length scales where nonlinear advection acts to smooth alongshore variability in the flow [Wilson et al., 2013]. Conversely, small \( R_w \) \(( \approx 4 \text{ for } 0.5 \text{ m/s flows})\) corresponds to long length scales for which the nonlinear advective terms can be neglected. Thus, although advection may be large locally, it may not be evident in field momentum balances [Feddersen and Guza, 2003; Apotsos et al., 2008] owing to the large alongshore distances (i.e., small \( R_w \)) over which gradients were computed [Wilson et al., 2013].

The numerical simulations indicate that the length scales for which \( R_w \) is small are as
short as 50 m at the southern edge of the domain \( y < 1100 \) m, but are greater than 400 m at \( y = 2000 \) m, corresponding to the strongest flows (Figure 9). These distances are mostly less than those used to estimate the field momentum balances. However, the numerical momentum balances presented here suggest the nonlinear advective terms are potentially important, but that their contribution is included in the parameterized drag coefficient and in the scatter (residuals) in the field-based momentum balances (Figure 8). Here, the length scales over which nonlinear advection is important appear to be determined primarily by the canyon-induced alongshore variability in the forcing (Figure 4), rather than by weakly varying inner surf zone bathymetry considered previously [Wilson et al., 2013]. For example, between \( y = 1250 \) and 2000 m there are large alongshore gradients in the sum of the forcing (Figure 7) corresponding to the large gradients in the wave field (Figures 4b and 4c). In this region, \( \frac{Dv}{Dt} \) is large and fluctuates with the sum of the forcing at length scales ranging from \( \sim 100 \) to \( \sim 250 \) m.

6. Conclusions

Numerical simulations with a depth-averaged model (SWAN and Delft3D) reproduce the forcing and flows observed onshore of complex inner shelf bathymetry that includes a submarine canyon. Modeled momentum balances indicate the primary forcing, consisting of the sum of the pressure and radiation-stress gradients, is balanced by the sum of bottom stress and nonlinear advection. The simulations suggest that much of the scatter (residuals) in a field-based balance between the total forcing and the bottom stress [Apotsos et al., 2008] may be owing to the neglected nonlinear advective terms. In addition, neglecting nonlinear advection can lead to incorrect estimation of the drag coefficient by attributing advective effects to bottom stress. Although advection does not affect the direction of alongshore currents, it can affect the spatial patterns of the flow field.
Appendix A: Model Performance

Model-data comparisons were made for ~30 days using hourly averages calculated from six instantaneous model outputs saved every 10 min and from six 8.5 min means of the observations (~51 min). The root mean square error normalized by the variance of the observations (NRMSE), bias, and squared correlation coefficient ($R^2$) between the hour-averaged modeled and observed water depth, wave height, mean direction, radiation stress, and Eulerian alongshore flows were calculated at each of the 26 instrument sites (Figure A1). Note that the water depth and flow are output from the hydrodynamic model, the wave height is computed using the roller module (which includes short-wave dissipation and the wave roller energy balance) within the hydrodynamic model, the mean direction is calculated from the SWAN output used to drive the roller module, and the bulk radiation stress is calculated from the water depth, the mean squared wave height, mean direction, and peak period (output by SWAN). In addition, the Willmott Skill [Willmott, 1981] and Murphy Skill [Murphy, 1988] metrics, which were developed for comparing time series of model output with observations, were evaluated. Willmott Skill (WS) is computed as:

$$WS = 1 - \frac{\sum_{i=1}^{N} |X_{mod} - X_{obs}|^2}{\sum_{i=1}^{N} (|X_{mod} - \bar{X}_{obs}| + |X_{obs} - \bar{X}_{obs}|)^2},$$

(A1)

where $X_{mod}$ and $X_{obs}$ are the modeled and observed variables of interest, respectively, and the overbars represent time averaging over the length of the time series with $N$ samples (the mean $N$ for all sites was 708 spanning about 30 days total including some data gaps). A skill of one indicates perfect agreement, whereas zero indicates “complete disagreement” [Willmott, 1981]. The Murphy Skill (MS) is:

Figure A1. Model predictions versus observations of hourly averaged root mean square wave heights (a and d), mean wave directions (b and e), and alongshore velocities (c and f) at the 2.5 (red triangles) and 1.0 m (black squares) depth sensors north of $y = 1700$ (Figures A1a–A1c) and south of $y = 1500$ m (Figures A1d–A1f). Units are given in the plot titles.
Table A1. Variance Normalized Root Mean Square Error (NRMSE), Wilmott Skill (WS), Murphy Skill (MS), Bias, and Squared Correlation Coefficient ($R^2$) Between the Hourly Averaged Modeled and Observed Water Depth and Eulerian Alongshore Velocity for Approximately 30 days (Mean $N=708$, Depends on Availability of Instrument Data) at the 1.0, 2.5, and 5.0 m Depth Sites at Each Transect (Identified by Alongshore y Distance)

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Table A2. Variance Normalized Root Mean Square Error (NRMSE), Wilmott Skill (WS), Murphy Skill (MS), Bias, and Squared Correlation Coefficient ($R^2$) Between the Hourly Averaged Modeled and Observed $H_m$, Mean Wave Direction, and $S_{xy}$ for Approximately 30 days (Mean $N=708$, Depends on Availability of Instrument Data) at the 1.0, 2.5, and 5.0 m Depth Sites at Each Transect (Identified by Alongshore y Distance)

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1900
An $MS$ of one indicates perfect agreement, a zero indicates the model predictive ability is equivalent to using a mean of the observations, while an $MS$ less than zero indicates the predictive ability is worse than using a mean of the observations. Further, (equation A2) can be shown to be equivalent to (Murphy, 1988; Ralston et al., 2010):

$$MS = 1 - \frac{\sum_{i=1}^{N} (X_{mod} - X_{obs})^2}{\sum_{i=1}^{N} (X_{obs} - \bar{X}_{obs})^2}.$$  

(A2)

where $\sigma$ indicates the standard deviation of the modeled ($mod$) or observed ($obs$) variable. In (equation A3) the first term is the squared correlation coefficient. The second term quantifies the ability of the model to reproduce the variance in the observations, and becomes zero if the least squares slope is one. The third term represents the disagreement of the model and observational means (bias) and corresponds to the linear regression intercept. Thus, the $MS$ provides insight into the model performance. Although the WS commonly is used [Li et al., 2005; Warner et al., 2005; Shi et al., 2011; Elias and Hansen, 2013], it can produce skill scores of ~0.4 from a random uncorrelated time series [Ralston et al., 2010]. In the comparison that follows skill scores are termed "moderate" for $0.5 < WS < 0.75$ or $0.0 < MS < 0.5$, with "high" and "poor" skills for higher and lower scores.

Water depths, which are related primarily to tidal fluctuations, are reproduced well by the model with high skills (both WS and MS) and no consistent bias between sites (Table A1). The model simulates the wave heights with moderate to high skills at all sites (Table A2 and Figures A1a and A1d). The largest errors in wave direction and $S_{xy}$ occur at $y = 1450$ and 1900 m, where alongshore variations in wave energy and incident directions are largest [Garrell et al., 2011]. Although WS scores for directions and $S_{xy}$ typically are "moderate", MS scores usually are "poor." Despite the sometimes-poor model skill, root mean square errors (not shown) and model bias for wave direction and MS bias usually are less than 5° and 50 N/m.

Alongshore flows in 1.0 and 2.5 m depth are reproduced well by the model for $1900 < y < 2450$ m with moderate to high skills (Table A1 and Figure A1c). Model skill is lower at the 5 m depth sites than at the shallower sites (Table A1) owing to a strong $M_2$ internal tide (which cannot be reproduced by the depth-averaged model) that resulted in observed flows as high as 0.30 m/s, particularly during spring tides [Lentz et al., 2004]. Harmonic analysis using $T_{Tide}$ [Pawlowicz et al., 2002] of the observed flows at the 5 m depth sites attributes less than 0.1 m/s of the observed ~12 h variable alongshore velocities to the barotropic tidal band energy. This flow magnitude is consistent with the modeled flows.

The model tends to overestimate the alongshore flows at the sites immediately north of the canyon (Table A1 and Figure A1f), biases ~0.05–0.10 m/s for $1100 < y < 1500$ m), where the currents usually are weaker and more spatially variable than those far from the canyon ($y > 1500$ m). In addition, the model-data correlations are low near the canyon (Table A1, $y < 1500$ m). However, model alongshore flow root-mean-square errors are of similar magnitude at sites near ($y < 1500$ m) and far ($y > 1700$ m) from the canyon, but the weaker flows result in decreased model skill (compare Figures A1c with A1f, and Table A1).

The hourly averaged model results were most sensitive to the breaker coefficient ($\gamma$) and the background eddy viscosity ($\nu_b$). The cross-shore wave height distribution was best reproduced with a $\gamma = 0.45$. However, the simulated wave heights were affected only slightly for $0.35 < \gamma < 0.50$, or for a $\gamma$ that depends on depth and wave number [Ruessink et al., 2003]. The flow patterns were not sensitive to $\gamma$ over this range, nor to the use of other dissipation formulations [Battjes and Janssen, 1978]. The spatial variability in the flows was best reproduced using a background eddy viscosity $\nu_b = 0.5$ m$^2$/s. However, the results were only weakly affected for $0.01 < \nu_b < 1.00$ m$^2$/s. Furthermore, cross-shore integrating the model results (as in Figures 6–8) reduces the sensitivity to horizontal mixing, and thus to the eddy viscosity.

References