Scaling Turbulent Dissipation in the Transition Layer

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ABSTRACT

Data from three midlatitude, month-long surveys are examined for evidence of enhanced vertical mixing associated with the transition layer (TL), here defined as the strongly stratified layer that exists between the well mixed layer and the thermocline below. In each survey, microstructure estimates of turbulent dissipation were collected concurrently with fine-structure stratification and shear. Survey-wide averages are formed in a “TL coordinate” $z_{TL}$, which is referenced around the depth of maximum stratification for each profile. Averaged profiles show characteristic TL structures such as peaks in stratification $N^2$ and shear variance $S^2$, which fall off steeply above $z_{TL} = 0$ and more gradually below. Turbulent dissipation rates $\varepsilon$ are 5–10 times larger than those found in the upper thermocline (TC). The gradient Richardson number $R_i = N^2/S^2$ becomes unstable ($R_i < 0.25$) within ~10 m of the TL upper boundary, suggesting that shear instability is active in the TL for $z_{TL} > 0$. $R_i$ is stable for $z_{TL} \leq 0$. Turbulent dissipation is found to scale exponentially with depth for $z_{TL} > 0$, but the decay scales are different for the TL and upper TC: $\varepsilon$ scales well with either $N^2$ or $S^2$. Owing to the strong correlation between $S^2$ and $N^2$, existing TC scalings of the form $\varepsilon \sim |S^2|N^2$ overpredict variations in $\varepsilon$. The scale dependence of shear variance is not found to significantly affect the scalings of $\varepsilon$ versus $N^2$ and $S^2$ for $z_{TL} \leq 0$. However, the onset of unstable $R_i$ at the top of the TL is sensitively dependent to the resolution of the shears.

1. Introduction

The uppermost layer of the ocean is typically associated with energetic motions and near-complete vertical mixing by turbulent processes and experiences constant fluxes of heat, gas, and momentum owing to interactions with the atmosphere. By contrast, the stably stratified thermocline below is shaped by weak vertical mixing processes, which slowly redistribute tracers and momentum within the oceanic interior. Between the actively mixing layer above and the thermocline below, there is often observed a region that is neither strongly mixing nor entirely quiescent. This so-called transition layer (TL) is associated with a strong concentration in vertical shear and stratification, although the respective maxima may be slightly offset in depth (Johnston and Rudnick 2009; Rahter 2010). Shear instability is a distinct possibility but is not guaranteed. Other dynamical processes that influence the TL include mixed layer processes, such as entrainment by turbulent eddies or penetrative convection, which “leak” through the mixed layer base. Over time scales from weeks to months, mesoscale processes also contribute to the TL by heating the upper part of the main thermocline into the actively mixing layer (Ferrari et al. 2008; Danabasoglu and Ferrari 2008).

Unlike the mixed layer, the stratified TL supports both interfacial and internal waves, which modify the local stratification and shear and also allow the exchange of wave energy and shear between the TL and the upper thermocline. Predominantly near-inertial waves...
propagating downward from the TL can contribute to thermocline mixing (Moum et al. 1989; D’Asaro et al. 1995; Alford and Gregg 2001; Alford 2003; Dohan and Davis 2011), and conversely, internal waves propagating upward from the thermocline can also increase mixing in the TL and deepen the mixed layer (Dohan and Davis 2011; Jochum et al. 2013).

Small-scale turbulent and wave processes are not resolved in general circulation models (GCMs). Instead, diapycnal mixing associated with these processes must be parameterized through a turbulent diffusivity that includes the effects of surface-forced boundary layer mixing, shear instability, and interior mixing due to breaking internal waves, among other processes. Shear instability at the base of the mixed layer is typically represented by switching on extra mixing when the local gradient Richardson number $R_i = N^2/S^2$ drops below a critical value $R_i^c$, where $N^2$ is the local buoyancy frequency, and $S^2$ is the shear variance (Price et al. 1986; Large et al. 1994; Kantha and Clayson 1994).

In the present study, we ask: do measured profiles of turbulent dissipation $\varepsilon$ support a picture of enhanced mixing in the TL? If so, is the dissipation in the TL consistent with local shear instability and associated with a low gradient Richardson number? Or do elevated values of $\varepsilon$ extend for some distance into the upper thermocline and more closely follow internal wave scalings for turbulent dissipation?

To answer these questions, we examine three month-long datasets, each containing approximately 75–150 profiles of turbulent dissipation. The data were collected in midocean Atlantic conditions, at latitudes between 10° and 30°, using the High Resolution Profiler (HRP) (Schmitt et al. 1988), which measured concurrent fine- and microstructure velocity, temperature, and conductivity.

We focus on features of the TL that are persistent, rather than varying in space and time, within each dataset. To strengthen this focus, our analysis takes place in a coordinate system referenced to the location of the transition layer, instead of the surface boundary. The main goals of the analysis are 1) to document the vertical structures of shear, stratification, and turbulent mixing and 2) to determine whether simple scalings relevant to shear instability and/or internal wave-driven mixing can adequately predict turbulent dissipation rates in the TL and into the upper thermocline. While these are limited goals, they are intended to provide guidance for future efforts to improve upper-ocean vertical mixing in coarse-resolution models.

The remaining sections of this paper address the following: 2) data and methods, 3) observations, 4) scalings of turbulent dissipation, and 5) discussion and conclusions.

2. Data and methods

a. Sampling locations and times

The sampling locations for the data considered in this study are shown on a map of the central Atlantic Ocean in Fig. 1. The first dataset was obtained in March–April 1992, west of the Canary Islands, as part of the North Atlantic Tracer Release Experiment (NATRE) (Ledwell et al. 1993; Toole et al. 1994; Polzin et al. 1995). The survey area was chosen as a relative minimum of wind forcing and eddy activity, so as to give a representative level of background thermocline mixing, and geostrophic velocities during the survey were estimated at 5–15 cm s$^{-1}$ (Polzin et al. 1995). Sampling consisted of a wide-area survey combined with sampling within a tight box. Although profiles were not taken at precisely
regular intervals, sampling was well distributed across all times of day.

The two remaining datasets were collected during successive years of the Brazil Basin Tracer Release Experiment (BBTRE) (Polzin et al. 1997; Toole et al. 1997; St. Laurent et al. 2001). The January–February 1996 survey (BBTRE96) spanned the breadth of the Brazil Basin, with about half of the samples concentrated above rough topography to the east, near the Mid-Atlantic Ridge (MAR) (Polzin et al. 1997). During March–April 1997 (BBTRE97), sampling was concentrated near the MAR because enhanced abyssal mixing rates were found there during the previous year. Notably, the two BBTRE surveys took place in different seasons (austral summer and fall, respectively).

b. The High Resolution Profiler

All data considered in this study were obtained using the HRP (Schmitt et al. 1988), a free-fall instrument carrying both fine-structure and microstructure sensors and capable of operating to full ocean depth (6000 dbar). An acoustic velocimeter measured relative velocities, which were referenced to absolute position using on-board accelerometers and a magnetometer. A Neil Brown Mark III CTD equipped with a custom fast-response temperature probe provided temperature and salinity specified to a 1-m resolution at typical fall rates of about 0.6 m s\(^{-1}\). The fine-structure data were sampled at 10 Hz and averaged into 0.5-dbar vertical bins.

Microstructure sensors included a Sea-Bird Electronics assembly with an FP07 thermistor and matching microconductivity sensor. Airfoil probes (Osborn 1974) provided measurements of velocity microstructure. Shear variances were computed by forming spectral estimates over the same 0.5-dbar depth bins as the finescale measurements, correcting for the electronic and sensor response functions, and integrating out in frequency (until noise began to dominate the variance). Under the assumption

![Fig. 2. NATRE depth–time profiles of \(N^2\), \(S^2\), and \(\epsilon\). Each profile is plotted with a time–width corresponding to the duration of a HRP cast. During days 88–106, profiles were collected at roughly regular intervals of 4 h; sampling during the last 5 days was repeated every 2 h. (top) The \(N^2\) peaks sharply between 150 and 200 m, in a band that we identify as the TL. (middle) The \(S^2\) is largest at the surface and decays with depth, before reappearing in the TL. (bottom) The \(\epsilon\) bursts in the ML, associated with diurnal convection events, often reach to the TL.](image-url)
of isotropy (Yamazaki and Osborn 1990) and using the averaged vertical shear variances for two horizontal velocity components $\langle u_z \rangle^2$ and $\langle v_z \rangle^2$, turbulent dissipation rates were estimated as

$$\nu = \frac{15}{2} \left( \frac{\langle u_z \rangle^2 + \langle v_z \rangle^2}{2} \right),$$

where $\nu$ is the kinematic viscosity. Further details of the instrument and data processing are given by Schmitt et al. (1988), Polzin et al. (1995), and Polzin and Montgomery (1996).

Fine-structure shears at the nominal resolution of the velocity measurements are estimated by first differencing of the 0.5-dbar binned velocities across a separation of $\Delta z = 1$ m. To allow an assessment of the scale dependence of $S^2 = \langle (u_z)^2 \rangle + \langle (v_z)^2 \rangle$ and $Ri$, and also to facilitate comparison with previous works on turbulent dissipation scaling in the thermocline (e.g., Gregg and Sanford 1988; Gregg 1989; Polzin et al. 1995), shears are also computed using a first difference across $\Delta z = 10$ m. The 10-m shear variances, denoted by a subscripted as $S_{10}^2$, have been multiplied by a factor of 2 to correct for the spectral transfer function of the differencing filter (Gregg and Sanford 1988 used the value 2.11).

Figures of the buoyancy frequency $N^2$ are derived from (finescale) salinity, temperature, and pressure using the adiabatic leveling method.

c. Transition layer-based vertical coordinate $z_{TL}$

Just as studies of near-surface and bottom-enhanced mixing have often used coordinate systems referenced to the relevant boundary, here we use a vertical coordinate that is referenced to the transition layer and denoted by $z_{TL}$. By shifting all data into $z_{TL}$ coordinates before averaging over time and space, we hope to minimize the blurring of sharp features due to changing TL depth.

We define the nominal center of the TL to be at the depth of maximum stratification $z_{N_{\text{max}}}$ and place the origin of our TL-based vertical coordinate, ($z_{TL} = 0$) at that depth. The sign convention is chosen such that $z_{TL}$ is
positive at the surface and negative below the TL. Buoyancy frequency $N^2$ is used instead of $S^2$ because profiles of stratification are more readily available than high-resolution velocity data. As will be discussed later, the main results of this study are not significantly affected by the choice.

It will also be convenient to define upper and lower boundaries for the TL. Johnston and Rudnick (2009) investigated several methods for defining a TL thickness from finescale $N^2$ and $S^2$ measurements and found that all produced similar results. We therefore take the upper TL limit as the depth where $N^2$ falls from its maximum value at $z_{TL} = 50$ to its depth-averaged value in the upper thermocline (taken over the range $2 < z_{TL} < 500$ m). The lower limit is set, somewhat arbitrarily, where the decreasing profile of $N^2$ in the TL first encounters either an abrupt change in slope or a local minimum.

Once the data have been transformed into “TL coordinates,” the quantities $N^2$, $S^2$, $S^2_10$, and $\varepsilon$ are depth-averaged into 10-m vertical bins and then averaged over all casts in each survey. Profiles of the gradient Richardson number $Ri = (N^2)/(S^2)$ are computed from the survey-averaged profiles, as indicated by the angle brackets. An $Ri_{10}$ is also computed using $(S^2_{10})$. The turbulent diffusivity is estimated using the Osborn (1980) relationship: $\kappa_\theta = \Gamma \varepsilon / N^2$, where for the mixing efficiency $\Gamma$ we use a nominal value of 0.2.

The 95% confidence intervals for each survey-averaged bin are estimated via a bootstrap method (Efron and Gong 1983), using 512 bootstrap replications. For $Ri$ and $\kappa_\theta$, which are each quotients of two averaged quantities, each replication is a quotient of averages formed from the same bootstrap sample of casts.

3. Observations

a. NATRE

Depth–time maps from NATRE of upper-ocean stratification ($N^2$), vertical shear variance ($S^2$), and turbulent kinetic energy dissipation rate ($\varepsilon$) are presented in Fig. 2. The vertical coordinate used here is true (measured) depth. In NATRE, a mixed layer (ML) of about 100–200-m thickness (blue values, top panel) is clearly visible. Completely unstratified patches and density
overturns ($N^2 \leq 0$) more than 2 m high, which occurred frequently, are here indicated by white patches (top panel). Separating the ML from the main thermocline (TC) is a high-$N^2$ (orange–red) band, which we associate with the TL.

The strongest shear variances (middle panel) are concentrated near the surface, tapering off by two orders of magnitude by about 100-m depth and leaving the bottom 50 m of the mixed layer relatively shear free. High values of $S^2$ reappear in the TL, with patchy shears found below in the TC. Turbulent dissipation (bottom panel) in the ML is concentrated in bursts, which were previously examined by Rahter (2010) and found to be associated with diurnal convection events. The most energetic events reach from the surface to the TL (e.g., on days 89–94). Within the TL, dissipation levels are moderately, but persistently, elevated. As with the shears, $\varepsilon$ is intermittent in the TC.

Figure 3 shows the same quantities after they have been shifted into the TL coordinate system, which emphasizes the vertical layering of the ML, TL, and TC. Above the TL, $S^2$ and $\varepsilon$ are highly variable on daily and multiday time scales. Below the TL, both quantities, along with $N^2$, are remarkably homogeneous, with little apparent dependence on the shear variability or bursts of turbulence above. This uniformity within the TL and below gives us some reassurance in our approach of taking survey-wide averages.

Survey-averaged profiles of 10-m binned stratification $N^2$, shear variance $S^2$, and turbulent dissipation $\varepsilon$ are presented in the first three panels of Fig. 4. For visual reference, bin averages from the TL ($10 \leq z_{TL} \leq 50$ m) are indicated in red, while TC bins ($z_{TL} \leq -30$ m) are marked in blue.

Stratification and shear variance both peak sharply in the TL and fall off steeply above $z_{TL} = 0$, about one order of magnitude by $z_{TL} = 20$ m, above which $S^2$ is increasing but $N^2$ settles to mixed layer values of less than $5 \times 10^{-6}$ s$^{-2}$. Shear variance $S_{10}^2$ has a profile similar to that of $S^2$, but the 1-m shear variances are about 50% larger in the TC and twice as large in the TL, which includes the peak at $z_{TL} = 0$. 

**FIG. 5.** As in Fig. 2, but for BBTRE96: (top) stratification is largest above 50 m, at the base of the mixed layer, but a second TL-like feature is also visible at 150–200-m depth; (middle) shear variance is enhanced at both stratification maxima, with a quiescent region in between those depths; (bottom) the turbulence dissipation rate is similarly enhanced.
Dissipation rates are generally decreasing through the ML and TC, with a “bump” of enhanced turbulence in the TL. Below the TL, $\varepsilon$ decreases more slowly, by at most a factor of 2 by $z_{\text{TL}} = -500$ m.

The 1-m gradient Richardson number $R_i$ is increasing with depth from the mixed layer into the TL. There is a transition from nominally unstable ($R_i < 1/4$) to stable ($R_i > 1/4$) values just above the TL, at about $z_{\text{TL}} = 20$ m, suggesting that shear instability is important in determining the upper TL limit. Here, the scale dependence of shear variance becomes significant, as $R_{i,10}$ does not indicate instability until $z_{\text{TL}} = 40$ m, or about 20 m closer to the surface than the 1-m gradient $R_i$. Throughout the TL and TC, $z_{\text{TL}} \approx 10$ m, $R_i$ (and $R_{i,10}$) is always stable on average, with a maximum value of about 0.6 (1.0) at $z_{\text{TL}} = 0$, and remaining larger than 0.5 (0.7) for $z_{\text{TL}} \approx -500$ m.

Vertical diffusivity $\kappa_p$ increases steadily with increasing $z_{\text{TL}}$ above the TL. By contrast, $\kappa_p$ in the TC is nearly constant at about $1 \times 10^{-5}$ m$^2$s$^{-1}$, with only a hint of the enhanced $\varepsilon$ in the depth range $0 \geq z_{\text{TL}} \geq -30$ m.

b. BBTRE96

Although the BBTRE96 survey covered nearly the entire width of the Brazil Basin (Fig. 1), the $N^2$ maximum (Fig. 5, top panel) is found at a relatively constant depth of less than 50 m. A second, more diffuse layer of elevated $N^2$ is also found between 150- and 200-m depth. Both high-$N^2$ layers are accompanied by elevated shears (middle panel). Since these profiles were taken near the beginning of austral summer, the lower $N^2$ maximum may be the remnant of a TL between the wintertime mixed layer and thermocline.

Compared to NATRE, the stratification and shear structures are more variable and less sharply defined in BBTRE96. There is some correspondence between $\varepsilon$ (bottom panel) and $S^2$ and $N^2$. Turbulent patches near the surface often reach to the upper $N^2$ maximum but not to the lower maximum, which appears to be relatively isolated.

Cruise-averaged profiles referenced to the upper $N^2$ maximum are shown in Fig. 6. Here, as in NATRE, the $N^2$ peak drops away sharply for $z_{\text{TL}} > 0$, to roughly thermocline levels within 20 m; however, $S^2$ is always increasing for $z_{\text{TL}} > 0$, to greater than $1 \times 10^{-3}$ s$^{-2}$ for $z_{\text{TL}} > 20$ m. Turbulent dissipation rates are largest for $10 \leq z_{\text{TL}} \leq 20$ m. All three quantities generally decrease with depth, although not monotonically, to the lower limit of the TL ($z_{\text{TL}} = -80$ m).

Associated with the increase in shear variance toward the surface is a Richardson number that is just stable
For \( z_{\text{TL}} = 10 \text{ m} \), \( \text{Ri} \) is stable, peaking strongly at \( z_{\text{TL}} = 0 \) before settling near 0.5 in the TC. The diffusivity \( \kappa_{\rho} \) decreases through the ML to a relative minimum of \( 2 \times 10^{-6} \text{ m}^2 \text{s}^{-1} \) in the TL, and increases slowly with depth for \( z_{\text{TL}} < 0 \). There is little indication of TL-enhanced diffusivity.

The secondary maxima in \( N^2 \) and \( S^2 \) appear near \( z_{\text{TL}} = -120 \text{ m} \). The \( \epsilon \) peak occurs at slightly greater depth, \( z_{\text{TL}} = -150 \text{ m} \), but is associated with a single high measurement in the bin average and is accompanied by relatively large error bars. A set of averages were also formed with \( z_{\text{TL}} \) referenced to the lower \( N^2 \) maximum. The resulting profiles (Fig. 7), show a structure that resembles the primary TL near the surface (Fig. 6), with sharp peaks in \( N^2, S^2, \) and \( \epsilon \). Here, as in the plots using the “upper TL” coordinate, the lower \( \epsilon \) peak appears about 30 m below the local \( N^2 \) peak. Unlike the upper TL and the TL in NATRE, \( \text{Ri} \) does not become unstable above the “lower TL.”

c. BBTRE97

Data from BBTRE97 are presented in Fig. 8. As in NATRE (Fig. 2), BBTRE97 is characterized by an unstratified ML, separated from the thermocline below by a sharp TL (top panel). Shear variances near the surface in BBTRE97 (middle panel) are smaller than those found in NATRE; instead, the largest shears are concentrated around the TL. As before, distinct turbulent dissipation “events” are observed near the surface, while patches of dissipation are seen in the thermocline; however, the TL itself is not as distinct, in terms of \( \kappa_{\rho} \), as in the previous two datasets.

Averaged profiles (Fig. 9) show TL boundaries identical to those in BBTRE96 (20 \( \geq z_{\text{TL}} \geq -80 \text{ m} \)), with a sharp cutoff above \( z_{\text{TL}} \) and a more gradual rolloff below. There is very little sign of a secondary \( N^2 \) and \( S^2 \) maximum near \( z = -120 \text{ m} \). If the lower TL seen in BBTRE96 was the remnant of a wintertime transition layer, then the corresponding structure in the following year has nearly disappeared by austral fall when the BBTRE97 profiles were collected. Here \( \epsilon \), which is generally decreasing with depth, has a dip at the bottom of the TL (\( z_{\text{TL}} = -80 \text{ m} \)). Depth bins with enhanced \( \epsilon \) are found between \(-40\) and \(-50\) m and around \(-120\) m, but with relatively wide error bars compared to surrounding bins.

4. Scalings of turbulent dissipation

In each of the datasets examined, \( \text{Ri} \) decreases below 0.25 within one vertical bin height (10 m) above the TL,
suggesting that the TL upper limit may be determined at least partly by shear instability. However, for all \( z_{\text{TL}} > 0 \), we find \( R_i > 0.25 \), (with \( R_i = 0.5 \) at the TL center). The shared stable regime motivates us to look for scalings of turbulent dissipation that smoothly connect the TL with the TC below.

a. NATRE scalings

A handful of trial scalings between the survey-averaged \( \varepsilon \) and fine-structure data from NATRE are presented in Fig. 10. We begin with simple scalings of \( \varepsilon \) with distance from the TL center, analogous to schemes proposed for abyssal tidal mixing, which have an exponential (St. Laurent et al. 2002) or rational (Polzin 2009) dependence upon a scale height \( \lambda \) above the bottom. Figure 10a re-plots \( \varepsilon \) versus \( z_{\text{TL}} \) for NATRE, with the TL indicated in red and the TC in blue as before. A single slope does not seem to fit the data in both the TL and TC. The slope of the log–linear least squares fit over TL points (0 \( \leq z_{\text{TL}} \leq 20 \) m, red line) is \( m = -2.37 \pm 1.13/100 \) m, corresponding to a scale height \( \xi \approx 20 \) m. The \( r^2 = 0.98 \) is high, but the error bars are wide because only a few points have been fitted. A fit over all data in 0 \( \leq z_{\text{TL}} \leq 500 \) m (gray line) gives a shallower slope, \( m = -0.07 \pm 0.03/100 \) m with \( r^2 = 0.38 \), but does not fit well the high \( \varepsilon \) values in the TL.

Figures 10b,c show log–log scalings of \( \varepsilon \) versus \( N^2 \) and \( S^2 \). In each plot, the fit is computed for all points 0 \( \leq z_{\text{TL}} \leq 500 \) m. The best-fit slope for \( \varepsilon \) versus \( N^2 \) is \( m = 0.98 \pm 0.31 \) with \( r^2 = 0.46 \). Shear variance \( S^2 \) has a somewhat larger slope, \( m = 1.40 \pm 0.36 \) with \( r^2 = 0.55 \). A fit for \( \varepsilon \) versus \( S^2 \) is also shown in Fig. 10c and has a slope similar to \( S^2 \) but a lower \( r^2 = 0.44 \).

A reason for the similarity between the fits \( \varepsilon \) versus \( N^2 \) and \( S^2 \) can be seen from Fig. 10d: \( S^2 \) and \( N^2 \) are highly correlated with \( r^2 = 0.95 \) and a best-fit slope of \( m = 0.75 \pm 0.05 \). Similarly \( S^2_{\text{TL}} \) scales with \( N^2 \), but with lower \( r^2 = 0.77 \). The relationship \( S^2 \sim N^{3/2} \) may also explain why \( R_i \) is a poor predictor for \( \varepsilon \) (\( r^2 = 0.13 \)), as shown in Fig. 10e.

It is possible to find a scaling \( \varepsilon \sim |S|^p|N|^q \) simply by letting \((p, q) = (1.40, 0.98)\), the slopes of the respective...
fits to $S^2$ and $N^2$. The resulting scaling, shown in Fig. 10f, has $r^2 = 0.52$, which falls between the $r^2$ values for the individual fits. Owing to the strong correlation between $S^2$ and $N^2$, however, $(p, q)$ are not well constrained.

b. BBTRE96 scalings

Figure 11 shows scalings from BBTRE96. As in NATRE, $\varepsilon$ versus $z_{TL}$ (Fig. 11a) is fitted in the TL only (red line) and for all points (gray line). The dip in $\varepsilon$ between the TL and upper TC is not captured by either fit. The fits with respect to $N^2$ and $S^2$ (Figs. 11b,c) have shallower slopes with $m = 0.76 \pm 0.13$ and $1.00 \pm 0.17$, but higher $r^2$ of 0.75 and 0.73, respectively, than the fits from NATRE. The fit with respect to $S^2$ has an even higher $r^2 = 0.80$. As in NATRE, $S^2$ scales nearly as $N^{3/2}$ (the actual exponent is $1.44 \pm 0.12$) with $r^2 = 0.93$ (Fig. 11d).

Figure 11e seems to show a positive correlation between $\varepsilon$ and $Ri$, with $r^2 = 0.45$. Intuitively, a lower (less stable) $Ri$ might be expected to correspond to higher $\varepsilon$, but here the relationship is the opposite, with higher $Ri$ corresponding to higher $\varepsilon$. The sign of the correlation can be explained by the fact that, for $z_{TL} \leq 0$, both $\varepsilon$ and $Ri$ take on their highest values near the top of the profile and tend to decrease with depth.

Figure 11f shows the scaling $\varepsilon \sim |S|^{1.00} |N|^{0.76}$ where, as before, the exponents are taken from the slopes of the fits to $S^2$ and $N^2$. Here $r^2 = 0.76$ is slightly higher than for the fits versus $S^2$ or $N^2$ taken separately.

A set of fits were also tried for the BBTRE96 profiles, which have been aligned around the “lower TL” (Fig. 7). We comment only briefly on the results, which are presented in Fig. 12. A single scaling versus distance (Fig. 12a) is more successful in the lower TL than in the upper TL (Fig. 11a). In general, the slopes for the lower TL fits (Figs. 12b–d) are similar to those from the upper, but the $r^2$ values tend to be somewhat lower, suggesting that a separate analysis of the lower TL is redundant.

c. BBTRE97 scalings

Scalings shown in Fig. 13 for BBTRE97 are similar to those found in NATRE and BBTRE96. As before, $\varepsilon$ versus $z_{TL}$ (Fig. 13a) is fitted separately for the TL (red), but the fit over all depths (gray line, scale height $\xi \approx 220$ m) is also reasonable.

The slopes for $\varepsilon$ versus $N^2$ and $S^2$ (Figs. 13b,c) are $m = 0.70 \pm 0.19$ and 0.89 $\pm 0.22$, which are indistinguishable within error limits from those found in BBTRE96, although with lower $r^2$ values of 0.54 and 0.59, respectively (and 0.55 for $S^2_{10}$). Part of the discrepancy is due to two
outliers in the TL (red, $\varepsilon \approx 10^{-8}$ W kg$^{-1}$), which have particularly large error limits.

Again $S^2$ and $N^2$ are well correlated (Fig. 13d) with $m = 0.80 \pm 0.03$ and $r^2 = 0.98$. The slope is slightly steeper than in BBTRE96, but the confidence intervals for the two datasets overlap. A positive correlation between $Ri$ and $\varepsilon$ is also present, as in BBTRE96, but here $r^2$ is about half as large (0.26 instead of 0.45). The last panel (Fig. 13f) shows the scaling $\varepsilon \sim |S|^{0.90} |N|^{0.70}$, $r^2 = 0.57$, where the exponents are indistinguishable (within confidence limits) from those used for BBTRE96. For easier comparison across the three surveys, the scaling results discussed in this section are summarized in Table 1.

5. Conclusions and discussion

Upper-ocean datasets from NATRE, BBTRE96, and BBTRE97 each exhibit clear transition layers—with elevated stratification, shear, and turbulent dissipation rates—between the mixed layer and the thermocline. The use of a “TL coordinate,” with $z_{TL} = 0$ at the depth of maximum stratification for each profile, allows us to form survey-averaged profiles that provide statistical stability while resolving TL features that vary over $O(10$ m) scales. Sharp peaks in both $N^2$ and $S^2$ are observed at $z_{TL} = 0$, falling off steeply above and more gradually below, and $\varepsilon$ attains values about 5–10 times higher in the TL than in the upper thermocline.

A main finding is that a transition from unstable gradient Richardson numbers (Ri $< 0.25$) in the mixed layer to stable Ri in the TL coincides ($+10$ m) with the upper TL limit as defined using $N^2$. If we adopt the view that the TL stratification peak is the result of vertical mixing into the stratified thermocline, then shear instability may be said to temper the $N^2$ maximum and set the TL upper boundary. Alternatively, the upper boundary of the TL could be viewed as the depth where...
the stabilizing effect of $N^2$, which is increasing, finally overcomes the destabilizing effect of $S^2$, which is decreasing. We simply conclude that, in the averaged sense, the TL upper boundary is the depth where the deepening effect of vertical mixing and the stabilizing effect of stratification are in balance.

The scale dependence of shear variance can be significant for assessing $Ri$. In NATRE, $Ri_{10} = 0.25$ about 20 m shallower than $Ri$; for BBTRE96 and BBTRE97, the depths where $Ri$ and $Ri_{10}$ become unstable are within 10 m of one another. Meanwhile, the choice of defining the TL using $N^2$ has only a small effect on $Ri$. If $z_{TL} = 0$ is instead defined at the $S^2$ maximum (see Fig. 3, middle panel), then the peak in averaged $S^2$ increases slightly while the $N^2$ peak decreases slightly, resulting in a net decrease in $Ri$ at $z_{TL} = 0$. The profiles of $N^2$, $S^2$, and $Ri$ are not changed significantly away from $z_{TL} = 0$.

While shear instability is likely important for $z_{TL} > 0$, $Ri$ is, on average, stable within the TL and below. However, elevated $\varepsilon$, $N^2$, and $S^2$ continue for tens of meters below $z_{TL} = 0$, extending up to 100 m from the shear-unstable region. The turbulent dissipation rate $\varepsilon$ does not exhibit a simple dependency on distance from the TL. A better fit can be obtained if two different length scales are assumed for the TL and TC.

Previous works have proposed a variety of scalings for turbulent dissipation in the TC, due primarily to breaking internal waves, as functions of stratification $N^2$ and the internal wave spectral level, $E \sim S^2$ (e.g., Gargett and Holloway 1984; Gregg 1987; Gregg and Sanford 1988; Gregg 1989; Polzin et al. 1995 and many others). Gregg and Sanford showed evidence of $\varepsilon \sim S^2$, while Gregg (1989) extended the scaling to $\varepsilon \sim S^2 N^2$, which was consistent with predictions for resonant energy transfers in a Garrett and Munk spectrum (e.g., GM81) (Munk 1981). Meanwhile, Gargett (1984) and Gargett (1990) argued that nonlinear energy transfers should scale closer to $\varepsilon \sim E N^{3/2}$ for GM-like wavefields and tend toward $\varepsilon \sim EN$ in more “monochromatic” wave fields. Polzin et al. (1995) showed evidence in favor of an $\varepsilon \sim E^2 N^2$ scaling with appropriate corrections for the frequency content of the wave spectrum.
Our results from NATRE, where we find $\epsilon \sim N^{1.96 \pm 0.60}$, are consistent with the $\epsilon \sim N^2$ scaling of Gregg and Sanford (1988). In BBTRE96 and BBTRE97, $\epsilon \sim N^{1.52 \pm 0.26}$ and $N^{1.40 \pm 0.38}$, respectively, suggesting a scaling closer to $\epsilon \sim N^{3/2}$. Our findings (Table 1) do not seem consistent with any of the other scalings.

We also found that $\epsilon$ scales as well or better with $S^2$, compared to $N^2$. In NATRE, BBTRE96, and BBTRE97, $\epsilon \sim S^p$ where $p = 2.80 \pm 0.72$, $2.00 \pm 0.34$, and $1.78 \pm 0.44$, respectively, with $r^2$ values similar to those for $N^2$. The similarity is associated with the strong correlation $S^2 \sim N^{3/2}$, $r^2 \geq 0.93$, and as a result, scalings of the form $\epsilon \sim |S|^p |N|^q$ tend to overpredict the variability in $\epsilon$. It is always possible to construct scalings, for example, $\epsilon \sim |S|^{1.40} |N|^{0.98}$ for NATRE, where the exponents have been taken from the slopes of the fits to $S^2$ and $N^2$, with $r^2$ values similar to those for the individual fits, but the exponents are not well constrained.

It is notable that all three surveys took place in relatively mild conditions, in the absence of strong wind events or mesoscale activity. Both the homogeneity in $S^2$ and $\epsilon$ below the TL and our scaling results seem consistent with “background” internal wave-driven turbulence. For mesoscale regimes with active eddies and fronts, Forryan et al. (2013) found a much different, Ri-dependent behavior, and a monotonic relationship but not a constant scaling between $\epsilon$ and $N^2$. Some aspects of the TL scalings that we observe may be easier than others to implement in models. In the region $z_{TL} \leq 0$, $\epsilon$ scales reasonably well with $S^2$ and $N^2$, and also with distance over a scale height, $\xi$, proportional to the vertical extent of the TL. For $z_{TL} > 0$, there can be significant missing mixing in the TL even though models include mechanisms for handling shear instability. This is because an accurate assessment of Ri depends on resolving the unstable scales. A recent study by Jochum et al. (2013) attempted to compensate for unresolved near-inertial wave (NIW) shears in a GCM by increasing the model-resolved shears near the upper boundary by 80%. As a result, they found significant changes to global SST patterns associated with deepening of the mixed layer. While our observations are not
directly comparable, the 1-m shear variances that we observe near depths where \( R_i \approx 0.25 \) are about 4 times larger than the 10-m shears (recall that \( S_{10}^2 \) has already been scaled by a spectral correction factor of 2), and thus the effect of missing shears near the boundary may be even larger than they accounted for.

Jochum et al. (2013) also simulated mixing due to downward-propagating NIWs by adding an exponentially decaying diffusivity (scale height \( \xi = 1000 \text{ m} \)) from the bottom of the boundary layer and found the effects to be minimal, relative to their amplification of near-boundary shears. Profiles from the three surveys that we

**Table 1.** Summary of scaling relationships. Shown are log–log fits of bin- and survey-averaged turbulent dissipation rate \( \epsilon \) to the finescale quantities in the leftmost column. Also included, in the rows set apart at the bottom, are scalings between \( S^2 \) (and \( S_{10}^2 \)) and \( N^2 \).

<table>
<thead>
<tr>
<th></th>
<th>NATRE</th>
<th>BBTRE96</th>
<th>BBTRE97</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon \sim (\cdot)^m )</td>
<td>( m )</td>
<td>( r^2 )</td>
<td>( m )</td>
</tr>
<tr>
<td>( z_{TL}/100 )</td>
<td>(-0.07 \pm 0.03 )</td>
<td>(-0.13 \pm 0.03 )</td>
<td>(-0.10 \pm 0.03 )</td>
</tr>
<tr>
<td>(TL only)</td>
<td>(-2.37 \pm 0.03 )</td>
<td>(-1.11 \pm 0.03 )</td>
<td>(-0.57 \pm 0.03 )</td>
</tr>
<tr>
<td>( N^2 )</td>
<td>( 0.98 \pm 0.31 )</td>
<td>( 0.76 \pm 0.13 )</td>
<td>( 0.70 \pm 0.19 )</td>
</tr>
<tr>
<td>( S^2 )</td>
<td>( 1.40 \pm 0.36 )</td>
<td>( 1.00 \pm 0.17 )</td>
<td>( 0.89 \pm 0.22 )</td>
</tr>
<tr>
<td>( S_{10}^2 )</td>
<td>( 1.25 \pm 0.36 )</td>
<td>( 1.10 \pm 0.17 )</td>
<td>( 1.12 \pm 0.22 )</td>
</tr>
<tr>
<td>( R_i )</td>
<td>( 1.69 \pm 1.29 )</td>
<td>( 1.78 \pm 0.57 )</td>
<td>( 2.11 \pm 1.03 )</td>
</tr>
<tr>
<td>( R_{i10} )</td>
<td>(-0.09 \pm 1.29 )</td>
<td>( 1.35 \pm 0.57 )</td>
<td>( 1.42 \pm 1.03 )</td>
</tr>
<tr>
<td>(</td>
<td>S</td>
<td>^p/N</td>
<td>^q</td>
</tr>
<tr>
<td>( (p, q) )</td>
<td>((1.40, 0.98))</td>
<td>((1.00, 0.76))</td>
<td>((1.89, 0.70))</td>
</tr>
</tbody>
</table>

\( S^2 \sim (N^2)^m \) \( 0.75 \pm 0.05 \) \( 0.95 \) \( 0.72 \pm 0.06 \) \( 0.93 \) \( 0.80 \pm 0.03 \) \( 0.98 \)

\( S_{10}^2 \sim (N^2)^m \) \( 0.67 \pm 0.05 \) \( 0.77 \) \( 0.67 \pm 0.06 \) \( 0.91 \) \( 0.61 \pm 0.03 \) \( 0.95 \)
examined showed evidence of enhanced turbulent dissipation $\epsilon$ through the TL and upper TC, associated with a changing scale height, depending on $N^2$ and $S^2$, but not always an enhanced diffusivity, $\kappa_\rho$ for $z_{TL} \leq 0$. Future studies like that of Melet et al. (2012), who compared the effects of differing vertical profiles of tidal mixing near the ocean floor, will be needed to assess whether different vertical profiles of TL dissipation and mixing are likely to have significant effects in the upper ocean.

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