Text S1 – Modeling the AMC

Our model AMC is a band of thickness h_{AMC} located at depth z_{AMC}, initially continuous and at temperature T_L. Melt content is initially set to 100% within this band, and zero elsewhere.

We use a modified form of the heat equation to account for the effect of crystallizing melt on the energy balance. In a closed volume V delimited by surface S, conservation of energy writes (see Table S1 for a summary of notations).

$$\frac{\partial}{\partial t}\int_V \overline{\rho}\overline{U}dV = -\int_S \rho U\vec{v}\cdot d\vec{S} - \int_S \vec{q}\cdot d\vec{S} + \int_V \frac{\partial x_L}{\partial t} L\rho_m dV$$

which, in differential form, after introducing Fourier's law and a thermodynamic parameter γ *becomes*

$$\frac{\partial}{\partial t}\left(\overline{\rho}c_P T\right) + \vec{\nabla}\cdot\left(\gamma T\vec{v}\right) = \vec{\nabla}\cdot\left(\kappa\vec{\nabla}T\right) - \frac{\partial x_L}{\partial t}\frac{L\rho_m}{\overline{\rho}\overline{c}_P}$$

Following Sinton and Detrick [1992], we assume that the melt fraction varies with temperature

$$\frac{\partial x_L}{\partial t} = \frac{\lceil x_L \rceil}{T_L - T_S}\frac{\partial T}{\partial t}$$

where the use of the ceiling function on x_L ensures that the latent heat term is applied only where $x_L>0$.

We then obtain a modified heat equation of the form

$$\left(1 + \lceil x_L \rceil B\right)\frac{\partial T}{\partial t} + \vec{\nabla}\cdot\left(\gamma T\vec{v}\right) = \kappa\nabla^2 T$$

with

$$B = \frac{L}{\left(T_L - T_S\right)}\frac{\rho_m}{\overline{\rho}c_P}$$

Using this equation requires constant tracking and updating of the x_L field. If between time t_n and t_{n+1} a point within the AMC undergoes a temperature change ΔT_n (<0), its melt content is updated by

$$x_L\left(t_{n+1}\right) = x_L\left(t_n\right) + \frac{\Delta T_n}{\left(T_L - T_S\right)}$$

At all other points x_L is kept at zero. We further impose that x_L be maintained between 0 and 1 to prevent any decoupling between x_L and T.