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Vol. XI, No. 3

AN ELECTROMAGNETIC METHOD FOR MEASURING  
THE VELOCITIES OF OCEAN CURRENTS  
FROM A SHIP UNDER WAY

BY

WILLIAM S. VON ARX

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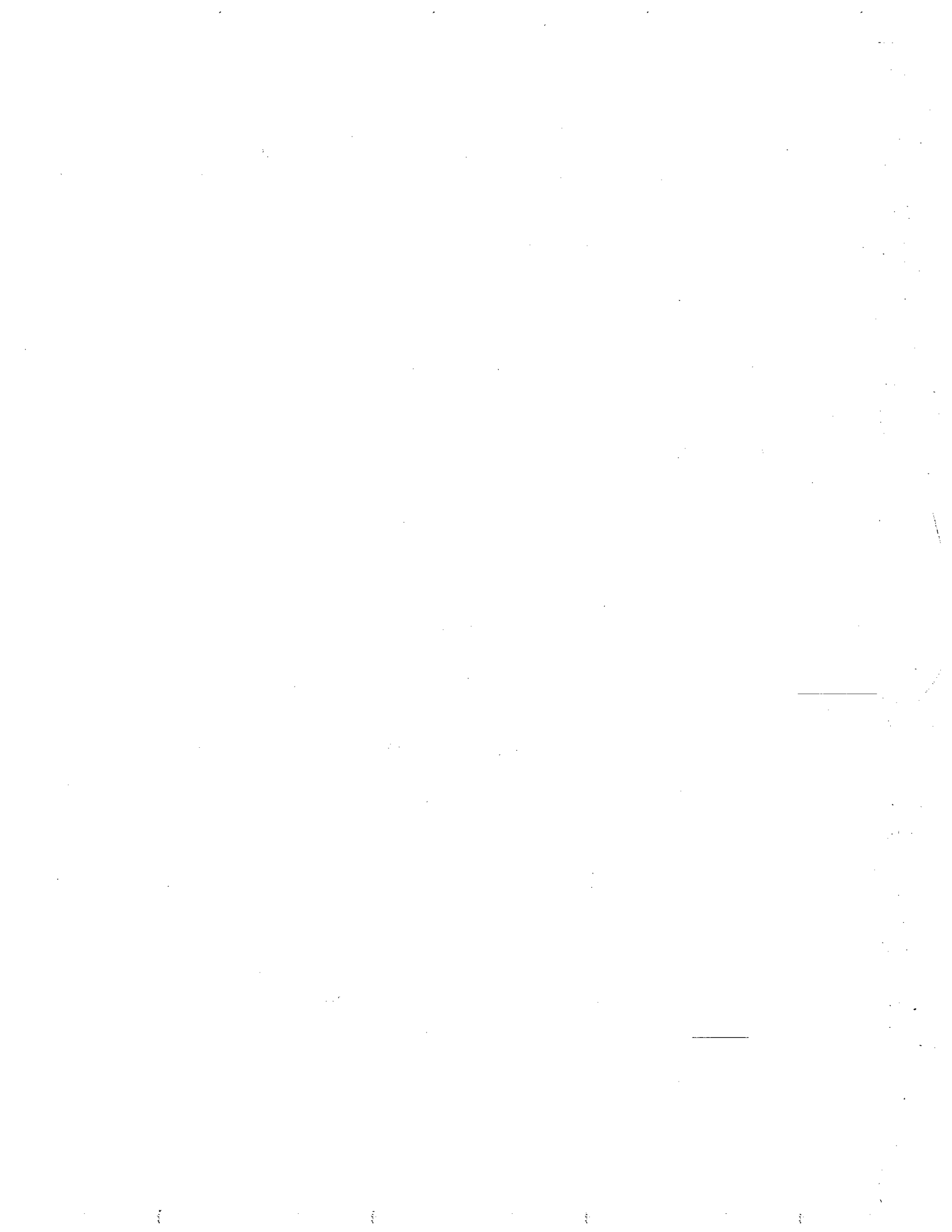
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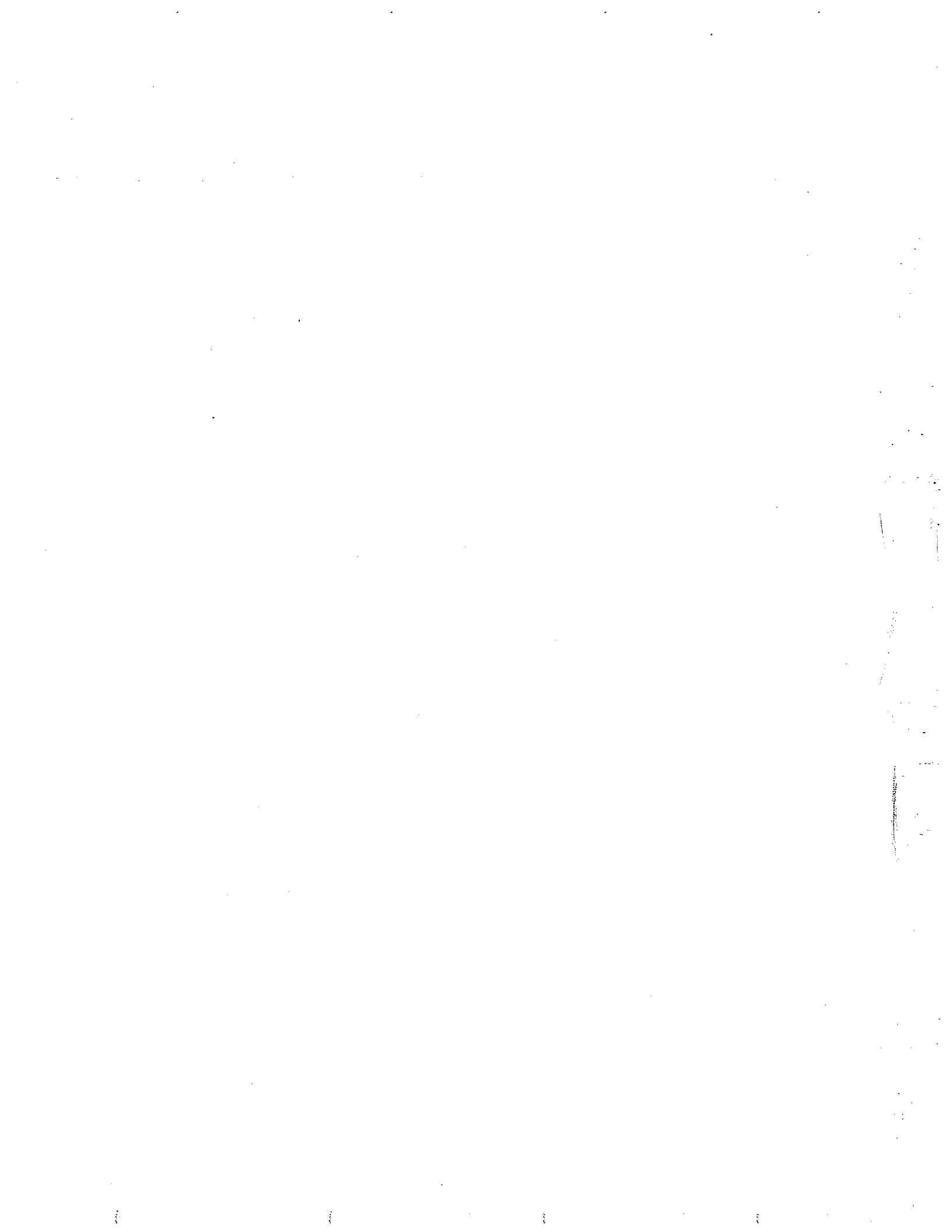
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## INTRODUCTION

During the past four years a deliberate effort has been made at the Woods Hole Oceanographic Institution to devise methods of kinematic observation generally suited to the needs of oceanographers. One result of this work, the electromagnetic method, has been brought from the experimental stage to one of useful maturity. Many of the theoretical potentialities of the method are still to be explored and developed. Nevertheless it seems likely that this remaining work may be done more soundly if present developments of the theory and instrumentation are made available for use and evaluation by others.

These studies in methods of kinematic observation have been supported mainly under the provisions of Bureau of Ships Contract NObs-2083, and Office of Naval Research Contract N6onr-277-1. This support and the assistance of the Naval Ordnance Laboratory, the Hydrographic Office (Oceanographic Division), the United States Coast Guard, and the David Taylor Model Basin of the United States Navy is gratefully acknowledged.





## I. THE GENERAL PROBLEM OF KINEMATIC OBSERVATION OF THE DEEP SEA

The task of describing the broad features of the circulation of the oceans has been accomplished largely by indirect methods and without the benefit of simultaneous observations. Except for a scattered few direct observations of water motion and the data from ships' logs, the bulk of the present kinematic description of the deep sea is inferred from dynamic observations. The method of dynamic sections, while fundamental to many phases of oceanographic work, is based on certain dynamic principles and is therefore incapable of yielding kinematic data free from the uncertainty of these principles. Direct observations made with conventional current meters suspended from anchored ships are so time-consuming that it is difficult to obtain synoptic information. These observations are also somewhat uncertain, for at an average distance of 4 kilometers from the anchor, the motion of the ship is ponderous and incessant. Measurements made from drifting ships are probably more reliable, but equally time-consuming and no less difficult to correlate in space and time. In view of the enormous volume of water that must be surveyed again and again, and the inaccessibility of the bottom as a frame of reference, it is easy to see why indirect methods have been used so widely and how urgent is the need for more rapid surveying techniques that yield sufficient kinematic data for essentially synoptic studies to be made of a modest portion of an ocean at a time.

During the past decade improved methods have been devised for observing ocean currents from moving ships. Thus far these methods are limited to the surface and near-surface layers. Nevertheless, the mobility of the ship permits something to be discerned of the synoptic pattern of flow and its changes. The simplest and most familiar of these methods takes its data from consecutive Loran fixes at 20 or 30 minute intervals while the ship is under way. The difference between the true geographic progress of the ship and the dead reckoning of the speed and course steered shows the combined effects of wind and current. The kinematic detail obtained by this method is improved if the ship is propelled slowly but it is found on slowing the ship, that it is more seriously drifted by the wind.

Greater kinematic detail free of the influence of the wind and the ship's speed has been obtained electromagnetically with respect to the reference frame provided by the earth's magnetic field. Theoretically, the electromagnetic principle allows continuous measurement of the mean and turbulent motions of the sea at any chosen level and at any rate at which the measuring apparatus can be propelled. The method is presently implemented by the Geomagnetic Electrokinetograph, a shipboard instrument capable of indicating and recording surface and near-surface velocity components normal to the direction of motion of the ship. These data collected on two courses at an angle to each other can be composed to fix the true horizontal velocity vector. The shortest practical run along each course is about 500 meters. This limitation is imposed partly by the apparatus and partly by the necessity of averaging out the turbulent irregularities of motion in the sea itself. The motion in what are thought to be turbulent systems as small as 100 meters across can be detected, but their internal details can be studied only if the systems are large enough to permit the ship to maneuver within their bound-

aries. Larger features of the sea can be studied by lengthening the run between course changes and arranging the pattern of true current determinations to reveal more clearly the changes observed in the record of one component of flow. Systematic changes of course are more often used to gather a uniformly distributed series of true current fixes.

The electromagnetic principle of measurement is purely kinematic in the sense that no assumptions are made regarding the causes of the water motion or the forces at work within it. The present techniques of measurement appear to be sound and practical at sea. The accuracy of the method is highest on the surface over deep water where the combined errors of measurement probably do not exceed ten per cent.

## II. INTRODUCTION TO THE DEEP SEA ELECTROMAGNETIC METHOD

The deep sea electromagnetic method is based on a suggestion made by Faraday in the Bakerian Lecture of 1832 before the Royal Society: "Theoretically, it seems a necessary consequence that where water is flowing, there electric currents should be formed: thus, if a line be imagined passing from Dover to Calais through the sea, and returning through the land beneath the water to Dover, it traces out a circuit of conducting matter, one part of which, when the water moves up or down the channel, is cutting the magnetic curves of the earth, whilst the other is relatively at rest." Faraday (1832) attempted to detect electric currents flowing over a short range in response to the water motions of the Thames, but the potentials developed at the copper electrodes he used masked the effect. Following the middle of the 19th century, however, the effect was noted repeatedly on the very long ranges provided by broken submarine telegraph cables. These observations are summarized by Longuet-Higgins (1949). Successful short range experiments were conducted in Dartmouth Harbor in 1918 by Young, Gerrard and Jevons (1920) who used both moored and drifting non-polarizing electrodes to reveal the existence of electric currents generated by tidal motions. More recently Barber and Longuet-Higgins (1948), Barber (1948) and Longuet-Higgins (1947, 1949) have detected and described theoretically the potential differences and associated magnetic effects produced by the flow of water in the English Channel.

Along with these natural observations, many applications of motional electromagnetic effects in liquids have been made in instruments for metering or detecting flow for biological and physical studies. Some of these are reviewed by Fabre (1932), Kolin (1936, 1944, 1945), Wetterer (1937, 1938), Einhorn (1939), Thürlemann (1941), and Guelcke and Schoute-Vanneck (1946). The latter developed an electromagnetic flow meter for use on the sea floor. The theory of this instrument has been investigated by Longuet-Higgins and Barber (1946). Instrumental applications of the principle of electromagnetic induction in liquids usually employ direct or alternating artificial magnetic fields. Since these are commonly generated by a component part of the instrument, observations of motion are made with regard to the instrument as a frame of reference. The Geomagnetic Electrokinetograph, as its name implies, makes use of the undisturbed magnetic flux of the earth as the frame of reference for observations at sea.<sup>1</sup>

The essential physical equipment that constitutes the surface Electrokinetograph is: (1) a pair of electrodes mounted a few tens of meters apart on a two-conductor cable long enough to stream them astern away from the influences of the ship and connect them to, (2) a recording potentiometer on board. With this equipment and the ship's compass, observations are made under way of the direction and the potential difference between the electrodes. These potential differences are due to the water motion at right angles to the course and are rigidly related to the drift and set of the ship and electrodes. The potential difference changes sign when currents set the ship to port or to starboard. The magnitude of the potential difference depends on the rate of drift

<sup>1</sup> The steadiness of the earth's magnetic flux as a reference frame has been studied and found to be sufficient for oceanographic observations by this method even during magnetic storms. Magnetic storms rarely involve fluctuations of magnetic intensity greater than one per cent of the total field intensity. Such fluctuations, though ineffective by themselves, may be rapid enough to produce strong earth currents by induction. It is found that potential differences accompanying earth currents are much smaller in the sea than on land. Thus far, no clear indications of interference from fluctuations of magnetic intensity or earth currents have been detected in Electrokinetograph records. The possibility of occasional interference exists, however, particularly in shoal waters near the land.

normal to the course, the length of wire between electrodes, the local strength of the vertical component of the earth's magnetic field, and to some extent on the vertical distribution of water velocities in the vicinity. Through measurements of the potential differences on two courses nearly at right angles, the drift or component velocities in these two directions are known. The vector sum or resultant of these velocities is the surface water current vector for that locality.

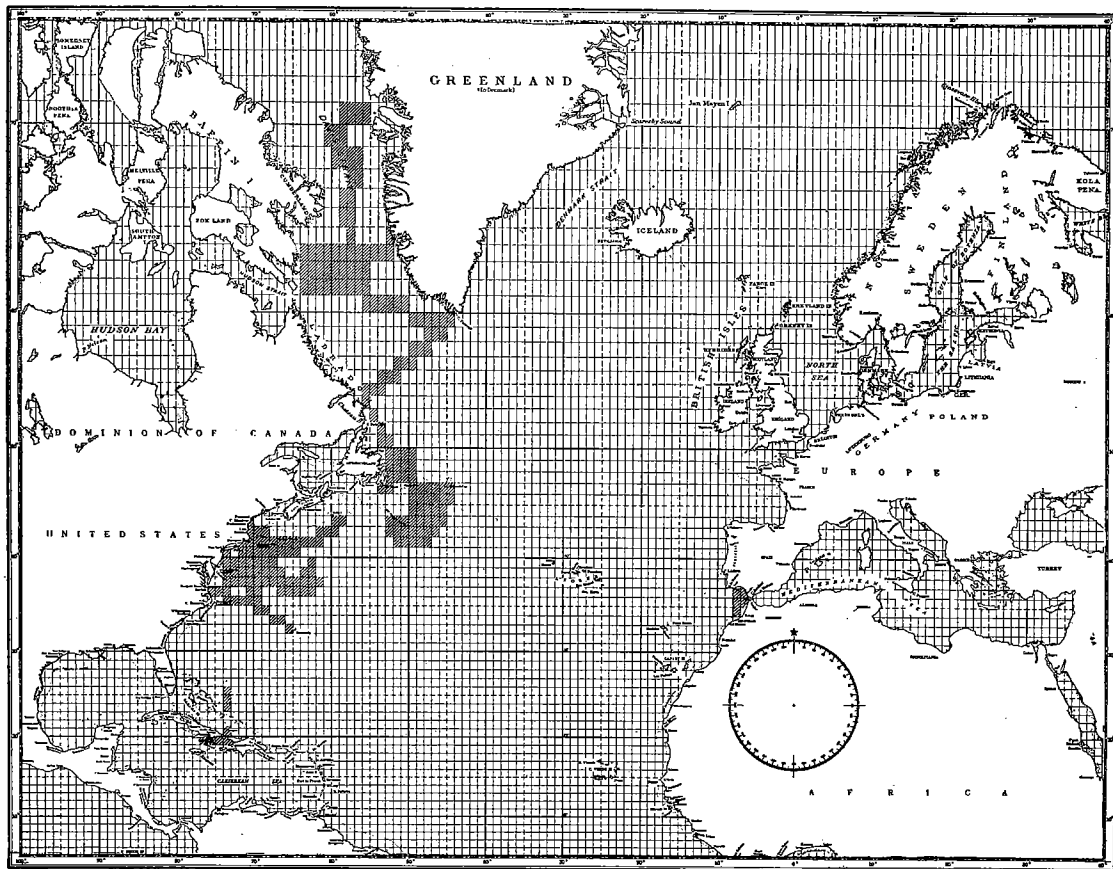
#### 1. THE DEVELOPMENT OF THE GEOMAGNETIC ELECTROKINETOGRAPH

The experiments which led to the development of the Electrokinetograph were begun in the autumn of 1946 when the experiments of Young, Gerrard and Jevons (op.cit., 1920) were repeated using a recording potentiometer instead of a marine galvanometer. Similar results were obtained, in that the direction of water flow was correctly indicated, but the signal strength was too small for the water motion. Later work in deeper water showed an improvement of the signal strength. Beginning in the summer of 1947 experiments were performed in water ranging in depth from 10 to 4,000 meters to examine the effect of depth on signal strength. It was found that in water over 100 meters deep tidal and ocean currents were measurable with acceptable reliability.

A year later the depth relationship was clarified by Stommel (1948) when he investigated the theory of the electric field induced in deep ocean currents. This study showed that the amount of shear in the flow and the proportionate thickness of moving water to the total depth determines the signal strength correspondence. In that the flow in the open sea is generally greatest at the surface and there is a vast body of highly conducting, relatively quiet water between the levels of most vigorous flow and the bottom, the Electrokinetograph finds its most favorable environment on the open sea. In shoal water the principal currents may be tidal and extend uniformly from the surface to within a short distance of the bottom. Thus, except for the water saturating the bottom materials, the quiet conducting layer is thin or absent and the electric circuit is closed mainly through the higher resistance of the sea bed. This causes a serious reduction of the signal strength received by moving apparatus. Even so, it is possible to determine the water motion with some accuracy by making sets of observations in the same locality both under way and with the cable and electrodes at rest on the bottom.

By spring of 1948 instrumentation of the electromagnetic method had been developed to essentially its present form. Through the interest of Floyd M. Soule, Oceanographer of the International Ice Patrol, an Electrokinetograph was installed on the U.S.C.G. Cutter *Evergreen* for the 1948 Ice Patrol Season; [Soule, Carter and Cheney (in press)]. The results showed sufficient promise for the instrument to be carried during the 1949 season, [Soule (in preparation)], and the practice may be continued. The results of comparisons of the currents observed electromagnetically with those inferred from the dynamic topography are discussed in section IV.

Earlier versions of the Electrokinetograph have been installed temporarily on R. V. *Balanus* and R. V. *Atlantis* of the Woods Hole Oceanographic Institution, and on the U. S. Navy Hydrographic Office oceanographic vessel U.S.S. *Rehoboth* (AGS-50). The latest model Electrokinetograph is at present undergoing exhaustive sea trials on R. V. *Atlantis* and several different towing systems are being tested. Altogether the electromagnetic method has been used at sea about 24 months during the past 4 years, and observations have been obtained in the hachured areas of figure 1.



Adapted from Misc. No. 9678

FIG. 1. The areas in which the Geomagnetic Electrokinetograph has been used to observe ocean currents. One or more sections have been made in some part of each of the one-degree areas hachured. The ships on which the instrument was temporarily installed are: R. V. *Atlantis* and R. V. *Balanus*, W.H.O.I., U.S.C.G. Cutter *Evergreen 295*, U.S.S. *Rehoboth AGS-50*. Approximately 15,000 miles have been covered in a total observing time of about 2,000 hours.

### III. PHYSICAL PRINCIPLES OF THE ELECTROMAGNETIC METHOD

Sea water contains an abundance of highly dissociated salts which make it an electrolyte of high conductivity. It is to be expected that the motion of an electrolyte through a magnetic field will produce electromagnetic effects similar to those well known for moving solid conductors. In moving electrolytes, as in solid conducting materials, the vector properties of electromagnetic effects are determined by the kinematic and external magnetic vectors rather than the geometrical configuration of the bodies, which, in both instances, restricts the measurable results but not the basic phenomena.

The motional magnetic fields of oppositely charged ions in an electrolyte moving with a velocity  $\mathbf{v}$  will interact with an extensive magnetic field at rest and of intensity  $\mathbf{H}$  to separate charges systematically within the electrolyte and set up in it an electric field of intensity  $\mathbf{E}$  fully determined by the vector cross product

$$\mathbf{E} = \mathbf{v} \times \mathbf{H} \quad (\text{III-1})$$

The direction of  $\mathbf{E}$  is perpendicular to the plane determined by  $\mathbf{v}$  and  $\mathbf{H}$ , and  $\mathbf{E}$  is positively directed in the sense of rotation of a right-hand screw turned from  $\mathbf{v}$  toward  $\mathbf{H}$  through the smaller of the angles between them. The electric field intensity,  $\mathbf{E}$ , is equal to the negative gradient of the potential. Thus, the steepest gradient of the motional potential will be everywhere colinear with  $\mathbf{E}$  and positively directed (plus to minus) in the same sense.

From equation (III-1) it can be said that a knowledge of  $\mathbf{v}$  and  $\mathbf{H}$  uniquely determines  $\mathbf{E}$ . In the oceanographic case  $\mathbf{v}$  is the unknown. A knowledge of  $\mathbf{E}$ , from measurements of component potential differences,  $\Delta V_s$ , in the sea<sup>2</sup>, and a knowledge of  $\mathbf{H}$  from published<sup>3</sup> data, permits  $\mathbf{v}$  to be found.

In that the oceans are approximately 1/1000 as deep as they are wide the most significant motions within them are horizontal. If measurements of  $\Delta V_s$  are restricted to the horizontal plane the intensity of a horizontal component of  $\mathbf{E}$  is indicated by each measurement, and with a knowledge of  $H_z$ , the vertical component of the magnetic field, a horizontal component of  $\mathbf{v}$  is uniquely determined. To examine this more closely;

$$\mathbf{v} \times \mathbf{H} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_x & v_y & v_z \\ H_x & H_y & H_z \end{vmatrix} \quad (\text{III-2})$$

and the components of  $\mathbf{E}$  are in a coordinate system arbitrarily set so that x is magnetic east, y magnetic north, z up;  $H_x = 0$ .

$$\begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} = \begin{Bmatrix} v_y H_z - v_z H_y \\ v_z H_x - v_x H_z \\ v_x H_y - v_y H_x \end{Bmatrix} \quad (\text{III-3})$$

<sup>2</sup>  $V$  is a scalar, but measured between two points  $s$  apart a quantity  $\Delta V_s$  is found having both direction and magnitude.  $\Delta V_s$  may be written  $\nabla V$  when the components  $(\Delta V_{sx}i + \Delta V_{sy}j)$  are known and composed. Similar notation will be used for  $\Delta \phi_s$  and  $s$  itself.

<sup>3</sup> The aspects of the earth's magnetic field at sea were surveyed mainly by the "Carnegie" during the years 1909-1929 and the "Galilee" in the years 1905-1908. The combined observational errors and the present accumulated secular change of the main magnetic field of the earth represented in these observations, and collected on H. O. magnetic charts 1700, 1701, 1702, 1703, 1704 and 1705, do not exceed 10 millioersted and are probably less than 5 millioersted at most points at sea according to E. H. Vestine (personal communication). (For details see Vestine [1947, 1948]. More general information on geomagnetism is presented by Fleming [1939] and Chapman and Bartels [1940].)

Since  $v_z = 0$  at the surface and bottom of the sea, and is probably small even in zones of sinking and upwelling, it is permissible to consider the special case for  $\mathbf{v}$ , the component in the horizontal plane ( $v_x\mathbf{i} + v_y\mathbf{j}$ ), as determined from measurements of  $(\Delta V_{sx}\mathbf{i} + \Delta V_{sy}\mathbf{j})$  and a knowledge of  $\mathbf{H}$  and its components, thus

$$\begin{cases} v_x = -E_y/H_z \\ v_y = E_x/H_z \end{cases} \equiv \begin{cases} \Delta V_{sy}/H_z \\ \Delta V_{sx}/H_z \end{cases} \quad \text{(III-4)}$$

composing  $v_x\mathbf{i}$  and  $v_y\mathbf{j}$

$$(v_x\mathbf{i} + v_y\mathbf{j}) = \mathbf{v} = (-\Delta V_{sy}\mathbf{i} + \Delta V_{sx}\mathbf{j})/H_z = \frac{\nabla V}{H_z} \quad \text{(III-5)}$$

As only  $H_z$  appears in equation (III-5) for measurements of the horizontal velocity of the sea, the x and y coordinates may be rotated around the z coordinate in any convenient direction, usually so as to lie parallel to the directions  $s_x$  and  $s_y$  of measurement. If these directions are measured with respect to the circles of either true or magnetic azimuth the direction of  $\mathbf{v}$  is also known with respect to either of the same systems. The direction of  $\mathbf{v}$  is horizontal and determined as a vector by

$$\mathbf{v} \times H_z = -\nabla V' \quad \text{(III-6)}$$

Measurements at sea show that  $\Delta V_s'^{(4)}$  is less than the motional electromotive force  $\text{Emf}_s$  expected from a velocity  $\mathbf{v}$  determined by independent means. This defect in magnitude arises from the less-than-perfect conductivity of the sea and the sea bed which restricts the flow of electric current in response to the motional  $\text{Emf}$  throughout the velocity field. The measuring apparatus, in motion through the magnetic field, has a motional  $\text{Emf}'_s$  induced in it which is precisely equal to  $\text{Emf}_s$  induced in the moving sea. The equivalent circuit is shown in figure 2 wherein,  $R + \sum_0^n r$  is small compared with  $r_s$ ,  $\text{Emf}'_s = \text{Emf}_s$ , and  $\phi_0 - \phi_s = \Delta\phi_s$

<sup>4</sup> Primes will denote instrumental quantities.

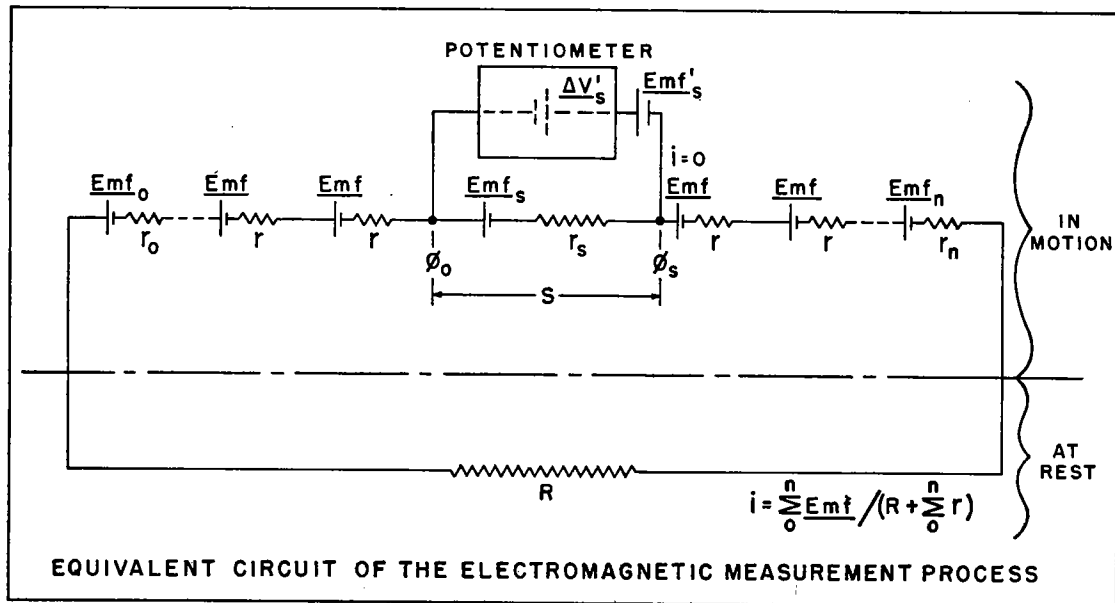


FIG. 2

The electric current  $i$  flowing in the sea circuit is

$$i = \frac{\sum \text{Emf}}{R + \sum_0^n r} \quad (\text{III-7})$$

and the potential drop across the directed interelectrode length  $s$  is

$$(\phi_0 - \phi_s) = \Delta\phi_s = \text{Emf}_s - ir_s \quad (\text{III-8})$$

Therefore the signal measured at the potentiometer is

$$\Delta V_s' = \text{Emf}_s' - \Delta\phi_s \quad (\text{III-9})$$

The value of  $\Delta\phi_s$  can be observed if the measuring apparatus is at rest so that  $\text{Emf}_s'$  in the measuring apparatus is zero. At sea there is no convenient means for making measurements with the cable at rest in the earth's magnetic field, hence the value of  $\Delta\phi_s$  must remain undetermined or be calculated from a knowledge of the velocity field.

Stommel has shown (op.cit., 1948) that the induced electric potential field in any arbitrary velocity field in the deep sea is defined by

$$\nabla^2\phi = \mathbf{H} \cdot \text{curl } \mathbf{v} \quad (\text{III-10})$$

This equation expressed in finite difference form permits the application of Southwell's relaxation method (1946) to find the value of the potential everywhere in the observed velocity field. At present a number of important assumptions must be made in practicing this form of solution, consequently it does not yet serve as a routine measure for correcting or determining the accuracy of the field observations.

Stommel's discussion of the electric potential fields in the deep ocean shows that the value of  $\Delta\phi_s$  depends directly on the ratio of the effective depth of a broad current to the total depth of water; that is, on the degree of perfection of the short circuit provided by the surrounding and subjacent water masses. His discussion also suggests that lateral extensions of the electric field adjacent to a major ocean current will cause small "fictitious" signals to be recorded. These "fictitious" signals have been studied in a preliminary way by means of the relaxation method. The potential field external to a broad but shallow ocean current is seen to die away rapidly in the horizontal direction normal to the flow but much less rapidly below the ocean current. Quantitatively these studies suggest that measurements of the surface velocity immediately adjacent to a boundary of a current may err from 3 to 5 per cent of the maximum velocity in the current due to extension of its electric field. Underneath the current and on the same level for a little distance either side these errors may be in the neighborhood of 10 per cent of the maximum velocity of the surface current. Exact evaluation of these errors requires a knowledge of the resistivity of the sea bed and rather detailed information concerning the velocity field in and around the ocean current. None of these data are available at present.

In the foregoing discussion, the vertical component of motion in the sea has been ruled out as negligible. While this is sufficiently true to be justified from a kinematic standpoint, it is possible for small vertical motions to have important electromagnetic effects near the magnetic equator where the vertical component of the magnetic field is



small and the horizontal component proportionately large. Occasion has not demanded a critical analysis of the problem but it seems likely that "fictitious" signals due to vertical motions of the sea may, if these are 1/100th as strong as the horizontal motions, produce east-west signals equal in strength to those due to horizontal motions within approximately 20 miles of the magnetic equator. In order for the east-west signals due to vertical motion to be less than 10 per cent of the horizontal motion signal, assuming 100:1 ratio of velocities, the site of observations would have to be removed approximately 200 miles from the magnetic equator.

It is unfortunate that both the electromagnetic method and the method of dynamic sections lose their effectiveness in the equatorial regions. However, the magnetic and geographic equators do not coincide but cross at two points, 25°W and 160°W longitude in mid-Atlantic and mid-Pacific respectively, leaving large areas where it is possible that supplementary data may be obtained by the two methods.

#### IV. ON THE VALIDITY OF THE ELECTROMAGNETIC METHOD

Reliable current measuring techniques that detect identical aspects of flow are so few it is very difficult to devise conclusive experiments which will demonstrate the validity of a new method by the process of comparison. Because of this difficulty the electromagnetic method has been tested against as many established methods as opportunity has permitted. It has also been used as a navigational aid and its effectiveness checked by Loran. The number of individual experiments is now so large that the results of the tests will be summarized in experimental classes rather than detailed. The experimental classes are as follows:

- Class I — Comparison of electromagnetic observations of current with simultaneous observations made with propeller-type current meters suspended from an anchored ship,
- Class II — Comparison of electromagnetic observations of current with simultaneous observations of the drift of current poles, current crosses, and dye trails,
- Class III — Comparison of electromagnetic observations of current with predicted currents from tables, charts and other published sources,
- Class IV — Comparison of electromagnetic observations of current with currents determined by means of a drifting ship whose drift was determined by Loran and corrected for windage by means of a chip log,
- Class V — Comparison of electromagnetic measurements of current with those inferred from simultaneous observations of the dynamic topography of a large area of the sea,
- Class VI — Use of the electromagnetic method to measure the current and to correct the steered heading of a ship in order to maintain a straight line of progress through a major ocean current.

##### I. THE "k" FACTOR

The results of most experiments comparing the water motion observed by the electromagnetic method and other methods have been expressed in terms of the ratio  $|\underline{Emf}_s|/|\underline{\Delta V}_s'|$  which is locally called the "k" factor. The value of  $\underline{Emf}_s$  used to calculate "k" is determined from the voltage anticipated from the water speed determined by non-electromagnetic methods, where  $\underline{Emf}_s = (H_z s v) \times 10^{-8}$ . The value of  $\underline{\Delta V}_s'$  is obtained from the Electrokinetograph record of the current. The "k" factor would be unity if  $\underline{\Delta \phi}_s$  were zero. Since the "k" factor ratio is dimensionless it may be calculated as well from the ratio of water speeds observed by non-electromagnetic and electromagnetic methods or from other equivalent expressions.

The range of values of "k" correlates with the depth of water to the extent shown in figure 3. In very shoal tidal reaches less than 10 meters deep "k" may vary from station to station in the range 1.5 to 15.0 and average near 10.0. On the continental shelf in depths of the order 10 to 100 meters the value of "k" averages less than 2.0 and seldom exceeds 3.0 in individual cases. Beyond the 150 meter isobath the value

of "k" is predominantly less than 1.10 and the value 1.05 may be representative of most situations in the open ocean. Recent evidence places the average open sea value of "k" at 1.04. It is thought that the correlation of "k" with the depth of water results from the dependence of the current regime on the depth. In areas where tidal currents

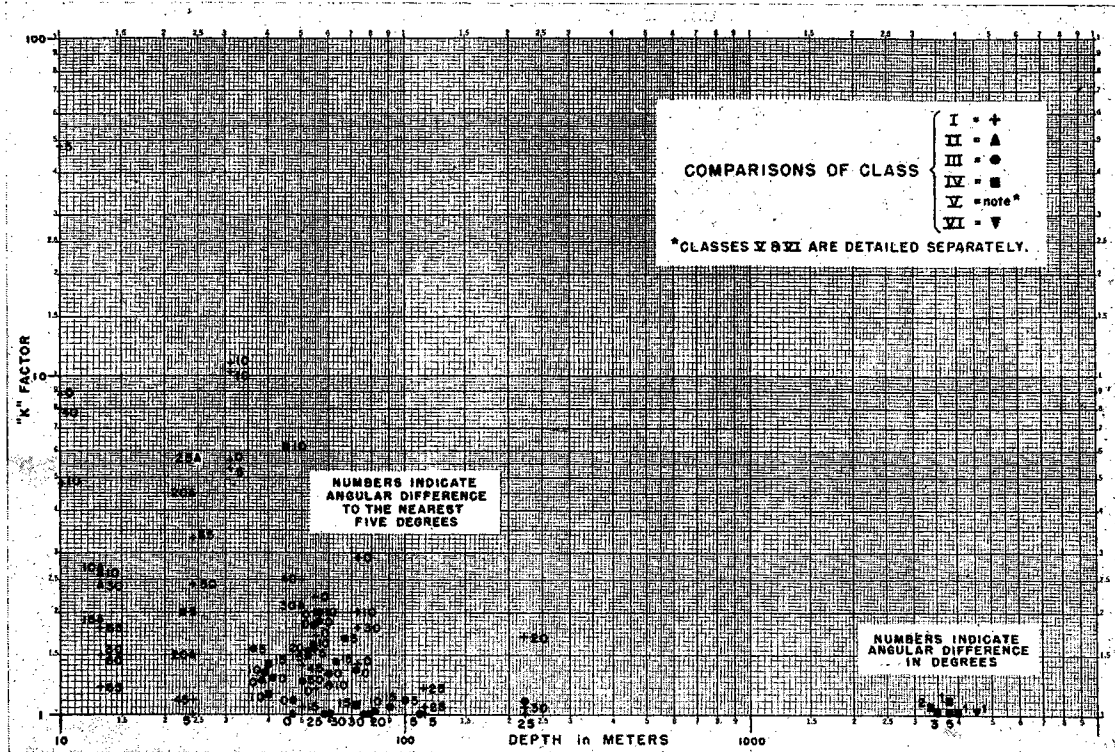


Fig. 3. The relationship between the "k" factor and depth of water determined during tests of the validity of the electromagnetic method. The decreasing scatter of "k" values with increasing depth of water is thought to be due, in part, to the diminishing uniformity of water motion with depth as tidal motions give way to wind currents and predominantly oceanic circulation, and, in part, to the methods used in making the comparisons in deeper water. Beyond the 100 meter isobath the "k" factor rarely departs more than 5 per cent from unity. The occasional values of "k" less than unity have been plotted as reciprocals.

predominate and the flow is more or less uniform to the bottom, the bottom sediments must provide the principal short circuiting medium. Since the electrical resistance of bottom sediments is distinctly higher than that of sea water the value of  $\Delta\phi_s$  is large and  $\Delta V_s'$  is correspondingly small. On banks and the distal portions of the continental shelf the throughgoing tidal regime is progressively less dominant. Oceanic circulation reaches inward over the shelf to some extent and the fetches to the distal portions are great enough for wind driven layers to become important. Both effects introduce shear and there is a corresponding reduction of "k" values. It is evident from both theory and experiment that shear will serve to reduce  $\Delta\phi_s$  in some instances quite as well as a thick subjacent mass of water at rest. This fact is thought to account for much of the variability of "k" since in shallow mixed water shear and turbulence go together, and both may be quite variable from time to time and place to place.

The "k" factor does not take direction into account. It has been found experimentally that, while the indicated magnitude of the flow may be deficient, the indicated direction is generally more or less correctly given by the electromagnetic method. Angular errors diminish in deep water along with magnitude errors. In shoal waters comparative measurements of Class I, using current meters, reveal angular disagreements often as high as  $40^\circ$  with occasional extremes of  $55^\circ$  or  $65^\circ$ . Simultaneous experiments of Class I and Class II, using current poles and dye, usually show better agreement to exist in the comparisons of Class II. Class II comparisons have an average angular disagreement of  $10^\circ$  with electromagnetic measurements and the extremes may reach  $20^\circ$  or  $30^\circ$ . Class III comparisons with charts and tables have so far turned out better than expected in that the average angular difference of 24 observations (see table 1) is slightly under  $10^\circ$ . Class IV comparisons made with drifting ships are on the average more consistent for the angular extremes have not exceeded  $15^\circ$  even in shoal water. Deep water experiments of Class IV have shown very small angular errors despite the large probable error in determining the windage on the ship. Except for one point representing 163 navigational measurements of Class VI, the Class V and VI experiments have not been plotted on figure 3 but are detailed in succeeding figures as individual cruises.

Experiments of Class I through IV are regarded as valuable mainly in revealing the variability of the "k" factor with depth and the current regime. None of them demonstrate the validity of the electromagnetic method quite as convincingly as the Class V and Class VI experiments.

## 2. DISCUSSION OF TESTS OF THE VALIDITY OF THE METHOD

### *Class I Experiments — Comparison with Current Meters*

Experiments of this class were conducted with two boats; one at anchor with a propeller-type current meter over the side, and the other sailing a square course about half or three quarters of a mile on each side around the first. The sites of these experiments were all in local waters: Cape Cod Bay, Nantucket Sound, Vineyard Sound, Buzzards Bay and Block Island Sound. In all 16 stations were occupied and 31 separate measurements taken. Each measurement required about an hour and consisted of four trips around the square with the Electrokinetograph and measurement of the current with the propeller-type current meter at intervals of about 10 minutes. The average of the latter was then compared with the average of the former measurements. The results are plotted in figure 3.

### *Class II Experiments — Comparison with Current Poles and Dye Trails*

These experiments were, in most instances, conducted simultaneously with Class I experiments but a few separate runs were made as well. In all there were 9 runs lasting about 3 hours apiece. Since surface currents were being measured electromagnetically, fluorescein dye was the principal surface tracer. Dye was dispensed on the surface either as a trail from small cloth sacks hung over the stern of the anchored boat or as a surface patch from the current poles and current crosses. When there was disagreement between the positions of the poles and crosses and the dye patches, the dye was plotted as the more nearly correct surface current indicator. Better agreement was

