A box model test of the freshwater forcing hypothesis of abrupt climate change and the physics governing ocean stability

Charles S. Jackson,1 Olivier Marchal,2 Yurun Liu,3 Shaoping Lu,3 and William G. Thompson2

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Observations and an ocean box model are combined in order to test the adequacy of the freshwater forcing hypothesis to explain abrupt climate change given the uncertainties in the parameterization of vertical buoyancy transport in the ocean. The combination is carried out using Bayesian stochastic inversion, which allows us to infer changes in the mass balance of Northern Hemisphere (NH) ice sheets and in the meridional transports of mass and heat in the Atlantic Ocean that would be required to explain Dansgaard-Oeschger Interstadials (DOIs) from 30 to 39 kyr B.P. The mean sea level changes implied by changes in NH ice sheet mass balance agree in amplitude and timing with reconstructions from the geologic record, which gives some support to the freshwater forcing hypothesis. The inversion suggests that the duration of the DOIs should be directly related to the growth of land ice. Our results are unaffected by uncertainties in the representation of vertical buoyancy transport in the ocean. However, the solutions are sensitive to assumptions about physical processes at polar latitudes.


1. Introduction

As hypotheses concerning the origin of abrupt climate change accumulate, there is an increasing need to synthesize data and to develop methods for assessing their strengths and inconsistencies. Understanding the mechanisms of abrupt climate change is particularly challenging because of the uncertainties in the paleoclimate observations and in our knowledge of the physics of the climate system. Among the techniques that can be used to combine noisy observations with incomplete knowledge of a physical system are so-called inverse methods. In particular, Bayesian stochastic inversion (BSI) provides a way to account for the uncertainties in data models and deals with the possibility of multiple solutions (inferences). Most notably, it can re-express the problem of abrupt climate change in terms of its implications for what we should see in observations if models provide adequate representation of the processes that exist in nature.

Here BSI is applied to estimate the changes in global sea level implied by the combination of a Greenland ice core paleotemperature record during the last glacial period with a highly simplified model that allows us to investigate the effects of different assumptions concerning the importance of high-latitude buoyancy forcing and low-latitude vertical mixing processes on the meridional overturning circulation (MOC). These changes are then compared to changes in global sea level inferred from independent geologic records. The comparison provides a test of the consistency between the various data sets and a test of the hypothesis that anomalies in the freshwater supply to the North Atlantic Ocean caused abrupt climate change through their effects on the ocean meridional overturning circulation (MOC) and attendant poleward heat flux (freshwater forcing hypothesis).

2. Interpretations of Dansgaard-Oeschger Interstadials

Dansgaard-Oeschger Interstadials (DOIs) are a series of apparent warm intervals first documented in Greenland ice core records that initiated abruptly and punctuated an otherwise cold glacial period during the last ice age. Interpretations of the origins and factors responsible for the DOIs have followed two tracks, (1) the refinement of observations of DOIs and (2) the development of models of physical processes perceived to be important to these events, particularly of the MOC in the Atlantic Ocean. Our objective here is to make use of Bayesian stochastic inversion (1) to synthesize observational constraints on the freshwater forcing hypothesis for the origin of the DOIs and (2) to document the effects of uncertainties in the representation of vertical mixing in the ocean which previous studies have shown to play a role in the stability of the MOC.

Our choice to focus on the freshwater forcing hypothesis to explain the DOIs in the Greenland record stems from numerous lines of observational evidence and model studies
that invoke the susceptibility of the MOC in the Atlantic Ocean to glacial meltwater discharges from Northern Hemisphere (NH) ice sheets [Stouffer et al., 2006]. These lines of evidence are reviewed in sections 2.1 and 2.2. There are significant uncertainties in each line of reasoning that leave open the interpretation that best explains abrupt climate change. Although we assume for testing purposes that the Greenland data primarily reflect changes in oceanic heat transport, we acknowledge that other components of the climate system could be important [Wunsch, 2003, 2006]. Other influences could include, for example, the changing size and shape of the Laurentide ice sheet [Jackson, 2000; Meissner et al., 2002], remote “teleconnections” to the tropical Pacific [Cane, 1998; Cane and Clement, 1999], and atmosphere–sea ice coupling [Gildor and Tziperman, 2003; Eisenman et al., 2009]. By making the assumption that the Greenland data solely reflect changes in ocean heat transport, we are likely ascribing too much of the observed variability to this single mechanism. The calculation will merely suggest the extent to which expected sea levels can reasonably be related to the forcing and the physics included in our inversion.

2.1. Observational Perspective

Abrupt warmings above Greenland during the last glacial period have first been postulated from ice core records of the oxygen isotopic composition (δ18O) of the ice and later supported by measurements on the isotopic composition of trapped air [Severinghaus et al., 1998; Lang et al., 1999; Huber et al., 2006]. A synthesis of the spatial extent of abrupt climate changes during this period is given by Voelker [2002]. Correlations with observations from marine sediments, such as the relative abundance of fossil shells of planktonic foraminifera and their oxygen isotopic composition, would suggest that the ocean was changing in concert with Greenland events, but with half the amplitude most likely due to feedbacks involving sea ice [Bond et al., 1993; van Kreveld et al., 2000; Weinelt et al., 2003].

Inferences of forcing have come mainly from the observation, prior to DOIs, of layers of ice rafted debris (IRD) in North Atlantic sediments, such as Heinrich events (for a review, see Hemming [2004]). This observation would fit the long-held notion that the MOC in the Atlantic Ocean could be reduced or even shut down if a sufficient amount of freshwater is supplied at the surface in the North Atlantic [Broecker et al., 1990; Clark et al., 2002].

Sudden changes in ice rafting inferred from paleoceanographic records are generally thought to reflect the evolving conditions of an ice sheet at its base, a place that is well insulated from external forcings. This has led some authors to postulate that DOIs originate from internal instabilities of the ice sheets that surrounded the North Atlantic Ocean [MacAyeal, 1993]. However, the apparent coordination of multiple ice sheets has called this hypothesis into question [Bond and Lotti, 1995], with the observed IRD record potentially explained as a response to observed climate variations [Marshall and Koutnik, 2006]. The significance of IRD layers in North Atlantic sediments remains unresolved in part because the presence or absence of IRD may have more to do with the mechanisms by which an ice sheet packages debris into bergs than it does about coastal berg production rates [Alley and MacAyeal, 1994; Hulbe 1997; van Kreveld et al., 2000]. Important sets of observations in this regard are records of relative sea level based on surface-dwelling corals [e.g., Chappell, 2002; Thompson and Goldstein, 2006], δ18O records from deep-sea sediments [Waebroeck et al., 2002], and δ18O records from the Red Sea [Siddall et al., 2003]. Reefs from the Huon Peninsula suggest that millennial scale changes in sea level were mainly coincident in time with Heinrich events [Chappell, 2002]. The largest Heinrich events are associated with a sea level rise in excess of 10 m, which is larger than the minimum sea level change of ±3 m that may be theoretically resolvable by coral records [Chappell, 2002]. On the other hand, there is a lack of evidence, at least at the Huon Peninsula, and indeed from the other reconstructions as well, that non–Heinrich related DOIs were associated with significant changes in sea level. Corals likely have the smallest uncertainties on past sea level and may be well constrained in terms of age; however, only a few observations exist.

The suggestion that changes in ocean circulation played a role in DOIs would be supported by other records from marine sediments. For example, records of benthic foramiferan δ13C suggest that the proportion of water masses from the northern North Atlantic and the Southern Ocean has varied during the last glaciation [e.g., Curry et al., 1999]. Important changes in deepwater δ13C can be simulated from changes in the MOC in the Atlantic Ocean [Marchal et al., 1998, 1999a, 1999b]. It has been difficult to draw a definitive picture of circulation changes from benthic δ13C records [Boyle, 2000]. The events that are best recorded are those associated with massive IRD layers originating from the Laurentide ice sheet [Boyle, 2000]. This raises the question of how climate events in Greenland with apparently similar amplitude could result from different freshwater forcings (as suggested by coral records of sea level) and be associated with different water mass distributions in the deep Atlantic (as suggested by benthic δ13C records).

2.2. Modeling Perspective

The existence of multiple equilibria in ocean models provides support to the idea that the MOC must be playing a key part if not the dominant role in DOIs (for a review, see Weaver and Hughes [1992]). These models have demonstrated that the ocean can suddenly make rapid transitions from one equilibrium state to another given a change in surface freshwater balance. This behavior is apparent when the equilibrium strength of the MOC is examined as a function of the strength of freshwater forcing. The corresponding hysteresis curve shows that the MOC can exhibit different states for the same transient or near-stationary forcing (e.g., Figure 2). These curves, however, do not give the whole sense of the nonlinear behavior of the MOC, as they only describe a set of equilibrium states. For instance, model experiments have shown that the MOC is also sensitive to the rate of change of freshwater forcing [e.g., Manabe and Stouffer, 1994; Marchal et al., 1999b; Lucarini and Stone, 2005a, 2005b].

Characterizing the sensitivity of ocean models to freshwater forcing is an important research topic, not only because of interest in past climate change, but also because of...
its potential relevance for global warming. Model studies show that the shape of the hysteresis curve that describes a model’s sensitivity to high-latitude freshwater input depends on a variety of factors such as (1) assumptions about buoyancy mixing in the ocean [Manabe and Stouffer, 1999; Schmittner and Weaver, 2001], (2) the location where freshwater anomaly is introduced to the ocean [e.g., Saenko et al., 2007], and (3) the processes controlling the fluxes of heat and momentum at the sea surface [e.g., Mikolajewicz and Maier-Reimer, 1994; Toggweiler and Samuels, 1998; Wunsch and Ferrari, 2004]. The latter is particularly important to understanding why the MOC in the Atlantic Ocean may have behaved differently during glacial epochs [Ganopolski and Rahmstorf, 2001; Schmittner et al., 2002]. Also some, but not all, models are able to maintain a reduced state of the MOC without a persistent source of freshwater to the North Atlantic [Stouffer et al., 2006]. The equilibrium solutions of coupled AOGCMs appear to only be temporarily stable because of diffusive processes that undermine the reduced MOC state [Liu et al., 2009].

[12] Nilsson and Walin [2001] use a box model of a single-hemisphere ocean to show that assumptions about vertical buoyancy transport are instrumental in the equilibrium response of the MOC to freshwater forcing. Consider vertical buoyancy transport as being independent of vertical density stratification. Then simple scaling arguments for the oceanic thermocline show that increasing the surface density contrast between the equatorial and high-latitude regions (Δρ) would increase the strength of the MOC, e.g.,

$$MOC \propto \Delta \rho^{1/3}.$$  

(1)

On the other hand, if vertical buoyancy transport increases with decreasing stratification – a plausible assumption – it is easier to generate vertical mass transport for the same amount of energy available for that transport when there is a reduced density contrast between the surface and deep oceans. Since the density contrast between the surface and deep oceans at low latitudes scales with the density contrast between the equator and the high latitudes at the surface (Δρ), increasing Δρ would decrease the strength of the MOC,

$$MOC \propto \Delta \rho^{-1/3}.$$  

(2)

This is what Nilsson and Walin [2001] termed the freshwater “boosting” hypothesis. Numerical solutions of a single-hemisphere box model replicate their arguments (see Appendix C). These arguments are also supported by numerical experiments with ocean circulation models [Nilsson et al., 2003; Mohammad and Nilsson, 2004; Marchal et al., 2007]. However, they may not apply in a straightforward way to a two-hemisphere basin with competing polar regions (water sinking in northern and southern hemispheres) [Saenko and Weaver, 2003; Mohammad and Nilsson, 2006; Marchal et al., 2007]. In particular, Marzeion et al. [2007] found there were additional feedbacks within a more realistic global AOGCM (CLIMBER-3α) that weakens the effects of freshwater forcing into the North Atlantic Ocean for producing a reduced stratification that is the basis for the “boosting” regime. Also, they describe equilibrium responses and may therefore be of limited value to understand transient changes of the MOC [e.g., Marchal et al., 2007].

[13] Box models also make the assumption that the MOC is related in a simple way to meridional density gradients through geostrophy and the thermal–wind relation. De Boer et al. [2010] provided several counterexamples where the relationship between the MOC and the meridional density gradient relationship may be affected by remote influences from the Southern Ocean winds and the Antarctic Bottom Water formation.

3. Methods

[14] In this section we describe our strategy for testing the freshwater forcing hypothesis using Bayesian stochastic inversion. A 4-box model of the Atlantic Ocean is combined with a paleotemperature record from a Greenland ice core in order to estimate a time series of freshwater flux anomaly in the North Atlantic and the attendant changes in the meridional transport of mass and heat, which would be required to explain a series of DOFs from 30 to 39 kyr B.P. The freshwater flux anomalies are converted into anomalies of global sea level, which are then compared with independent evidence from the geologic record. This exercise is repeated with different assumptions about vertical mixing in the model in order to address its influence on the response of ocean circulation to freshwater forcing [e.g., Nilsson and Walin, 2001].

3.1. Bayesian Stochastic Inversion

[15] Stochastic inverse modeling using Bayesian inference is a tool to interpret observational data in the presence of a model [Gelman et al., 2004]. In particular it estimates the probability distribution of a forcing which permits a model to reproduce what is observed. It should be understood at the outset, however, that the solution is expressed as a conditional probability, meaning that all probability measures are relative to the best forcing identified to explain the data.

[16] For instance, the time series of freshwater forcing that permits a climate model to reproduce a paleoclimate record from a Greenland ice core may be inferred by stochastically testing various forcing functions (scenarios) and quantifying the relative likelihood of each solution through a measure of model-data mismatch. Markov Chain Monte Carlo (MCMC) provides sampling rules that both ensures sampling is not biased toward a particular region of solution space and improves efficiency by avoiding regions that are unlikely to provide an acceptable match to the Greenland data. However, standard MCMC approaches to stochastic inversion tend to be inefficient or impractical for applications that are computationally expensive. The solution to this challenge has been to create a statistical model of the physics-based numerical model, also called a “statistical emulator,” which approximates the numerical model based on a limited number of sensitivity experiments [e.g., Kennedy and O’Hagan, 2001; Higdon et al., 2004].

[17] Here we take a different approach by keeping the physics-based model within a stochastic sampling algorithm, but relying on innovations in our sampling strategy for efficiently identifying optimal solutions and quantifying uncertainties. This alternate strategy is particularly useful with
3.2. Ocean Model

[19] The present work considers a box model of the Atlantic Ocean (2 Hemispheres). The fluxes of heat, salt, and mass between different oceanic boxes are parameterized. A whole hierarchy of box models has been used to explore aspects of the MOC [Thual and McWilliams, 1992]. Here we use a model that can exhibit several modes of the MOC broadly corresponding to some of the multiple equilibria found in ocean general circulation models [e.g., Manabe and Stouffer, 1988; Marotze and Willebrand, 1991].

[20] The model includes (1) two high-latitude boxes (one in each hemisphere) and (2) two low-latitude boxes, one for the warm thermocline water and one for the cold subthermocline water (Figure 1). The effects of the Southern Ocean on the Atlantic’s MOC are not included (see section 5.3 for an investigation of these effects). Details about the box model are reported in Appendix B.

[21] Several versions of a box model are considered with different parameterizations of vertical volume transport between the two low-latitude boxes. In general, this vertical volume transport \( q_m \) is a function of the density contrast between the two boxes \( \rho_3 - \rho_2 \) and the depth of the thermocline \( D \),

\[
q_m = q_{mv} \left( \frac{\rho_3 - \rho_2}{\rho_1 - \rho_2} \right)^\zeta \left( \frac{D}{h} \right)^\eta.
\]

Physically this relationship means that the mass flux that is required to maintain an advective–diffusive equilibrium is smaller if there is a greater density contrast and/or larger depth to the thermocline. The latter relationship implies less diffusive mass fluxes with deeper, more gradual density gradients. The exponents \( \zeta \) and \( \eta \) in equation (3) can be set to represent different mixing assumptions [Nilsson and Walin, 2001]. For instance \( \zeta = 0, \eta = 1 \) represents constant mixing, \( \zeta = 0.5, \eta = 0.5 \) represents a stability–dependent mixing, and \( \zeta = 1, \eta = 1 \) represents constant mixing energy. The model parameters are listed in Table 1.
Table 1. Parameters of the 4-Box Model Ocean

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_i)</td>
<td>area box 1</td>
<td>(0.8 \times 10^{13} \text{ m}^2)</td>
</tr>
<tr>
<td>(A_2)</td>
<td>area box 2</td>
<td>(6.4 \times 10^{13} \text{ m}^2)</td>
</tr>
<tr>
<td>(A_3)</td>
<td>area box 3</td>
<td>(6.4 \times 10^{13} \text{ m}^2)</td>
</tr>
<tr>
<td>(A_4)</td>
<td>area box 4</td>
<td>(6.4 \times 10^{13} \text{ m}^2)</td>
</tr>
<tr>
<td>(A_o)</td>
<td>area of global ocean</td>
<td>(3.61 \times 10^{14} \text{ m}^2)</td>
</tr>
<tr>
<td>(D_i)</td>
<td>initial depth box 1-4</td>
<td>(500 \text{ m})</td>
</tr>
<tr>
<td>(H)</td>
<td>depth of ocean</td>
<td>(4000 \text{ m})</td>
</tr>
<tr>
<td>(V_j)</td>
<td>volume box 1-4</td>
<td>(3.2 \times 10^{16} \text{ m}^3)</td>
</tr>
<tr>
<td>(V_i)</td>
<td>volume box 1-4</td>
<td>(9.6 \times 10^{15} \text{ m}^3)</td>
</tr>
<tr>
<td>(T_{pi})</td>
<td>temperature of bath for poles</td>
<td>(0\degree C)</td>
</tr>
<tr>
<td>(T_e)</td>
<td>temperature of bath for equator</td>
<td>(30\degree C)</td>
</tr>
<tr>
<td>(k)</td>
<td>thermal exchange velocity</td>
<td>(1.7 \times 10^{-6} \text{ m s}^{-1})</td>
</tr>
<tr>
<td>(q_{ref})</td>
<td>reference volume flux box 1-4</td>
<td>(6 \text{ Sv})</td>
</tr>
<tr>
<td>(q_{me})</td>
<td>reference volume flux box 4</td>
<td>(6 \text{ Sv})</td>
</tr>
<tr>
<td>(q_{mv})</td>
<td>reference vertical mixing flux</td>
<td>(12.2 \text{ Sv})</td>
</tr>
<tr>
<td>(\phi_1)</td>
<td>NH atmospheric moisture flux</td>
<td>(0.1 \text{ Sv})</td>
</tr>
<tr>
<td>(\phi_2)</td>
<td>SH atmospheric moisture flux</td>
<td>(0.1 \text{ Sv})</td>
</tr>
<tr>
<td>(T_{ps})</td>
<td>initial temperature box 1-4</td>
<td>(2.75\degree C)</td>
</tr>
<tr>
<td>(S_{is})</td>
<td>initial salinity box 1-4</td>
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</tr>
<tr>
<td>(\rho_0)</td>
<td>mean ocean density</td>
<td>(1028 \text{ kg m}^{-3})</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>thermal expansion coefficient</td>
<td>(1.5 \times 10^{-3} (\degree C)^{-1})</td>
</tr>
<tr>
<td>(\beta)</td>
<td>salinity expansion coefficient</td>
<td>(0.8 \times 10^{-3})</td>
</tr>
<tr>
<td>(dt)</td>
<td>time step</td>
<td>(1.58 \times 10^7 \text{ s})</td>
</tr>
</tbody>
</table>

3.4. Cost Function

The stochastic inversion searches for freshwater forcing scenarios that minimize a cost function with two contributions. A first contribution quantifies the mismatch between predicted and observed sea surface temperatures in the polar region in the North Atlantic. A second contribution ensures that sea level is not appreciably altered by the end of the experiment. The reason for the latter constraint is discussed below. The ocean model predicts a volume-average temperature for each box. We presume that the fluctuations of ice \(\delta^{18}O\) in the Greenland record [Grootes and Stuiver, 1997] are related in a simple, linear way to variations in sea surface temperature in the polar region in the North Atlantic (i.e., to temperature in box 1). This is likely only qualitatively correct [Lindas et al., 2004; Huber et al., 2006]. To be specific, we presume that a 15°C warming above Greenland [e.g., Lindas et al., 2004] corresponds to a 4°C warming of the northern polar ocean box [Bond and Lotti, 1995]. The cost function is thus defined as,

\[
E(m) = \frac{1}{1101} \sum_{i=1}^{1101} \left( \frac{(T_i(t) - T_{\text{ps}}) - (T_{\text{obs}}(t) - T_{\text{obs}}))}{(1\degree C)^2} + \frac{(\Delta \text{MSL})^2}{(30 \text{ m})^2} \right).
\]

Here \(T_{\text{ps}}\) is the model temperature in the northern polar box and \(T_{\text{obs}}\) is the temperature interpreted from the Greenland

Figure 2. Equilibrium solutions of the 4-box ocean model. The meridional overturning circulation in the Northern Hemisphere (\(q_1\)) is shown as a function of the meridional moisture transport in the atmosphere (\(\phi_1\)). Arrows indicate the direction of hysteresis. The different lines refer to different assumptions about vertical buoyancy transport: constant vertical mixing (solid line), stability dependent mixing (short dashed line), and constant mixing energy (long dashed line).
record. The overbar indicates a time average (1101 evenly spaced points between 30 and 39 kyr B.P.). Note that we have not smoothed the Greenland temperature estimates. The error in making this model-data comparison has an assumed uncertainty of 1°C. Finally, ΔMSL is the change in mean sea level at 30 kyr B.P.

[25] The second contribution to the cost (equation (4)) constrains the net change in mean sea level by the end of the inversion to ±30 m, although most solutions considered with that constraint are within a few meters of zero change. This choice reflects an interpretation of sea level reconstructions which show that the continental ice sheets were not growing or shrinking appreciably between 30 and 39 kyr B.P. This constraint helps the inversion reject unrealistic freshwater forcing scenarios. Indeed, we found that without this constraint, the inversion preferentially selects solutions that require global ice volume to grow and shrink by more than the equivalent of 120 m sea level on millennial time scales. Note that this constraint does not specify what sea level must do between the beginning and end of the experiment. It merely specifies that all changes must approximately cancel.

3.5. Freshwater Forcing

[26] The stochastic inversion estimates a time series of freshwater forcing \( ff \) at 81 evenly spaced intervals from 39 to 30 kyr B.P. A continuous time series is made by linear interpolation between these points. Candidate values of \( ff \) are selected from a uniform “prior” distribution with a minimum of \(-0.3 \text{ Sv}\) and maximum of \(0.3 \text{ Sv}\) \((1 \text{ Sv} = 10^6 \text{ m}^3 \text{ s}^{-1})\). As discussed in section 4, the choice of this range does matter. The stochastic inversion takes a random step between these minimum and maximum values for each of the 81 unknown forcing values, with a variable average step size that is determined by the MVFSA algorithm. This step size gradually becomes smaller through the course of the inversion. Although the freshwater forcing values are selected individually, their effect on the model is only evaluated as a whole through the impact of the time series on the cost function. This selection process takes into account possible nonlinear dependencies in the selection of the best candidate solutions.

3.6. Solution Averaging

[27] The inversion consists of 400 independent attempts to minimize the cost function, with each attempt involving approximately 3600 experiments. The best performing experiment in each attempt is then systematically averaged together, only combining experiments that result in a further reduction in the cost function. This process tends to reduce the noise or random component that may exist in each of the individual solutions. There were multiple groups of solutions that were identified through this selection and averaging process. Within each attempt to minimize the cost function, we selected the solution with the smallest cost value for analysis.

4. Results

[28] A set of three freshwater forcing scenarios are obtained through inversion of the Greenland paleotemperature record, one for each configuration of the 4-box model representing constant vertical mixing, stability-dependent mixing, and constant mixing energy (Figure 3b). These forcing scenarios modify the meridional overturning circulation and poleward heat transport in such a way as to recreate features observed in the Greenland record (Figure 3a). The periods of sustained warmth correspond to periods of (1) freshwater removal from the ocean (growth of continental ice sheets) and (2) increased meridional flow in the NH \( (q_1) \), and (3) decreased meridional flow in the SH \( (q_4) \) (Figure 3c).

[29] Uncertainties in vertical mixing, which had such a dramatic effect on the equilibrium MOC (Figure 2), have very little effect on the solutions that reproduce the transient sequence of abrupt climate change (Figure 3). Section 5.1 summarizes the reasons why different assumptions about vertical buoyancy transport are not important to these solutions.

[30] A time integral of the freshwater forcing obtained from the inversion solutions provides predictions of global sea level anomalies that we assume are related to the decay or growth of the NH ice sheets. Again, the differences in mixing formulation make little difference to the predicted sea level anomalies (Figure 4). The sea level anomalies obtained from the three freshwater forcing scenarios are compared against the three independent estimates of sea level change from 39 to 30 kyr B.P. based on corals and \( \delta^{18}O \) records (Figure 4). The feature that is in broad agreement among the modeled and observed sea level anomalies is the initial drop of sea level of 15 to 20 m. It corresponds to the longest period of sustained warmth in the Greenland paleotemperature record following Heinrich event 4 at 39 kyr B.P. According to our inversion, the subsequent interstadial events with shorter duration can be explained by smaller variations in sea level (less than 5 m). The general relationship that emerges is that the duration of Greenland warmth is approximately proportional to the amount of ice sheet growth (or sea level fall), with ratios ranging from 112 years to 167 years sustained warmth for every meter of sea level fall. Given the uncertainties in the sea level data, we cannot draw firm inferences about model-data disagreement for the shorter DOI events. Given the crudeness of our ocean model, the level of agreement between the various estimates of sea level and the model predictions is perhaps surprisingly good. The agreement is independent of the uncertainties in vertical mixing formulation as indicated by the relatively minor differences among the three modeled curves of sea level anomaly (Figure 4).

[31] In another inversion (not shown) we select candidate freshwater forcing values from a wider range of possible values \((-0.4 \text{ Sv} \text{ to } 1.3 \text{ Sv})\). The inversion in this case tends to favor temperature jumps on the order of 9°C (not the 4°C jumps that we imposed as the “target”) and required sea level variations on the order of 60 m. The ocean circulation remains equatorially asymmetric. In contrast, the solutions with the smaller \((-0.3 \text{ Sv} \text{ to } 0.3 \text{ Sv})\) freshwater forcing (Figure 3) come mainly from the equatorially symmetric circulation state (sinking at both poles). We infer that a threshold exists for freshwater forcing values in excess of 0.3 Sv that makes it difficult for the inversion to identify the symmetric-only solutions. We know from equilibrium experiments that the asymmetric states are particularly stable. With regards to the realism of the symmetric solutions, they were required to
obtain a match to the observational data, which, admittedly, could be occurring for the wrong reasons. On the other hand, very little is known of the circulations and balances that close the global ocean circulation. The symmetric solutions allow a kind of seesaw response between northern and southern sinking rates which is consistent with higher-dimensional OGCMs [Knutti et al., 2004].

5. Discussion

5.1. Insensitivity to Formulation of Vertical Mixing

Vertical mixing at low latitudes is important to setting the stability thresholds of the meridional overturning circulation in ocean models [e.g., Prahl et al., 2003; Dijkstra and Weijer, 2005]. However, these results mainly pertain to the equilibrium states of the MOC. The insensitivity of our inversions to vertical mixing formulation highlights one of the primary shortcomings of scaling arguments derived for the steady state and of inferences drawn from model experiments on equilibrium response of MOC to slowly evolving freshwater forcing. Indeed, the hysteresis obtained from such experiments has been used as a conceptual model of abrupt climate change (e.g., Figure 2). The MOC, however, can have a transient response to a sudden change in freshwater forcing that is quite different from its equilibrium response. Moreover, the stability thresholds may depend on the rate of change in freshwater forcing in addition to its amplitude [e.g., Manabe and Stouffer, 1994; Marchal et al., 1999b; Lucarini and Stone, 2005a, 2005b].

To illustrate this point, Figure 5 shows the adjustment to equilibrium of the ocean model to a 0.3 Sv step increase in freshwater forcing that starts at year 500. The initial model state is an asymmetric circulation with northern sinking. Initially all versions of the model, no matter which formulation of vertical mixing is assumed, show decreasing overturning strength in the Northern Hemisphere (Figure 5a). Only after ∼600 years do the different mixing assumptions lead to divergent responses. The increase in overturning strength with increased freshwater forcing, which occurs for the stability-dependent and fixed mixing energy assumptions, does not occur until 500 years after the freshwater forcing is initiated (Figure 5a).

The above results are interpreted as follows. With the reduction in overturning strength, less mass is exported from the surface equatorial box and the thermocline quickly deepens (Figure 5b). There is virtually no difference in the time evolution of thermocline depth among the different

Figure 3. Inversion solutions. (a) Comparison of the temperature of the polar northern box (black lines) with the sea surface temperature inferred from the Greenland record (gray lines) [Grootes and Stuiver, 1997]. (b) The changes in surface freshwater forcing for the polar northern box. (c) The changes in the northern (black) and southern (gray) components of the ocean’s meridional circulation. (d) The changes in thermocline depth. The solid, short, and long dashed lines correspond to different assumptions about vertical mixing: constant vertical mixing (solid line), stability dependent mixing (short dashed line), and constant mixing energy (long dashed line).
mixing assumptions until after 1000 years of the initiation of freshwater forcing. The similarity of the initial response of the model for all mixing assumptions is due to the fact that the adjustment is dominated within the first 500 years by changes in the horizontal flow in the NH \(q_1\), with virtually no change in vertical flux at low latitudes \(q_{vm}\). It is this latter flux that leads to different equilibrium sensitivities for the different assumptions. The vertical flow \(q_{vm}\) responds slowly because the effect of a deepening thermocline (from reduced horizontal flow \(q_1\)) (Figure 5b) counteracts the effect of a reduced density stratification in the initial stage of the adjustment (Figure 5c). After \(\sim 500\) years, however, the latter effect overcomes the former for the cases with stability-dependent mixing and constant mixing energy. The model then approaches the “boosting” equilibrium state [Nilsson and Walin, 2001].

[35] The above analysis pertains to a comparison of results for the asymmetric circulation mode with northern sinking. Thus, one may question its applicability to the solutions in Figure 3, which are mostly for the equatorially symmetric circulation state. We find, however, that the different mixing assumptions have little impact on inversion solutions that require transitions among asymmetric circulation states (not shown). In particular, if we change the inversion target to find solutions for a larger temperature jumps of 9°C (instead of 4°C), requiring the model to transit among its asymmetric states, nearly identical freshwater forcing scenarios are found.

5.2. Role of Ice Sheets

[36] The presumption has been that ice sheets were mainly important to abrupt climate change as a source of freshwater [Broecker et al., 1990; MacAyeal, 1993; Clark et al., 2002]. This is logical given their main contribution to the geologic record is in the form of ice rafted debris observed during the cold phases between DOIs [Bond et al., 1993; Alley and MacAyeal, 1994; Bond and Lotti, 1995]. Our inversion results, however, emphasize the importance of ice sheet growth to sustaining the observed warmth of the DOIs. According to our results, ice sheet collapse is important for shutting down the MOC. But without the subsequent ice sheet growth and reduced freshwater input to the ocean, the MOC would not be able to prolong the warmth at high northern latitudes. Thus, the longer the warmth the larger the predicted drop in global sea level. The coral and \(\delta^{18}O\) records of sea level change from 39 to 30 kyr B.P. tend to support this inference: they consistently show \(\sim 15\) to 20 m drop after the Heinrich event 4 at 39 kyr B.P., with smaller variations during the subsequent series of shorter DOIs.

[37] Heinrich event 5, at around 46 kyr B.P., was followed by a \(\sim 2\) kyr long warming similar to Heinrich event 4 described above [Grootes et al., 1993; Meese et al., 1997]. There is very little change in the deep-sea \(\delta^{18}O\) record [Waelbroeck et al., 2002], which may be a consequence of the

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**Figure 4.** Comparison between sea level anomalies inferred from the Greenland record and estimated from (a) coral records [Thompson and Goldstein, 2006], (b) \(\delta^{18}O\) records from deep-sea sediments [Waelbroeck et al., 2002], and (c) \(\delta^{18}O\) records from Red Sea sediments [Siddall et al., 2003]. The Red Sea data have been smoothed by a three-point running mean. The uncertainty of \(\pm 5.5\) m (1σ) indicated for the Red Sea data reflects a smoothing of about 500 years which removes some of the intersample variability [Siddall et al., 2003; Rohling et al., 2004]. The solid, short, and long dashed lines correspond to different assumptions about vertical mixing: constant vertical mixing (solid line), stability dependent mixing (short dashed line), and constant mixing energy (long dashed line).
data smoothing applied to this record. Although existing coral records are not ideal to identify the amplitude of sea level change between 48 and 39 kyr B.P., they do suggest the possibility of a 30 m drop in sea level between 48 and 44 kyr B.P. (near the end of the interstadial warming). The sea level record from the Red Sea [Siddall et al., 2003] (not shown) is consistent in timing and amplitude with features in the Greenland δ18O record following Heinrich events 4 and 5. A recent review of sea level estimates generally supports our primary finding that periods of sustained warmth are correlated with falling sea levels [Siddall et al., 2008].

[38] One of the important implications of our results is the suggestion that abrupt climate change appears to be more of a forced phenomenon than one that is triggered and maintained by the existence of multiple circulation states in the ocean. Given the uncertainties in sea level data, we can only make this inference for DOI 8 (starting at 38 kyr B.P.) and not necessarily for DOIs 7, 6, and 5 that can be explained by much smaller changes in freshwater forcing. A different and intriguing interpretation of the role of ice melt is to regulate the periodicity of DOI events undergoing self-sustained relaxation oscillations with less ice melt corresponding to longer duration interstadial events [Schulz et al., 1999; Schulz 2002; Schulz et al., 2002; Timmermann et al., 2003; Sima et al., 2004]. Because the 4-box model does not exhibit internal instability behavior, we cannot test this hypothesis without building upon or altering the model to include this possibility.

[39] Our interpretation is also supported by several other studies that require a sustained reduction of the hydrologic balance of the North Atlantic to explain interstadial warmth. The AOGCM modeling work of Knutti et al. [2004] summarize the relationship between modeled Greenland temperature and anomalous freshwater forcing to the North Atlantic Ocean with a fairly simple relationship $T_n \sim \tanh(\delta \cdot ff)$ and was able to fit a “thermal-freshwater seesaw” conceptual model to simultaneously connect Greenland and Antarctic temperature anomalies with benthic δ18O sea level proxy record between 60 to 25 kyr B.P. While the 4-box model does not have as simple a relationship between temperature anomalies in the Northern box (1) and freshwater flux, the differences do not make a lot of difference to the implied sea level anomalies (see auxiliary material Figure S1). 1 Liu et al. [2009] was only able to simulate the Greenland record of the Bolling-Allerod warming around $\sim 14.5$ kyr B.P. with a large and sudden reduction in freshwater supply to the North Atlantic, which also allowed favorable comparison with proxy records from the Cariaco Basin, Iberian Margin, and Antarctica.

[40] The amount of ice melt during Heinrich event 4 as inferred from surface ocean oxygen isotopes is the equivalent

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1Auxiliary materials are available in the HTML. doi:10.1029/2010PA001936.

Figure 5. Adjustment of the 4-box ocean model to a 0.3 Sv step increase in poleward moisture transport ($\phi_i$) starting at year 500 and maintained for the duration of the experiment. Shown are the anomalies relative to the values prior to year 500. (a) The northern sinking volume flux ($q_1$). (b) The depth of the thermocline ($D_i$). (c) The bottom (box 3) to surface (box 2) density difference ($\rho_3-\rho_2$). The solid, short, and long dashed lines correspond to different assumptions about vertical mixing: constant vertical mixing (solid line), stability dependent mixing (short dashed line), and constant mixing energy (long dashed line).
of (2 ± 1) m [Roche et al., 2004], an amount that is roughly consistent with the amounts associated with observed IRD thicknesses [Alley and MacAyeal, 1994]. While these results are consistent with our results in that little or no freshwater forcing was required to shut down the MOC at the beginning, presumably it is the collapse of an ice sheet that allows for an extended period of ice growth. Thus it is not clear to us how we could then explain a sustained 15 to 20 drop in sea level that seems to be required by multiple sea level records without a larger collapse of the ice sheet during the Heinrich event. This question could be better addressed with a coupled climate-ice sheet model.

5.3. Sensitivity to Modeling Assumptions

[41] The Nilsson and Walin [2001] scaling arguments that form the basis of the 4-box model presents a set of hypotheses for the balance of buoyancy and vertical mixing processes that govern the strength of the MOC. At the request of a reviewer, we present the results of two additional experiments that test the effects of different sets of modifications to the model that explore alternate hypotheses for processes that govern the global MOC. Given the specialized derivation of the 4-box model, it is not entirely clear that these modifications make the model more realistic. Nevertheless, the experiments present an opportunity to use paleoclimate proxy archives to evaluate the plausibility of different modeling assumptions.

5.3.1. Southern Ocean

[42] Our two-hemisphere box model is equatorially symmetric and does not represent the Southern Ocean. In order to test the potential importance of the Southern Ocean on our results, we parameterized its dynamical influences according to the scaling arguments presented by Gnanadesikan [1999] [see also Saenko and Weaver, 2003]. It is important to emphasize, however, that the role of the Southern Ocean to powering the Atlantic’s overturning circulation is an open question [Bugnion et al., 2006a, 2006b; Kuhlbrodt et al., 2007].

[43] Following Gnanadesikan [1999], the net volume flux \( q_4 \) from the Southern Ocean to the equator would be driven by zonal wind stress and transient eddies. With the parameterization of the dynamical influences of these effects in our model, we found that the MOC tends to be stabilized, such that the model becomes fairly insensitive to freshwater forcing at high latitudes in the NH (see auxiliary material Figure S2). In particular the model lacks a symmetric circulation state that was important to the inversion solutions discussed previously. The stochastic inversion fails to find any freshwater forcing scenario that replicates the Greenland paleotemperature record and does not come close to predicting observed changes in sea level (Figure 6).

[44] These results may be taken to mean that inferences of freshwater forcing as attempted here can depend strongly on

Figure 6. Inversion solutions for the 4-box model with a representation of the Southern Ocean. (a) Comparison of the model solutions for temperature for the polar northern temperature (black lines) to the scaled Greenland record (gray lines) [Grootes et al., 1993; Meese et al., 1997]. (b) The corresponding solutions for MOC strength [Sv]. (c) Comparison of sea level anomalies inferred from the Greenland record and estimated from \( \delta^{18}O \) records from the Red Sea [Siddall et al., 2003]. The solid and dashed lines correspond to different assumptions about vertical mixing: constant vertical mixing (solid line) and stability dependent mixing (dashed line).
the treatment of the Southern Ocean. Clearly, this assertion would need to be tested with more realistic ocean models.

5.3.2. High-Latitude Convection

Our model lacks a series of potential feedbacks introduced by the stratification when the polar box is flooded with freshwater, such as an increase in sea ice cover, a reduced heat flux from the ocean to the atmosphere, and an increase of precipitation over evaporation, all of which reinforce a polar halocline catastrophe [Welander 1982; Marotzke and Willebrand, 1991]. While the model represents a kind of polar halocline catastrophe, it does so largely without any of these other feedbacks.

The box model is extended to represent polar stratification by subdividing each polar box into a surface box (of 500 m depth) and a deep box (3500 m depth) [Welander, 1982]. When the density difference between the surface and deep boxes results in stable stratification, no mixing is specified between these boxes. In contrast, when density in the surface box exceeds that of the deep box, a bidirectional volume flux is prescribed at a fixed rate of 5 Sv (we also tested the results with a fixed rate of 1 Sv). All other aspects of the model remain unchanged.

With polar stratification inversion solutions indicate that a much larger amount of freshwater (equivalent to ~70 m of sea level for DOI 8) is required to force abrupt transitions in meridional overturning circulation as compared to the 4-box model and sea level records (see auxiliary material Figure S3). Equilibrium sensitivity of the MOC to freshwater forcing indicates the absence of equatorially symmetric solutions that were so important to the 4-box model solutions. Thus, processes at high latitudes (Southern Ocean or polar stratification) lead to significantly different inferences about freshwater forcing from the Greenland record. However, whether these processes can aptly be represented in a box model is unclear.

5.3.3. Higher-Dimensional Model

The fluxes of mass, heat, and salt are represented very crudely in an ocean box model. The extent to which this crudeness influences our results, in particular our estimate of freshwater forcing, warrants therefore further consideration. In an effort to assess the accuracy of our results, the freshwater forcing derived from the 4-box model inversion of the Greenland data (e.g., Figure 3b) is used as boundary conditions for a more physically realistic, zonally averaged climate model [Schmittner and Stocker, 2001]. This model comprises three components: (1) an ocean circulation model with three basins (Atlantic, Pacific, and Indian Oceans) connected to a Southern Ocean [Wright and Stocker, 1992], (2) a zonally averaged energy and moisture balance model of the atmosphere [Stocker et al., 1992], and (3) a thermodynamic model of sea ice [Wright and Stocker, 1993]. After tuning, the model is able to capture important features of observed modern climate such as the seasonal variations in surface air temperature, the meridional circulation in different oceanic basins, and the meridional heat and moisture fluxes in the ocean and atmosphere [Schmittner and Stocker, 2001; Stocker et al., 1992].

We find that the climate model, when forced with the freshwater water anomalies deduced from the combination of Greenland data and the box model, reproduces the
paleotemperature record from the Greenland ice core to a reasonable degree (Figure 7).

This result provides some additional support to the relevance of the box model to interpret the Greenland data. Use of the zonally averaged model for the purpose of inferring freshwater forcing from these data is the subject of a separate study (S. Lu et al., Consistent observational and modeling support for ice sheet forcing of abrupt climate change, submitted to *Paleoceanography*, 2010).

6. Conclusions

[51] Bayesian stochastic inversion is used to test the freshwater forcing hypothesis of abrupt climate change. A time series of freshwater flux anomaly in the North Atlantic Ocean between 30 and 39 kyr B.P. is inferred by combining a paleotemperature record from a Greenland ice core with an ocean box model. Reconstructions of global sea level from independent geological records are then compared to the sea level changes implied by these anomalies. The comparison gives some support to the freshwater forcing hypothesis for the time interval from 30 to 39 kyr B.P. The largest drop in sea level present in observations and in our inversion is associated with ice sheet growth which sustained freshwater removal from the ocean and enhanced meridional circulation and heat flux to maintain the observed warmth during the long (~2.5 kyr) DO18. According to our results, the longer the period of warmth the larger the drop in sea level. Thus, both our results and observations show an approximately 15 to 20 m drop in sea level following Heinrich event 4 at 39 kyr B.P., the feature of greatest commonality.

[52] We have considered three fundamentally different assumptions about vertical buoyancy transport at low latitudes corresponding to constant vertical mixing, stability-dependent mixing, and fixed mixing energy. These different assumptions produce, for the steady state, qualitatively different sensitivities of meridional mass transport to freshwater forcing, including the possibility that an increase in freshwater forcing leads to an increase in the transport. Our inversion results, however, are robust against these assumptions. Thus, the sensitivity to the nature of vertical mixing is most apparent for circulation states at equilibrium and much less so for the adjustment of circulation to time-dependent freshwater forcing.

[53] Some of the major limitations of this work should be stressed. For example, it is assumed that the Greenland paleotemperature record reflects changes in oceanic poleward heat transport and that an equatorially symmetric 4-box ocean model can represent this transport adequately. More realistic ocean models would be required to test other potentially important assumptions such as the treatment of the Southern Ocean and polar stratification.

Appendix A

[54] The objective of stochastic inversion using Bayesian inference is to estimate a joint probability for selecting a set uncertain parameters (e.g., freshwater forcing) that allow a process model to approximate observations within the uncertainty of the observations and model. The result is known as a posteriori probability density function (PPD).

\[
P_{PPD}(m|d_{obs}, g(m)) \propto \exp \left[ -\frac{1}{2} (g(m) - d_{obs})^T C_{noise}^{-1} (g(m) - d_{obs}) \right] \cdot \text{prior}(m)
\]

(A1)

[55] The error covariance matrix \( C_{noise} \) quantifies the uncertainty in comparing model output to data and \( \text{prior}(m) \) represents prior probability for the freshwater forcing (i.e., a uniform probability from ~0.3 to 0.3 Sv) independent of a new set of observations \( d_{obs} \) represented here by the scaled Greenland ice core proxy data. The quantity within the square brackets is typically referred to as the “cost” function or metric of errors in the model relative to what is observed with superscript \( (T) \) referring to the matrix transpose. Even with the Gaussian assumption for observational and modeling uncertainties, the PPD is generally not Gaussian because of nonlinearities represented in the ocean model \( g(m) \).

[56] Equation (A1) can only be estimated stochastically by selecting alternate freshwater forcing time series from the \( \text{prior}(m) \) distribution, simulating the combined effects of these parameters within the ocean model, and estimating the likelihood of this selection by evaluating the cost function specified by equation (4) in section 3.4. In general this requires a large number of experiments.

[57] We use Multiple Very Fast Simulated Annealing (MVFSAs) to select uncertain freshwater forcing time series [Sen and Stoffa, 1996; Jackson et al., 2004]. The rules for selecting samples is similar to a Metropolis/Gibbs sampler insofar as candidate parameter set values are either accepted or rejected (for stepping through parameter space) in proportion to a probability

\[
P = \exp \left( -\frac{\Delta E}{T_k} \right).
\]

(A2)

where \( \Delta E \) is the change in the metric of model-data discrepancies, also called the “cost function,” for going from a model with parameter set values \( m_k \) to model with parameter set values \( m_{k+1} \). The selection of model parameters given an initial selection \( m_1 \) within MVFSAs are chosen such that

\[
m_j^{k+1} = m_j^k + y_j (m_j^{\text{max}} - m_j^{\text{min}}),
\]

(A3)

\[
y_j \in [-1,1],
\]

(A4)

and

\[
m_j^{\text{min}} \leq m_j^{k+1} \leq m_j^{\text{max}}
\]

(A5)

where \( y_j \) is generated according to a Cauchy distribution

\[
y_j = \text{sgn}(\text{RND - 0.5}) T_k \left( 1 + \frac{1}{T_k^2} \right)^{\lfloor \text{RND - 1} \rfloor} - 1.
\]

(A6)
Within equations (A3)–(A6), subscript \((i)\) is the parameter number, \((k)\) is the iteration number, \(RND\) is a random number generator with a uniform distribution between 0 and 1, and \(\text{sgn}\) is the sign operator. The cooling schedule at iteration \((k)\) is

\[
T_k = T_o \exp\left(-0.9(k - 1)^{1/2}\right). \tag{A7}
\]

\(T_o\) is the initial annealing temperature and is equal to 3.5. The acceptance criterion for successive model selections is the same as for the Metropolis rule. The sampling process is stopped if successive iterations fail to reduce the cost function after a predetermined number of experiments. This convergence criterion is 5.5 times the total number of uncertain parameters [Jackson et al., 2004]. This algorithm has been applied to estimate the uncertainty in selecting parameters important to climate sensitivity within an Atmospheric GCM [Jackson et al., 2004, 2008; Jackson, 2009] as well as processes affecting the exchange of energy and moisture at the land surface [Jackson et al., 2003].

Appendix B

[58] The formulation of a two-hemisphere 4-box model is given here. The box volumes are (for a definition of symbols, see Table 1):

\[
V_1 = A_1H, \tag{B1}
\]

\[
V_2 = A_2D, \tag{B2}
\]

\[
V_3 = A_2(H - D), \tag{B3}
\]

and

\[
V_4 = A_4H. \tag{B4}
\]

These volumes can vary with time when the depth of the thermocline \(D\) or the total depth of the ocean \(H\) changes (Figure 1). The equations governing the conservation of heat, salt, and mass for the case when there is sinking in the northern box 1 \((q_1 > 0)\) and upwelling in the southern box 4 \((q_4 < 0)\) (asymmetric circulation) are:

\[
\frac{dT_1}{dt} = \frac{1}{V_1} \left[ q_1(T_2 - T_1) + \kappa A_1(T_p - T_1) \right] \tag{B5}
\]

\[
\frac{dS_1}{dt} = \frac{1}{V_1} \left[ [S_2 - S_1] - \varphi_1 + \varphi_f]S_1 \right] \tag{B6}
\]

\[
\frac{dT_2}{dt} = \frac{1}{V_2} \left[ -q_4(T_4 - T_2) + q_m(T_3 - T_2) + \kappa A_2(T_p - T_2) \right] \tag{B7}
\]

\[
\frac{dS_2}{dt} = \frac{1}{V_2} \left[ -q_4(S_4 - S_2) + q_m(S_3 - S_2) + \varphi_1 + \varphi_f]S_2 \right] \tag{B8}
\]

\[
\frac{dT_3}{dt} = \frac{1}{V_3} \left[ (q_1 + \varphi_f)(T_1 - T_3) \right] \tag{B9}
\]

\[
\frac{dS_3}{dt} = \frac{1}{V_3} \left[ (q_1 + \varphi_f)(S_1 - S_3) \right] \tag{B10}
\]

\[
\frac{dT_4}{dt} = \frac{1}{V_4} \left[ -q_4(T_3 - T_4) + \kappa A_4(T_p - T_4) \right] \tag{B11}
\]

\[
\frac{dS_4}{dt} = \frac{1}{V_4} \left[ -q_4(S_3 - S_4) - \varphi_4S_4 \right]. \tag{B12}
\]

The vertical volume flux \(q_m\) only acts to move mass from the subthermocline box (box 3) into the thermocline box (box 2) by redefining the depth of the thermocline \(D\). This allows the exchange to be unidirectional while conserving mass and heat in the process.

[59] The heat exchange between the equatorial or polar boxes and the atmosphere is proportional to the temperature difference with respect to reference values \(T_e\) and \(T_p\), respectively. The value for the thermal relaxation velocity \(\kappa = 1.7 \times 10^{-6} \text{ m s}^{-1}\) implies a surface heat loss of 6.6 W m\(^{-2}\) for each degree difference (the heat loss/gain per degree change in a m\(^2\) column is \(\rho C_p \kappa = 1000 \text{ kg m}^{-3} \cdot 3850 \text{ J kg}^{-1} \text{C}^{-1} \cdot 1.7 \times 10^{-6} \text{ m s}^{-1} = 6.6 \text{ W m}^{-2} \text{C}^{-1}\)). This value is smaller than the values of 10 to 80 W m\(^{-2}\)C\(^{-1}\) that are typically associated with this parameter [e.g., Chu et al., 1998]. However, it is consistent with other box-type “climate” models [Scott et al., 1999; Lucarini and Stone, 2005a, 2005b]. We also neglect to include in the heat balance for the northern polar box the heat that would be required to melt icebergs that likely accompany Heinrich events. One needs approximately 0.03 petawatts of heat to melt 0.1 Sv of ice mass being discharged into the ocean. The total poleward heat being transported by the modeled ocean is 1 petawatt.

[60] In equation (B6), we choose to impose negative freshwater forcing anomalies to the northern box in the same way as positive anomalies. Because this choice was confusing to more than one reviewer, we discuss our thinking at length here: Note that freshwater fluxes \((f_f)\) are anomalies on top of the atmospheric moisture flux \(\varphi_f\). Thus, in most cases, the total flux is still positive, and much reduced during times of ice growth and mimics the variations in river discharge one may find through a glacial cycle [Marshall and Clarke, 1999]. This choice is consistent with the treatment taken by a number of other authors who have considered ice sheet mass balance influence on the MOC circulation [Wang and Mysak, 2001; Wang and Mysak, 2002; Knutti et al., 2004; Liu et al., 2009]. Most treatments of ice sheet forcing of climate focus on periods of enhanced ice sheet discharge/melt and are usually imposed in their models through enhanced river discharge or placed directly across a latitude band of the Atlantic Ocean [e.g., Liu et al., 2009]. Moreover, the Liu et al. [2009] experimental design does not include an ice sheet model. Its land hydrology scheme, therefore, does not account for the possibility that less modeled precipitation might end up discharging into rivers through its accumulation within an ice sheet or a proglacial lake (Z. Liu et al., personal communication, 2010). Far fewer studies have considered the representation of the coupled ice sheet-ocean system during periods of ice growth [Marshall and Clarke, 1999; Wang and Mysak, 2001, 2002]. In these examples, precipitation was
used to grow land ice mass at the expense of river discharge rates. Knutti et al. [2004] considered the effects of ice sheet growth and decay on Stage 3 climate through anomalous positive freshwater fluxes independent of the model’s hydrologic cycle. However, Knutti et al. [2004] interpreted the mean value of the imposed freshwater fluxes during their Stage 3 coupled AOGCM experiment to represent the hydrologic cycle and climate that would maintain a steady ice volume. They interpreted periods of reduced freshwater flux as periods of ice sheet growth and falling sea level in order to compare their results to the observational data (their Figure 3). They argue that because significant uncertainties exist concerning the freshwater budget of the Atlantic basin, it is justifiable to shift the zero level in the freshwater flux in order to account for periods of ice growth. Knutti et al. [2004] acknowledged that this choice would affect their model’s AMOC thresholds to freshwater forcing, but estimated this detail would not affect their conclusions.

[62] The rate of change of the thermocline depth is

$$\frac{dD}{dt} = \frac{qw - q_1 - q_4 - q_3 - q_4}{A_2}$$  

where the vertical volume flux $qw$ balances the horizontal volume fluxes $q_1, q_4$, and the atmospheric moisture fluxes $q_3$ and $q_4$. On the other hand, the rate of change of total ocean depth $H$ is only affected by freshwater forcing $ff$ from ice sheet growth or decay,

$$\frac{dH}{dt} = ff/A_o,$$  

where $A_o$ is the area of the whole ocean (not just the area over the Atlantic). Because the freshwater forcing and poleward moisture transports are treated as volume fluxes, they appear as damping terms in equations (B6), (B8), and (B12). This is the correct form for conserving mass, although this form is slightly different from what is traditionally done in ocean models where freshwater transports are treated as fluxes of negative salinity. Note that even though the freshwater transports and freshwater forcing are independent of salinity, their effect on the circulation (via equations (B6), (B8), and (B12)) is dependent on the salinity of the boxes where the forcing is being applied.

[63] The volume fluxes between the equatorial and polar boxes are assumed to be proportional to the depth-integrated density contrasts relative to a reference “$r$”:

$$q_1 = q_1' \left( \frac{\rho_1 H - \rho_2 D - \rho_3 (H - D)}{\rho_1 H_{b} - \rho_2 D_{b} - \rho_3 (H_{b} - D_{b})} \right)$$  \hspace{1cm} (B15)$$

and

$$q_4 = q_4' \left( \frac{\rho_4 H - \rho_2 D - \rho_3 (H - D)}{\rho_4 H_{b} - \rho_2 D_{b} - \rho_3 (H_{b} - D_{b})} \right).$$  \hspace{1cm} (B16)$$

The density of each box (kg m$^{-3}$) $\rho$ is a linear function of temperature ($T$) and salinity ($S$),

$$\rho = \rho_o (1 + \beta S - \alpha T)$$  \hspace{1cm} (A2.17)$$

where $\rho_o = 1028$ kg m$^{-3}$, $\alpha = 1.5 \times 10^{-4}$ (°K)$^{-1}$, and $\beta = 0.8 \times 10^{-3}$. If vertical density stratification is stable ($\rho_3 > \rho_2$), the vertical volume transport $q_m$ is upward and a function of both vertical density contrast and thermocline depth,

$$q_m = q_m' \left( \frac{\rho_u H_{b} - \rho_2 D_{b} - \rho_3 (H_{b} - D_{b})}{\rho_4 H_{b} - \rho_2 D_{b} - \rho_3 (H_{b} - D_{b})} \right) \frac{D_{b}}{D} \right)^{\eta}.$$  \hspace{1cm} (B18)$$

This formulation does not represent convective processes that would occur if the lower layer becomes less dense than the upper layer ($\rho_3 \leq \rho_2$). However, this unstable situation never occurs in our model experiments. The exponents $\zeta$ and $\eta$ in equation (B18) can be set to represent different mixing assumptions (see text).
Equations (B5)–(B12) for an asymmetric circulation with sinking in the north and upwelling in the south can be re-expressed in a form that covers all possible circulation states (asymmetric flow with northern sinking, asymmetric flow with southern sinking, and equatorially symmetric flow):

\[
\frac{dT_1}{dt} = \frac{1}{V_1} \left[ 0.5(|q_1| + q_1)(T_2 - T_1) + 0.5(|q_1| - q_1)(T_3 - T_1) + \kappa A_1 (T_p - T_1) \right]
\]

\text{(B19)}

\[
\frac{dS_1}{dt} = \frac{1}{V_1} \left[ 0.5(|q_1| + q_1)(S_2 - S_1) + 0.5(|q_1| - q_1)(S_3 - S_1) \right] - (q_1 + f)S_1
\]

\text{(B20)}

\[
\frac{dT_2}{dt} = \frac{1}{V_2} \left[ 0.5(|q_1| - q_1)(T_1 - T_2) + 0.5(|q_4| - q_4)(T_4 - T_2) + 0.5(|q_m| + q_m)(T_1 - T_2) + \kappa A_2 (T_e - T_2) \right]
\]

\text{(B21)}

\[
\frac{dS_2}{dt} = \frac{1}{V_2} \left[ 0.5(|q_4| - q_4)(S_2 - S_2) + 0.5(|q_4| - q_4)(S_4 - S_2) + 0.5(|q_m| + q_m)(S_1 - S_2) + (q_4 + q_4)S_2 \right]
\]

\text{(B22)}

\[
\frac{dT_3}{dt} = \frac{1}{V_3} \left[ 0.5(|q_1| + q_1 + q_1 + q_1)(T_1 - T_3) + 0.5(|q_4| + \phi_4 + q_4 + \phi_4)(T_4 - T_3) \right]
\]

\text{(B23)}

\[
\frac{dS_3}{dt} = \frac{1}{V_3} \left[ 0.5(|q_1| + \phi_1 + q_1 + \phi_1)(S_1 - S_3) + 0.5(|q_4| + \phi_4 + q_4 + \phi_4)(S_4 - S_3) \right]
\]

\text{(B24)}

\[
\frac{dS_4}{dt} = \frac{1}{V_4} \left[ 0.5(|q_4| + q_4)(S_2 - S_4) + 0.5(|q_4| - q_4)(S_3 - S_4) \right] - \phi_4 S_4
\]

\text{(B26)}

Appendix C

The single–hemisphere 3–box ocean model is designed to replicate the scaling arguments about the effects of freshwater forcing on MOC under different mixing assumptions [Nilsson and Walin, 2001]. We present those results here. Figure C1 shows the equilibrium responses of MOC to changes in freshwater forcing in the 3–box model (numerical results) as compared to scaling theory. The slight differences in slope in Figure C1b come from mathematical simplifications that were used by Nilsson and Walin [2001] who assume \(q_m = q_1\) in the mass conservation equation, whereas we show the accurate expression is \(q_m = q_1 + \phi_1\).

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Jackson et al.: Box Model Test of Abrupt Climate Change


C. S. Jackson, Institute for Geophysics, University of Texas at Austin, 10100 Burnet Rd., ROC-196, Mail Code: R2200, Austin, TX 78758, USA. (charles@ig.utexas.edu)

Y. Liu and S. Lu, Department of Physics, University of Texas at Austin, 1 University Stn. C1600, Austin, TX 78712, USA.

O. Marchal and W. G. Thompson, Department of Geology and Geophysics, Woods Hole Oceanographic Institution, 266 Woods Hole Rd., MS 23, Woods Hole, MA 02543, USA.