NOTES AND CORRESPONDENCE

The Coastal Bottom Boundary Layer: A Note on the Model of Chapman and Lentz

JOSEPH PEDLOSKY

Physical Oceanography Department, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts

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ABSTRACT

The bottom boundary layer of a stratified flow on a coastal continental shelf is examined using the model of Chapman and Lentz. The flow is driven by a surface stress, uniform in the alongshore coordinate, in a downwelling-favorable direction. The stress diminishes in the offshore direction and produces an Ekman pumping, as well as an onshore Ekman flux. The model yields an interior flow, sandwiched between an upper Ekman layer and a bottom boundary layer. The interior has a horizontal density gradient produced by a balance between horizontal diffusion of density and vertical advection of a background vertical density gradient. The interior flow is vertically sheared and in thermal wind balance. Whereas the original model of Chapman and Lentz considered an alongshore flow that is freely evolving, the present note focuses on the equilibrium structure of a flow driven by stress and discusses the vertical and lateral structure of the flow and, in particular, the boundary layer thickness. The vertical diffusivity of density in the bottom boundary layer is considered so strong, locally, as to render the bottom boundary layer's density a function of only offshore position. Boundary layer budgets of mass, momentum, and buoyancy determine the barotropic component of the interior flow as well as the boundary layer thickness, which is a function of the offshore coordinate. The alongshore flow has enhanced vertical shear in the boundary layer that reduces the alongshore flow in the boundary layer; however, the velocity at the bottom is generally not zero but produces a stress that locally balances the applied surface stress. The offshore transport in the bottom boundary layer therefore balances the onshore surface Ekman flux. The model predicts the thickness of the bottom boundary layer, which is a complicated function of several parameters, including the strength of the forcing stress, the vertical and horizontal diffusion coefficients in the interior, and the horizontal diffusion in the boundary layer. The model yields a boundary layer over only a finite portion of the bottom slope if the interior diffusion coefficients are too large; otherwise, the layer extends over the full lateral extent of the domain.

1. Introduction

The nature of the flow of a stratified fluid along a sloping bottom poses a problem of particular relevance to coastal oceanographic dynamics. Beginning with the pioneering work of MacCready and Rhines (1993) and Garrett et al. (1993), it became clear that the dynamics of the flow outside the boundary layer cannot be separated from the bottom boundary layer's dynamics and thermodynamics. This appears to be another example of the general control of the interior (i.e., exterior to the boundary layer) by the thermodynamic forcing by buoyancy fluxes in the boundary layer (see, e.g., Whitehead and Pedlosky 2000).

The problem of coastal flows along a continental slope is rendered intrinsically difficult by the advection of density in the bottom boundary layer (bbl). When the cross-shelf flow in the bbl is offshore, as in the case when the alongshore flow has shallow depths on its right looking downstream in the Northern Hemisphere, light water is advected beneath heavier fluid and turbulent mixing occurs. This tends to produce a bbl that is well mixed vertically in the density field. The resulting horizontal density gradient in the boundary layer produces a vertical shear in the alongshore flow due to a thermal wind balance and, in many of the cases studied, tends to bring the alongshore flow to rest at the bottom, arresting the frictionally induced cross-shelf flow. This has been suggested as a mechanism to de-

Corresponding author address: Joseph Pedlosky, Physical Oceanography Department, Clark 363, MS 21, Woods Hole Oceanographic Institution, Woods Hole, MA 02543.
E-mail: jpedlosky@whoi.edu

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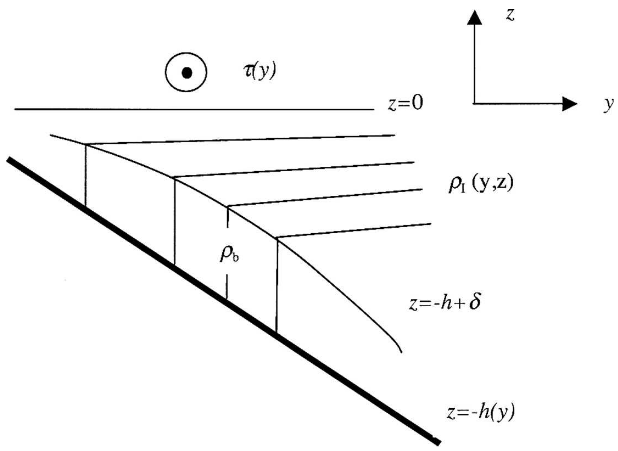

FIG. 1. A schematic of the flow field. The fluid is driven by an alongshore (x direction) stress τ and a uniform surface heating that produces a vertical density gradient as well as a horizontal cross-stream (y direction) gradient. The bottom boundary layer extends a distance $\delta(y)$ off the sloping bottom. The density in the boundary layer is well mixed vertically and matches the interior density at $z = -h(y) + \delta$.

couple the alongshore flow from the frictional effects of the bottom, allowing the flow to evade the damping effects of bottom stress.

Chapman and Lentz (1997, hereinafter CL) introduced an ingenious simplification of the dynamics in which the bottom boundary layer density is completely well mixed vertically and whose density distribution matches continuously the density field in the interior at the boundary between the interior and the bbl, as shown in Fig. 1. In CL the flow is driven by an upstream inflow of a alongshore current and its development downstream is examined. A final equilibrium state is reached in which the bottom velocity in the bbl is zero, expunging any frictional influence on the current. The current outside the boundary layer is depth independent and the density field possesses a horizontal variation only in the bbl. The width of the bbl in the cross-shelf (y) direction is limited. That is, the boundary layer thickness goes to zero, roughly speaking, at each edge of the alongshore current. This is, in principle, an inconsistency of the model since the bottom boundary condition of no-normal density gradient always requires a boundary layer adjustment of the interior density field. It is assumed (S. J. Lentz 2006, personal communication) that this apparent inconsistency can be conceptually removed by imagining a very thin bbl outside the current that is more weakly driven by the thermal condition than by the cross-shelf frictionally driven flow under the alongshore current.

The question of the structure of the final equilibrium state, and, in particular, the question of whether, in equilibrium, the bottom frictional interaction is eliminated by the thermal wind of the bbl is an essential one for the dynamics of coastal currents. In this paper the problem is approached by examining *only* the equilibrium state, independent of time and alongshore distance, of a current driven at its upper surface by a stress that is variable only in the offshore direction. In the presence of a vertical density gradient driven by surface heating, the onshore flow in the upper Ekman layer is generally divergent and this produces a vertical velocity in the interior and a consequent horizontal density gradient in the interior as well as an offshore flow in the bottom boundary layer. I employ the bbl model of CL to discuss the equilibrium situation that arises. By considering a purely two-dimensional flow, it is possible to otherwise enrich the physical context with respect to CL by considering the role of dissipation in the interior and surface forcing by an applied wind stress. The flow in the bbl is driven both by the applied stress and by the thermal condition of no vertical density flux through the bottom. Section 2 describes the basic model and boundary conditions. Section 3 describes the momentum and buoyancy balances in the interior, and section 4 describes the same budgets for the bbl. Section 5 is devoted to a discussion of the determination of the boundary layer thickness. In section 6 the results are discussed and compared with CL.

2. The model

The model describes flow driven by a stress in the alongshore direction parallel to the x axis. All variables are assumed to be independent of x but are functions of offshore distance y and of the vertical coordinate z. The range of z is given by $-h(y) + \delta \leq z \leq 0$, while the depth to the bottom is $h(y) = \alpha y$. The local boundary layer thickness is δ. The flow is hydrostatic and in the downstream direction is in geostrophic balance. An f-plane model is used. The governing equations are linearized in the momentum balance as in CL, but the buoyancy equation is fully nonlinear:

$$fu = -\frac{1}{\rho_o}\frac{\partial p}{\partial y}, \tag{2.1a}$$

$$-fv = \frac{1}{\rho_o}\frac{\partial \tau}{\partial z}, \tag{2.1b}$$

$$\frac{\rho}{\rho_o}g = -\frac{1}{\rho_o}\frac{\partial p}{\partial z}, \quad \text{and} \tag{2.1c}$$

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{2.1d}$$

In (2.1b) τ is the turbulent stress in the x direction *within* the fluid. The Boussinesq approximation allows us to replace the density in the horizontal momentum equations with the mean constant density ρ_o, while ρ is the density anomaly.

The buoyancy equation balances advection of density and its diffusion by turbulent mixing. Thus,

$$v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} = \frac{\partial}{\partial y}\left(\kappa_H \frac{\partial \rho}{\partial y}\right) + \frac{\partial}{\partial z}\left(\kappa_V \frac{\partial \rho}{\partial z}\right). \quad (2.1e)$$

We will assume that the depth of the region of interest is great enough to allow a standard Ekman surface boundary layer. This yields an onshore flux $M_E = \tau^w/\rho_o f$ and an Ekman pumping velocity

$$w = w_e = -\frac{1}{\rho_o f}\frac{\partial \tau^w}{\partial y}, \quad (2.2)$$

which serves as the upper boundary condition for the fluid vertical velocity below the surface Ekman layer. At the apex of the wedge representing the coastal shelf region the total depth goes to zero and the separation of the flow into an upper Ekman layer, an interior, and a boundary layer flow will fail and so the model will not be valid all the way to $y = 0$; therefore, a small tip of the wedge must be excluded from our analysis.

The buoyancy flux normal to the bottom vanishes there and all perturbations to the background density vanish as y becomes large, along with the applied stress. As in Fig. 1, we insist that the density be continuous across the upper boundary of the bbl at $z = -\alpha y + \delta$.

3. The interior

In the interior of the fluid, that is, beneath the surface Ekman layer and above the bbl, it is assumed that the turbulent momentum stresses are negligible. From (2.1b) that leads immediately to

$$v_I = 0, \quad (3.1)$$

where the subscript I labels interior variables. From (2.1d) this implies that the interior vertical velocity is independent of z. Since w_I must match the Ekman pumping velocity at the upper boundary, $z = 0$, it follows that

$$w_I = w_e(y) = -\frac{1}{\rho f}\frac{\partial \tau^w}{\partial y}. \quad (3.2)$$

The buoyancy equation becomes

$$w_e\frac{\partial \rho_I}{\partial z} = \frac{\partial}{\partial y}\left(\kappa_{H_I}\frac{\partial \rho_I}{\partial y}\right) + \frac{\partial}{\partial z}\left(\kappa_{V_I}\frac{\partial \rho_I}{\partial z}\right), \quad (3.3)$$

and we note that the turbulent diffusion coefficients may differ in the interior from their values in the boundary layer. Indeed, we will insist that they do and imagine that the mixing is much stronger in this region near the bottom boundary. Suppose that at the upper boundary a heat or density flux is imposed that is uniform in y; that is, suppose that

$$\kappa_{V_I}\frac{\partial \rho_I}{\partial z} = -H \quad \text{at} \quad z = 0. \quad (3.4)$$

We can find solutions to (3.3) in the interior in the form

$$\rho_I = \rho_o + \overline{\rho}_I(z) + \tilde{\rho}_I(y), \quad (3.5)$$

where

$$\frac{1}{\rho_o}\frac{\partial \overline{\rho}_I}{\partial z} = -\frac{H}{\kappa_{V_I}\rho_o} \equiv -\frac{1}{g}N^2; \quad (3.6)$$

N is constant and such that the y-dependent part of the interior density field satisfies

$$-w_e\rho_o\frac{N^2}{g} = \frac{\partial}{\partial y}\left(\kappa_{H_I}\frac{\partial \tilde{\rho}_I}{\partial y}\right). \quad (3.7)$$

Using (2.2) and the condition that both τ^w and $\tilde{\rho}_I$ vanish for large y, it is easy to obtain

$$\frac{\rho_I}{\rho_o} = -z\frac{N^2}{g} - \int_y^\infty \frac{\tau^w}{\rho_o f}\frac{N^2}{g\kappa_{H_I}}\,dy' + 1. \quad (3.8)$$

Note that the total density approaches ρ_o at the surface ($z = 0$) for large y. From the thermal wind equation for u_I

$$\frac{\partial u_I}{\partial z} = \frac{g}{f\rho_o}\frac{\partial \tilde{\rho}_I}{\partial y} = \frac{\tau^w N^2}{\rho_o f^2 \kappa_{H_I}} \quad (3.9)$$

or

$$u_I = \frac{z}{\kappa_{H_I}}\frac{N^2}{f^2}\frac{\tau^w}{\rho_o} - \frac{1}{\rho_o f}\frac{\partial p_s}{\partial y}. \quad (3.10)$$

The laterally varying pressure field at the upper surface, p_s, is an unknown of the problem and will be determined only after an analysis of the bottom boundary layer. Now that the interior fields are determined up to the unknown surface pressure, we turn our attention to the boundary layer.

4. The bottom boundary layer balances

In the bottom boundary layer the vertical mixing is hypothesized to be so intense that to leading order the dominant term in the buoyancy equation is, locally, the second term on the right-hand side of (2.1e), leading to a solution for ρ_b that is a function only of y. If the density at the upper edge of the boundary layer is continuous with the interior density, as in the CL model,

$$\rho_b(y) = \rho_I[y, z = -h(y) + \delta], \quad (4.1)$$

so that

$$\frac{\rho_b}{\rho_o} = (h - \delta)\frac{N^2}{g} - \int_y^\infty \frac{\tau^w}{\rho_o f}\frac{N^2}{g\kappa_{H_I}}\,dy' + 1. \qquad (4.2)$$

Using the hydrostatic relation and using the condition that the pressure at $z = -h + \delta$ is continuous between the interior and the bbl leads to

$$\frac{p_b}{\rho_o} = -(h - \delta)[z + (h - \delta)/2]N^2 + z\int_y^\infty \frac{\tau^w}{\rho_o f}\frac{N^2}{g\kappa_{H_I}}\,dy' - gz + \frac{p_s}{\rho_o}, \qquad (4.3)$$

while the geostrophic balance for the alongshore velocity yields

$$u_b = \frac{N^2}{f}\frac{\partial}{\partial y}\{[z + (h - \delta)/2](h - \delta)\} + z\frac{\tau^w}{\rho_o f^2}\frac{N^2}{\kappa_{H_I}} - \frac{1}{\rho_o f}\frac{\partial p_s}{\partial y}. \qquad (4.4)$$

The momentum equation (2.1b) requires a detailed knowledge of the distribution of the turbulent stresses within the bbl, but, assuming that the stress vanishes on the boundary with the interior, the integral of (2.1b) yields the offshore flux in the bbl:

$$V_b\delta = \frac{\tau(z = -h)}{\rho_o f}, \qquad (4.5)$$

where we have defined $V_b = (1/\delta)\int_{-h}^{-h+\delta} \upsilon_b\,dz$. Note that in (4.5) τ is the turbulent stress in the fluid and not the applied wind stress. As in CL, we assume a simple, linear relation between the bottom stress in the fluid and the bottom alongshore velocity so that

$$V_b\delta = \frac{\tau(-h)}{\rho_o f} = ru_b(-h)/f; \qquad (4.6)$$

thus

$$-V_b\delta = \frac{r}{f}\left[\frac{N^2}{f}\delta\frac{\partial(h - \delta)}{\partial y}\right] + h\frac{r}{f}\frac{\tau^w}{\rho_o f^2}\frac{N^2}{\kappa_{H_I}} + \frac{r}{\rho_o f^2}\frac{\partial p_s}{\partial y}. \qquad (4.7)$$

If the offshore mass flux can be determined, (4.7) can be thought of as the equation that determines the surface pressure gradient for a given boundary layer thickness. Of course, both δ and the frictionally driven mass flux must be determined. It is natural to anticipate from a simple mass balance that the offshore flux in the bottom boundary layer must balance the onshore flow in the surface Ekman layer, which is a known quantity. However, it is illuminating to show this directly from a local, integral mass budget for the boundary layer and that is shown in the appendix, where the expected result

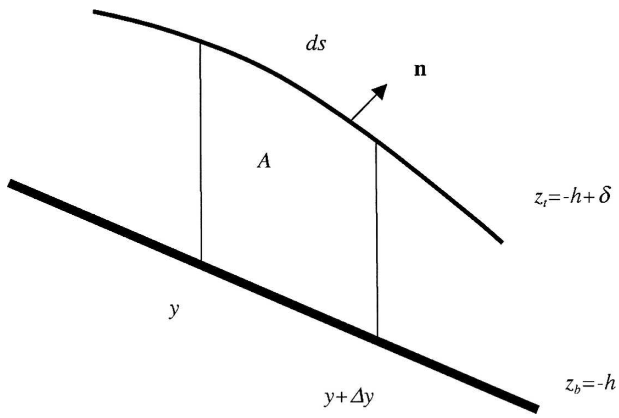

FIG. 2. The control volume over which the buoyancy equation for the bottom boundary layer is integrated.

$$V_b\delta = \frac{\tau^w}{\rho_o f} \qquad (4.8)$$

is obtained.

The final balance involves a similar integral of the buoyancy equation, and this leads to an equation to determine the boundary layer thickness. We first integrate over the differential area A as shown in Fig. 2. If $\Im_b$ is the buoyancy flux vector in the bbl, an application of (A.3) and (A.4) to the advective contribution yields

$$dyV_b\delta\frac{\partial\rho_b}{\partial y} = \Im_b\cdot\mathbf{n}ds + dy\frac{\partial}{\partial y}\left(\kappa_{H_b}\delta\frac{\partial\rho_b}{\partial y}\right). \qquad (4.9)$$

With (A.2) we can rewrite this as

$$V_b\delta\frac{\partial\rho_b}{\partial y} = \left(\kappa_{V_b}\frac{\partial\rho_b}{\partial z} - \kappa_{H_b}\frac{\partial\rho_b}{\partial y}\frac{\partial z_t}{\partial y}\right)_{z=z_t} + \frac{\partial}{\partial y}\left(\delta\kappa_{H_b}\frac{\partial\rho_b}{\partial y}\right). \qquad (4.10)$$

The term on the left-hand side of (4.10) represents the total advection of density in the bbl. The vertical advection across the top of the bbl is balanced by the divergence of the horizontal mass flux. The first term on the right-hand side is the diffusive flux across the top of the bbl, while the final term on the right-hand side is the horizontal, turbulent diffusion of buoyancy in the bbl.

We have already supposed that the vertical mixing is so strong, $\kappa_{V_b} \to \infty$, that the vertical density gradient in the bbl is driven to zero, so the first term on the right-hand side is the product of the very large vertical mixing and the negligible vertical density gradient, and this does not allow a direct evaluation of that term. However, by considering a small pillbox straddling the top of the bbl, it is easy to show that the diffusive mass flux across the top of the bbl must be continuous (this relies on the continuity of both the mass flux and density across $z = z_t$). Therefore, this diffusive term can be rewritten in terms of the *interior* diffusive flux, allowing (4.10) to be rewritten:

$$V_b\delta\frac{\partial\rho_b}{\partial y} = \kappa_{V_I}\frac{\partial\rho_I}{\partial z} - \kappa_{H_I}\frac{\partial\rho_I}{\partial y}\frac{\partial z_t}{\partial y} + \frac{\partial}{\partial y}\left(\delta\kappa_{H_b}\frac{\partial\rho_b}{\partial y}\right). \tag{4.11}$$

With (4.2) and (4.8), (4.11) becomes a single equation for the boundary layer thickness $\delta(y)$. After some algebra, the equation becomes

$$\frac{\partial}{\partial y}\left[\kappa_{H_b}\delta\frac{\partial}{\partial y}(h-\delta) + \kappa_{H_b}\delta\frac{\tau^w}{\rho_o f\kappa_{H_I}}\right] = \kappa_{V_I} + \left(\frac{\tau^w}{\rho_o f}\right)^2/\kappa_{H_I}. \tag{4.12}$$

It is important to note that the term in the flux across the top of the bbl that depends on the slope of that boundary is canceled by a similar term in the lateral advection of density, this balance reflecting the interior balance (3.7).

The left-hand side represents the turbulent lateral diffusion of density in the bbl. The first term on the right-hand side reflects the effect of vertical diffusion through the top of the bbl and the last term is the effect of the lateral advection in the density gradient in the bbl. Before proceeding to solutions of (4.12) it is important to note that once δ is known, the surface pressure gradient can be determined and is given by (4.7) or

$$-\frac{1}{\rho_o f}\frac{\partial p_s}{\partial y} = \frac{N^2}{f}\delta\frac{\partial(h-\delta)}{\partial y} + h\frac{\tau^w N^2}{\rho_o f^2\kappa_{H_I}} + \frac{\tau^w}{\rho_o r}; \tag{4.13}$$

so the alongshore velocity in the bbl is

$$u_b = \frac{N^2}{f}(z+h)\left[\frac{\partial(h-\delta)}{\partial y} + \frac{\tau^w}{\rho_o f\kappa_{H_I}}\right] + \frac{\tau^w}{\rho_o r}, \tag{4.14}$$

which satisfies the condition that at $z = -h$, the bottom stress ru_b matches the applied wind stress. For comparison, the determination of p_s allows us to write the interior alongshore flow:

$$u_I = \frac{N^2}{f}(z+h)\left(\frac{\tau^w}{\rho_o f\kappa_{H_I}}\right) + \frac{N^2}{f}\delta\frac{\partial}{\partial y}(h-\delta) + \frac{\tau^w}{\rho_o r}. \tag{4.15}$$

Note that the alongshore velocity is continuous at $z = -h + \delta$ and is more strongly sheared in the bbl due to the stronger lateral density gradient. Indeed, if the applied wind stress is zero, the alongshore velocity is sheared sufficiently to bring the velocity to zero at the bottom; the cross-shelf transport is also zero and the model predicts the "arrested" boundary layer state predicted by MacCready and Rhines (1993). The presence of the applied stress leads to a nonzero bottom velocity, bottom stress, and cross-shelf, frictionally driven velocity.

5. The boundary layer thickness

To investigate the solutions of (4.12) for δ it is useful to nondimensionalize the equation. We will consider an idealized bottom geometry of the form $h = \alpha y$. Horizontal lengths are scaled by the constant L, characteristic of the wind stress variation in y, while depths and the boundary layer thickness are scaled with αL. For simplicity a stress pattern of the form

$$\tau^w = \tau_o e^{-a(y/L)} \tag{5.1}$$

is chosen. Then (4.12) can be put in the form

$$\frac{d}{dy}\left[\delta\frac{d\delta}{dy} - (1 + Fe^{-ay})\delta\right] = -\Sigma_V - F^2\Sigma_H e^{-2ay}, \tag{5.2}$$

where the parameters

$$F = \frac{\tau_o}{\alpha\rho_o f\kappa_{H_I}}, \quad \Sigma_V = \frac{\kappa_{V_I}}{\kappa_{H_b}\alpha^2}, \quad \text{and} \quad \Sigma_H = \frac{\kappa_{H_I}}{\kappa_{H_b}}, \tag{5.3a–c}$$

and depends on only the wind stress, the ratio of the diffusion coefficients within and exterior to the boundary layer, and the slope of the bottom. All variables are now nondimensional. Since the right-hand side of (5.2) is known, it can be integrated once in y to obtain the nondimensional equation

$$\delta\frac{d\delta}{dy} - (1 + Fe^{-ay})\delta = -\Sigma_V(y - y_o) - \frac{F^2}{2a}(e^{-2ay} - e^{-2ay_o})\Sigma_H + C, \quad \text{with} \tag{5.4a}$$

$$C = \{\delta[d\delta/dy - (1 + Fe^{-ay_o})]\}_{y=y_o}. \tag{5.4b}$$

a)

b)

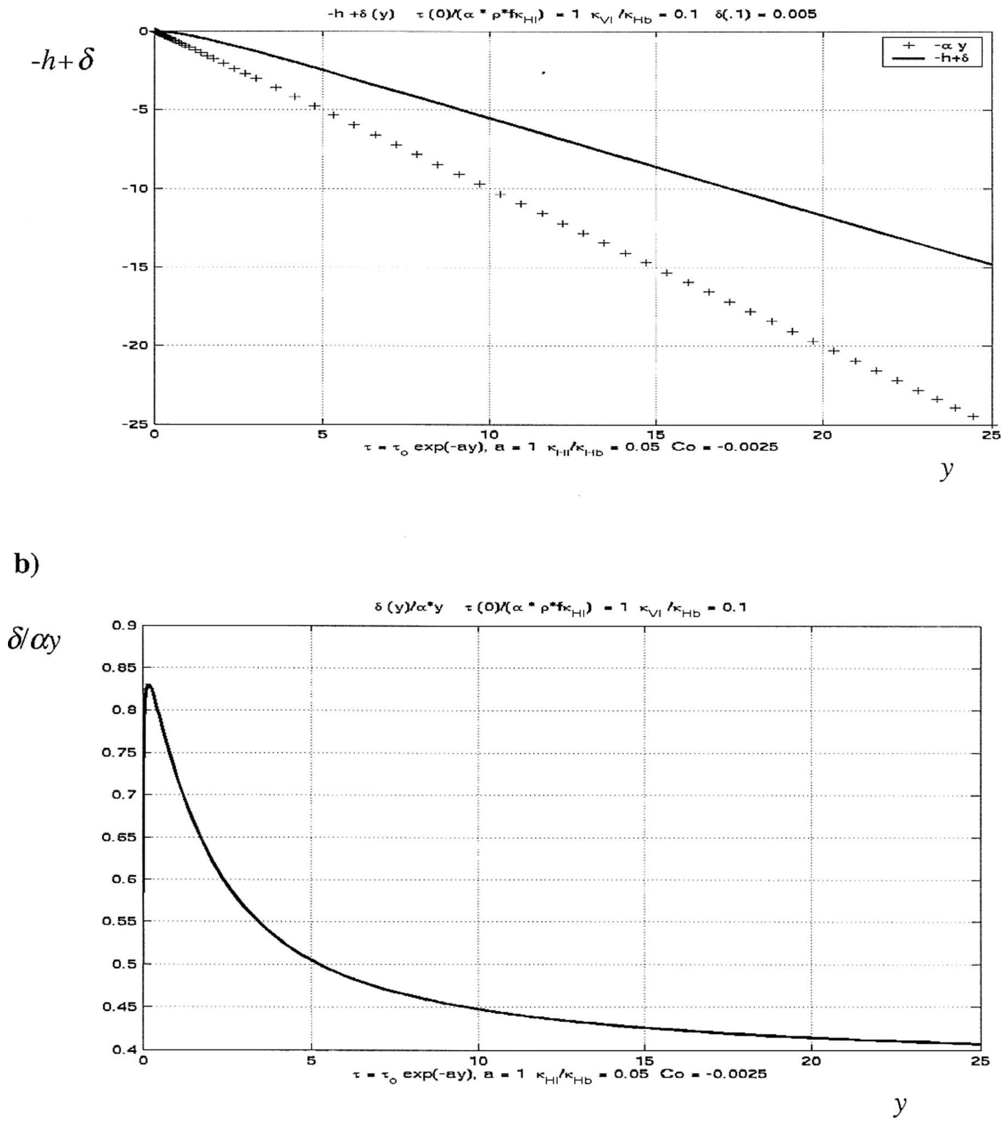

FIG. 3. (a) The boundary layer thickness with respect to the sloping bottom for a=1, $\Sigma_V = 0.1$, and $\Sigma_H = 0.05$. A starting value of δ of one-half of the depth at $y = y_o = 0.01$ is chosen and C is -0.0025; $F = 1$ has been used. (b) The boundary layer thickness as a fraction of the local depth, $h = \alpha y$.

We start the integration from a point $y = y_o > 0$ since the model has assumed a separation in depth between the upper Ekman layer, the interior, and the bbl, which certainly does not apply to the vertex of the wedge-shaped region being considered. The calculations performed below are not sensitive to changes in the (small) value of y_o. The constant C can be shown to be

$$C = \left[\delta\left(-\frac{g}{\alpha N^2 \rho_o}\frac{\partial \rho_b}{\partial y}\right)\right]_{y=y_o}. \quad (5.5)$$

Except for the fact that C is plausibly negative, we have no way within the current theory to specify it precisely. However, the qualitative results that follow are not terribly sensitive to its exact value.

Equation (5.4) is slightly transformed in terms of the variable $q = \delta^2$:

$$\frac{dq}{dy} = 2(1 + Fe^{-ay})q^{1/2} - 2\Sigma_V(y - y_o) - F^2\Sigma_H(e^{-2ay} - e^{-2ay_o})/a + 2C. \quad (5.6)$$

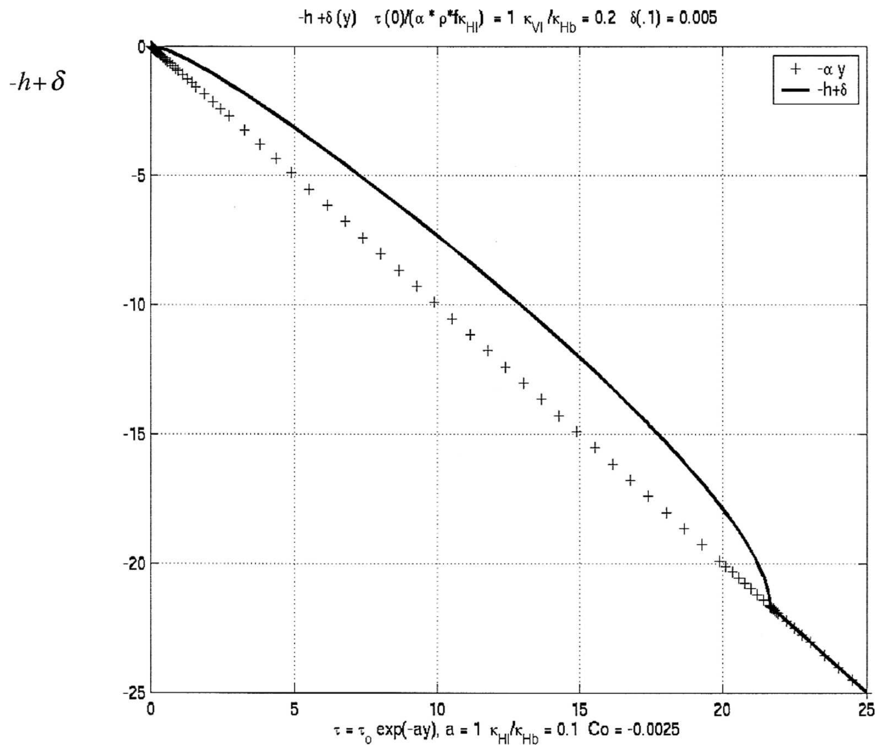

FIG. 4. As in Fig. 3a except that $\Sigma_V = 0.2$ and $\Sigma_H = 0.1$.

A typical calculation of δ is shown in Fig. 3. We have chosen the parameters $a = 1$ (the characteristic length scale is thus the scale of the seaward decay of the wind stress) and $\Sigma_V = 0.1$, assuming that the interior vertical mixing is small relative to the lateral mixing in the bbl and Σ_H is similarly small, 0.05. A starting value of δ of one-half of the depth at $y = y_o = 0.01$ is chosen and C is -0.0025. A nominal value of $F = 1$ has been used. For these values of the parameters the boundary layer thickness grows until it eventually occupies about half the local total depth. At distances $y \gg 1$, the applied stress is negligible, the interior horizontal density gradient is also small, and the flow consists of an interior, depth-independent flow in balance with the surface pressure gradient $\partial p_s/\partial y$ under which the bbl brings the long-shore velocity to rest.

If the size of the interior diffusion coefficients is greater, then, as shown in Fig. 4, the boundary layer thickness reaches a maximum size at a midpoint along the shelf and then diminishes in size and finally vanishes in a manner reminiscent of the calculations in CL. In Fig. 4 Σ_V is 0.2 and $\Sigma_H = 0.1$; that is, we have doubled the strength of the interior buoyancy diffusion while all other parameters are the same. We note that in this case the boundary layer shrinks to zero well beyond the region strongly driven by the wind stress. Note that at those values of y for which δ is zero, the interior alongshore velocity (4.15) itself satisfies the bottom stress condition. If this occurs, as in Fig. 4, in a region where the surface wind stress is negligibly small, the bottom velocity as determined by the interior flow is itself zero. Although the stress condition is satisfied the condition of no-normal density flux through the lower boundary is violated. However, since the interior vertical diffusivity is assumed small, this could be argued to merely reflect the fact that a thinner boundary layer, reflecting weaker interior mixing not driven by the surface stress might be added to satisfy the density flux condition but this is not a natural feature of the CL model.

In the preceding calculations we have assumed that the horizontal diffusivity is at least an order of magnitude greater in the bbl than in the interior. Figure 5 shows a calculation when this ratio $\Sigma_H = 0.65$. Larger values of the ratio cause the boundary layer to ground as in Fig. 4.

6. Discussion

In general, the CL model is able to represent an equilibrium state, one growing neither in time nor in downstream coordinate, that represents the turbulent interaction of the alongshore current with a sloping bottom.

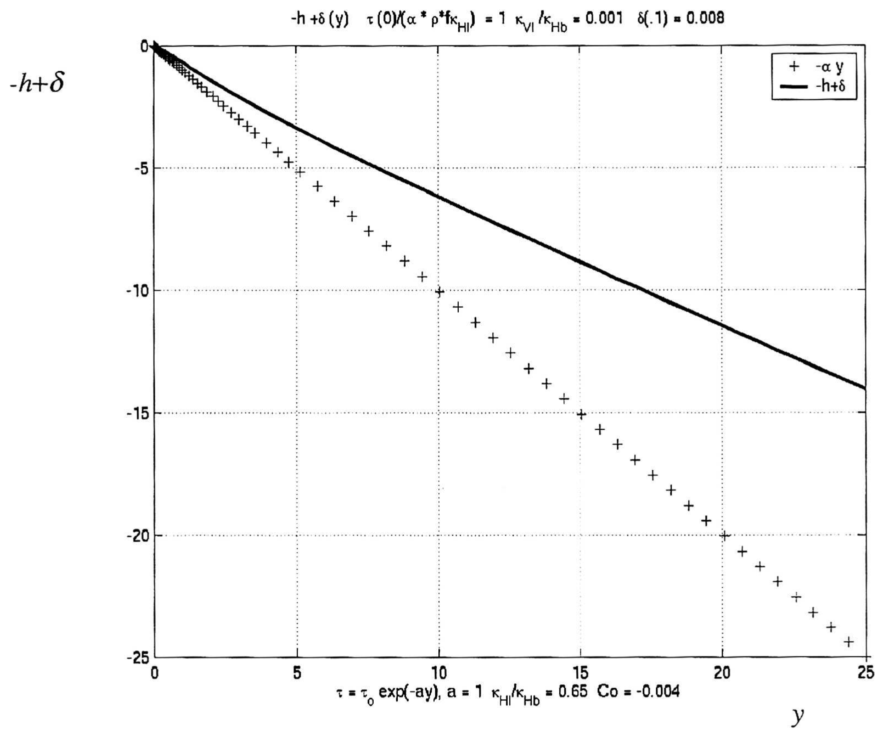

FIG. 5. The boundary layer thickness for $\Sigma_H = 0.65$, $\Sigma_V = 0.001$, $a = 1$, and $F = 1$. In this calculation the horizontal diffusion in the boundary layer is equal to its interior value.

The heuristic model of the turbulent boundary layer satisfies the stress condition and the zero density flux condition on the sloping bottom. As is common in the dynamics of rotating stratified flows with horizontal boundaries, in this case the sloping bottom, the specification of the interior flow is not independent of the interaction with the boundary. In the case studied here the pressure field p_s is determined and is so arranged that the bottom velocity can satisfy the stress condition. In the absence of local stress the interior velocity itself satisfies the condition of no slip on the bottom in a manner reminiscent of the results found in the laboratory experiments reported in Whitehead and Pedlosky (2000) and in agreement with the predictions of the numerical models of MacCready and Rhines (1993) and CL. This strengthens the possibility that alongshore coastal currents can substantially evade the dissipative effects of bottom friction and maintain their form over large alongshore distances.

Of course there are several somewhat artificial features of the basic mode, including the assumption that the density in the bbl is absolutely depth independent. Both observations (K. Brink 2006, personal communication) and detailed numerical studies, for example, Chapman (2000), show a small residual to the vertical density gradient, but the major qualitative feature of the reduced bottom velocity still obtains. The assumption in the model of constant interior N is clearly unrealistic but was assumed to allow a simple solution of the density equation. It is not believed that the inclusion of variable buoyancy frequency will alter the qualitative nature of the response. Perhaps the greatest weakness of the model is the assumption of linearity in the momentum balance. Especially in the presence of strong alongshore flows and concomitant density fronts, such an approximation is likely to be problematic.

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APPENDIX

The Boundary Layer Mass Balance

To obtain the mass balance for the bbl, we integrate the continuity equation over the interval $z_b = -h \le z \le -h + \delta = z_t$, that is, from the bottom to the top of the bbl and over a infinitesimal distance in y to obtain

$$\frac{\partial}{\partial y}\int_{z_t}^{z_b} \upsilon_b\, dz + w_b(z_t) - \upsilon_b(z_t)\frac{\partial z_t}{\partial y} - w_b(z_b) + \upsilon_b(z_b)\frac{\partial z_b}{\partial y} = 0. \tag{A.1}$$

The last two terms in (A.1) cancel as a result of the no-normal flow condition at the bottom. If one recognizes that the unit normal to the upper boundary of the bbl is

$$\mathbf{n} = \frac{\mathbf{k} - \mathbf{j}\partial z_t/\partial y}{(dy^2 + dz_t{}^2)^{1/2}}\, dy, \tag{A.2}$$

the second and third terms in (A.1) represent the total flux of fluid, w_*, across the curved upper boundary of the bbl in a lateral distance dy. This leads to the intuitive statement

$$\frac{\partial V_b \delta}{\partial y} + w_* = 0, \tag{A.3}$$

where

$$w_* = w_b(z_t) - \upsilon_b(z_t)\frac{\partial z_t}{\partial y}. \tag{A.4}$$

The mass flux across the line $z = -h + \delta$ must be continuous between the interior and the boundary layer, but in the interior υ_I is zero so that

$$\frac{\partial V_b \delta}{\partial y} + w_e = 0$$

or

$$\frac{\partial V_b \delta}{\partial y} - \frac{\partial \tau^w/\rho_o f}{\partial y} = 0. \tag{A.5a,b}$$

Integrating once and using the condition that the wind stress and the frictionally driven offshore flow vanish for large y leads to the expected result (3.8):

$$V_b \delta = \frac{\tau^w}{\rho_o f}. \tag{A.6}$$

Note that comparing (4.6) with (4.8) implies that the stress applied at the upper surface is communicated directly to the bottom.

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