NOTES AND CORRESPONDENCE

Stommel’s Box Model of Thermohaline Circulation Revisited—The Role of Mechanical Energy Supporting Mixing and the Wind-Driven Gyration

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ABSTRACT

The classical two-box model of Stommel is extended in two directions: replacing the buoyancy constraint with an energy constraint and including the wind-driven gyre. Stommel postulated a buoyancy constraint for the thermohaline circulation, and his basic idea has evolved into the dominating theory of thermohaline circulation; however, recently, it is argued that the thermohaline circulation is maintained by mechanical energy from wind stress and tides. The major difference between these two types of models is the bifurcation structure: the Stommel-like model has two thermal modes (one stable and another one unstable) and one stable haline mode, whereas the energy-constraint model has one stable thermal mode and two saline modes (one stable and another one unstable). Adding the wind-driven gyre changes the threshold value of thermohaline bifurcation greatly; thus, the inclusion of the wind-driven gyre is a vital step in completely modeling the physical processes related to thermohaline circulation.

1. Introduction

Our current knowledge of the thermohaline circulation (THC), the prevailing concept of deep ocean currents (Wunsch 2002; Rahmstorf 2003), is fitted into a general dynamical framework through Stommel’s pioneer work (Stommel 1961, hereinafter S61). The most important results of S61 are the existence of multiple equilibria of THC and the thermohaline catastrophe. These phenomena are closely linked to climate changes, and they have been confirmed by numerical models (Bryan 1986; Manabe and Stouffer 1988).

Stommel made use of a fundamental assumption that the strength of THC is linearly proportional to the equator–pole density difference. This assumption implies that thermohaline circulation is driven by surface thermohaline forcing. Stommel’s box model has been extended and modified in many studies, but its basic idea remains unchanged. Recently, Stommel’s hypothesis has been challenged by the theory of mechanical energy balance (Huang 1999, hereafter H99; Lyle 1997; Nilsson and Walin 2001, hereafter NW01; Nilsson et al. 2003; Walin 1990), and there is paleoclimate evidence supporting the energetic argument (Adkins and Pasquero 2004; Broecker et al. 2004; Keigwin 2004; Shackleton et al. 1988). In addition, laboratory experiments also suggested that surface thermohaline forcing alone may not be able to sustain the thermohaline circulation observed in the world’s oceans (Wang and Huang 2005). According to the energetic arguments, the amount of mechanical energy available for sustaining mixing is a key factor in regulating THC (H99; Huang 2004; Munk and Wunsch 1998; Wunsch and Ferrari 2004; Kuhlbrodt et al. 2007).

In this note we will explore how to use a simple two-box model to study two important issues related to THC: the dynamical roles of mechanical energy and the wind-driven gyre. Instead of the classical approach of either adopting a buoyancy constraint or using a fixed
diapycnal diffusivity, a new energy constraint has been explored in recent studies. For example, H99 has tested the assumption of a fixed energy for mixing in a simple ocean general circulation model (OGCM). The basic point is that the THC is a heat conveyor belt rather than a heat engine, and external mechanical energy sources are required for sustaining the circulation. Similarly, NW01 used an analytical two-layer model to explore the sensitivity of THC to the parameterization of vertical mixing. In particular, their model study suggested that a decline in surface density contrast can result in a stronger THC.

On the other hand, numerous studies (e.g., Toggweiler and Samuels 1995) have shown that wind-driven circulation (WDC) is not only a key factor in maintaining THC but also a key factor in regulating the stability of THC. Stommel and Rooth (1968) made the first attempt to explore the influence of wind stress on THC, but their contribution has seldom been cited and little attention has been paid to studying the dynamical role of wind stress in simple box models. However, many recent studies have explored the importance of wind stress forcing, for example, Longworth et al. (2005), MacMynowski and Tziperman (2006), and Pasquero and Tziperman (2004). Clearly, given the strong nonlinear interplay between THC and WDC, a model for the THC without the wind-driven component seems incomplete.

What can happen if the balance of mechanical energy is used as a major constraint in the model? We will reexamine the classical Stommel two-box model by including a wind-driven gyre and replacing the buoyancy constraint with the energy constraint. Although WDC and external sources of mechanical energy supporting mixing are closely related to each other, details of such a connection remain unclear at this time; thus, we will treat them as external and independent parameters of the model.

2. Model formulation

a. A two-box model including the wind-driven gyre

We start from a slightly modified version of S61 (Huang et al. 1992) and add the wind-driven gyre (Fig. 1). For the thermal mode depicted in the figure, water flows from the low-latitude box to the high-latitude box and returns through the pipe; the temperature $T$ balance in box 1 obeys

$$\rho_0 c_p \rho_w L^2 \frac{dT_1}{dt} = -\rho_0 c_p \rho_w L u T_1 + \rho_0 c_p \rho_w L^2 w T_2$$

$$+ \rho_0 c_p L^2 (T_0 - T_1) - \rho_0 c_p \rho_w L \omega (T_1 - T_2),$$

where $\rho_0$ is the mean density, $c_p$ is the specific heat under constant pressure, $L$ and $H$ are the width and depth of each box, $u$ and $w$ are the horizontal and vertical velocities, $\omega$ is the strength of gyration, and $\Gamma = 0.78$ m day$^{-1}$ is the surface relaxation constant (Haney 1971); there are similar equations for box 2 and for salinity $S$. The control equations are

$$\frac{d\Delta T}{dt} = -2w \Delta T + (p - 1 - 2\omega) \Delta T + (p + 1) T_0,$$  \hspace{1cm} (1a)

$$\frac{d\Delta S}{dt} = -2w \Delta S + (p - 2\omega) \Delta S + 2p S_0.$$  \hspace{1cm} (1b)

For the haline mode, the direction of transport between these two boxes is reversed, but the contribution due to WDC is unchanged, so that

$$\frac{d\Delta T}{dt} = -2w \Delta T - (p + 1 + 2\omega) \Delta T + (p + 1) T_0,$$  \hspace{1cm} (2a)

$$\frac{d\Delta S}{dt} = -2w \Delta S - (p + 2\omega) \Delta S + 2p S_0.$$  \hspace{1cm} (2b)

where $S_0$ and $T_0$ are the mean (reference) salinity and temperature of the model ocean, $\Delta T = T_1 - T_2$, $\Delta S = S_1 - S_2$, $T_1$ and $T_2$, and $S_1$ and $S_2$ are temperature and salinity in each box, and $p (-p)$ is the precipitation
(evaporation) at the high-latitude (low latitude) box (S61; Huang et al. 1992). Note that this volume transport is independent of the energy for mixing and temperature/salinity, so that our formulation is different from Longworth et al. (2005). Equations (1) and (2) are the principal governing equations of THC. To calculate the circulation, we need one more constraint that determines the vertical velocity \( w \).

### b. Constraints regulating the circulation rate

The constraint regulating the meridional overturning rate’s (MOR’s) \( w \) is the remaining critical part of the model formulation. S61 assumed that circulation rate is linearly proportional to the density difference, that is, 

\[
w_s = c(\rho_0 \alpha \Delta T - \rho_0 \beta \Delta S),\tag{3}
\]

where the subscript \( s \) stands for Stommel hypothesis and \( c \) is a constant. Accordingly, the circulation is regulated by the surface buoyancy difference, implying that surface thermohaline forcing drives THC. A model based on such a constraint will be called a Stommel-like model.

There are clearly many questions related to the traditional theory of THC based on Stommel’s framework. A careful examination of Stommel’s assumption using Eq. (3) reveals the following: first, Stommel’s choice of such a constraint is not obvious; second, this is not the only possible constraint; and third, this may not be the best constraint for modeling THC under different climate conditions. Actually, an obvious weakness is relating the meridional flow to the meridional density gradient \( \Delta \rho \), which is contrary to the notion of the geostrophic balance (Olbers 2001).

Stommel’s assumption has been widely adapted in most existing box models. More critically, the parameter \( c \) has been treated as a constant intrinsic to each model, and it is assumed to be invariant under different climate conditions. According to the new hypothesis based on the balance of mechanical energy, however, the THC is driven by external sources of mechanical energy from wind stress and tides. Although cold and dense deep water formed at high latitudes has been traditionally identified as the driving force for the THC, cooling of surface water at high latitudes does not increase mechanical energy in the mean state. Instead, cooling can induce great loss of gravitational potential energy (GPE) in the mean state through convective adjustment. During the convective adjustment a small part of the GPE loss from the mean state can be converted into kinetic energy of the mean state, but the total amount of mechanical energy in the mean state is greatly reduced during the convective adjustment induced by cooling; therefore, cooling does not create mechanical energy of the mean state in the ocean, and it does not drive THC energetically.

To maintain a steady circulation in the ocean, on the other hand, dense deep water must be brought upward through the thermocline, and heat absorbed in the upper ocean must be mixed downward against the upwelling of the cold and dense water through diapycnal mixing. Diapycnal mixing in a stably stratified fluid pushes light parcels downward and heavy parcels upward, so GPE in the mean state is increased. The increase of GPE through this process is due to the conversion of kinetic energy of turbulence and internal waves, and this energy source is ultimately related to the external mechanical energy sources imported by wind stress and tidal dissipation (Wunsch 1998; Munk and Wunsch 1998; H99; Wunsch and Ferrari 2004; Wang and Huang 2004).

Thus, the balance of mechanical energy, that is, the external sources and the dissipation of the mechanical energy, is what really controls the THC. As an alternative for the commonly used buoyancy constraint, a constraint based on mechanical energy sustaining diapycnal mixing can be formulated as follows: Assuming a one-dimensional balance of density in the vertical direction

\[
w \rho_z = \kappa \rho_z z,
\]

where \( \kappa \) is the vertical (diapycnal) diffusivity, we obtain the following scale for the vertical velocity

\[
w = \kappa / D,
\]

where \( D \) is the scale depth of the main thermocline. The differences in temperature, salinity, and density between the surface and abyssal ocean are \( \Delta T, \Delta S \), and \( \Delta \rho \). Note that \( \Delta \rho = \rho_1 (\alpha \Delta T + \beta \Delta S) = \rho_1 - \rho_2 < 0 \). In a two-layer model with density difference \( \Delta \rho \) the rate of GPE increases (per unit area) because vertical mixing \( \kappa \) is \(-g \kappa \Delta \rho\). Using the notations introduced above, the rate of GPE created by mixing in the box model is

\[
E_m = -g \kappa \Delta \rho L^2 = -g DL^2 w \Delta \rho = g DL^2 \rho_0 \alpha \Delta T - \rho_0 \beta \Delta S).
\]

Therefore, the scale of MOR satisfies

\[
w_e = \frac{e}{\rho_0 \alpha \Delta T - \rho_0 \beta \Delta S},
\tag{4}
\]

where the subscript \( e \) denotes the energy constraint, and

\[
e = \frac{E_m}{gDL^2} = \frac{E}{D} \text{ where } E = \frac{E_m}{gL^2}
\]

has a dimension of density multiplied by velocity and represents the strength of the external source of me-
mechanical energy sustaining mixing. Here, $E$ is the rate of external mechanical energy supply per unit area. Constraint Eq. (4) is similar to the formula used by NW01 for a two-layer model. A model based on this constraint will be called an energy-constraint model. A very important point is that external sources of mechanical energy can vary with climate changes; thus, $e$ in the energy-constraint model is treated as an external parameter. Note that this parameter can vary over a certain range as climate conditions change, although determining the exact connection between climate conditions and energy parameter $e$ remains one of the grand challenges for the new hypothesis of THC and climate system.

The magnitude of $e$ in the world's oceans can be estimated from mechanical energy sources sustaining diapycnal mixing. Using a one-and-a-half-dimensional model, Munk and Wunsch (1998) estimated that about 2 TW is required for sustaining the stratification in the world oceans. Assume the total rate of energy supporting diapycnal mixing in the ocean interior is on the order of 1 TW; thus, for the world's upper ocean with horizontal area of $3.4 \times 10^{14}$ m$^2$ and a depth of 1 km, a suitable range of $e$ is on the order of $1 \times 10^{-7}$ to $3 \times 10^{-7}$ kg m$^{-2}$ s$^{-1}$.

3. The role of mechanical energy in THC

The external sources of mechanical energy for the world oceans have certainly changed greatly over time. Under different climate conditions, the meridional temperature gradient varies, and thus wind stress may greatly increase or decline. In fact, energy input due to wind stress has increased on the order of 10%–20% over the past half century, according to the National Centers for Environmental Prediction (NCEP) and European Centre for Medium-Range Weather Forecasts (ECMWF) datasets (Huang et al. 2006). Similarly, wind stress during the Last Glacial Maximum may have been much stronger than that of present day, and tidal dissipation in the open ocean may have been stronger than that of present day because of the disappearance of shallow seas. Egbert et al. (2004) studied the paleotidal circulation during the Last Glacial Maximum, and their results suggest that tidal dissipation at that time was 50% higher than that at present. The implication of these factors has not been fully explored in previous models based on either the buoyancy control or fixed diffusivity.

Allowing change of the energy parameter is a dramatic departure from the common practice in previous box models based on the buoyancy constraint, where the constant $c$ is treated as an invariant regardless of changes in climate conditions. The prominent differences between these two types of models are: the Stommel-like model possesses a thermal saddle-node bifurcation and the corresponding critical value of buoyancy parameter $c$ is a minimum (Fig. 2a), but the energy-constraint model has a haline saddle-node bifurcation and the corresponding threshold of energy parameter $e$ is a maximum (Fig. 2b). The sensitivity to the parameter $c$ in the Stommel-like model seems to resemble the
sensitivity to energy parameter $e$ in the energy-constraint model. Although the meaning of increasing the energy parameter is clearly linked to climate conditions, the dynamic meaning for increasing the buoyancy parameter $c$ is unclear.

4. The role of gyration to THC

a. How does gyration affect THC

For the Stommel-like model, the increase of WDC enhances MOR in the thermal mode and reduces MOR in the haline mode (Fig. 3a). This is consistent with previous results from OGCM experiments that THC is intensified as the wind stress or gyration is enhanced (Toggweiler and Samuels 1995). A similar phenomenon has also been reproduced with the aid of a box-loop coupled model by Pasquero and Tziperman (2004). However, the role of WDC to MOR is reversed in the energy-constraint model (Fig. 3b).

Note that when gyration is smaller than a critical value, the energy-constraint model has three steady solutions: a stable thermal mode, a stable haline mode, and an unstable haline mode; beyond this critical value the model has one stable solution, only a thermal mode. For the present-day ocean, $T_0 = 15^\circ C$, $S_0 = 35$, $p = 1$ m yr$^{-1}$, and $e = 2.5 \times 10^{-7}$ kg m$^{-2}$ s$^{-1}$; the basin is 4000 km long and 4 km deep; and the critical value is $\omega = 3.1$ Sv (1 Sv = 10$^6$ m$^3$ s$^{-1}$) (Fig. 3b). Therefore, if the model starts from an initial state in the thermal mode and with a small WDC, the solution will remain in the stable thermal mode. However, the situation can be quite different if the model starts from an initial state of a stable haline mode with a weak WDC. When WDC is increased beyond the critical value, the haline mode is no longer a viable state and the system must switch to the thermal mode through a catastrophic change.

b. The strength of wind gyre required for maintaining the current THC

Longworth et al. (2005) found that given a sufficiently strong wind-driven gyre, multiple equilibria in both the Stommel model and the Rooth (1982) model disappear. On the other hand, WDC is credited as a key factor for maintaining the THC under current climate conditions (e.g., Timmermann and Goosse 2004). An interesting question is, what is the strength of WDC to maintain the THC in the current climate state? As will be shown shortly, the estimation may depend on the choice of the model.

To show the relation between WDC and THC, we take a specific sample point in the parameter space: $T_0 = 15^\circ C$, $S_0 = 35$, and $p = 1$ m yr$^{-1}$. For the Stommel-like model, the stable haline solution disappears when $\omega \geq 7.2$ Sv, and the unstable thermal solution disappears when $\omega \geq 7.8$ Sv (Fig. 4a). Overall, there is an upper limit beyond which there are no multiple solutions, and this upper limit is insensitive to the buoyancy parameter $c$. For the energy-constraint model, there is an upper bound, too. In fact, for this set of $T_0$, $S_0$, and $p$, two haline solutions (a stable one and an unstable
one) disappear when gyration is stronger than 6.7 Sv for any value of the energy parameter $e$. However, for a given value of energy parameter $e$, the exact upper boundary, which consists of the parabola $E_1E_2$, for the existence of multiple solutions, can be much lower than this upper bound (Fig. 4b). In general, more available energy for mixing leads to a smaller upper limit of WDC beyond which there are no multiple solutions.

In the MOR–constraint–gyration parameter space (Fig. 4, upper panels), we have a clear view of THC bifurcation. For the Stommel-like model (Fig. 4a) there are three regions: in region A there are three steady states (one stable thermal mode, one unstable thermal mode, and one stable haline mode); in region B there are two steady states (one stable thermal mode and one unstable thermal mode); and in region C there is a single steady state (region $C_1$ has a single stable thermal mode and region $C_2$ has a single stable haline mode). For the energy-constraint model (Fig. 4b) there are two regions: in region A there are three steady states (one stable thermal mode, one stable haline mode, and one unstable haline mode) and in region C there is a single stable thermal mode only. From these three-dimensional figures we can take some section views. For example, Figs. 3a,b can be obtained when the energy parameter is fixed and a cutoff is made.

It is interesting to note that even if both gyration and energy parameters approach zero in the energy-constraint model, there is still a nonzero overturning circulation, which corresponds to the theoretical limit of

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**Fig. 4.** Bifurcation diagram of THC in the three-dimensional parameter space, with $T_0 = 15^\circ C$ and $S_0 = 35$. (top) The MOR–constraint–gyration space, $p = 1$ m yr$^{-1}$: (a) the Stommel-like model, where there is a threshold of gyration beyond which there is no multiple steady solutions of THC; (b) the energy-constraint model, where the upper bound for the existence of multiple steady states of THC is a parabola $E_1E_2$ in the $e$–$\alpha$ plane. “A” denotes a region with three steady states, “B” denotes a region with two steady states, and “C” denotes a region with a single steady state. (bottom) The freshwater–gyration–MOR space (note that freshwater increases from right to left): (c) the Stommel-like model, where $e = 5 \times 10^{-7}$ m$^4$ kg$^{-1}$ s$^{-1}$, line $A$–$B$ is the watershed or the lower limit of freshwater flux, and multiple states of THC exist when freshwater flux is larger than the values corresponding to this boundary; (d) the energy-constraint model, where $e = 2.5 \times 10^{-7}$ kg m$^{-2}$ s$^{-1}$ and point P is a watershed.
zero meridional density difference $\alpha \Delta T = \beta \Delta S$ (Fig. 4b). However, when the buoyancy parameter $c$ approaches zero, there is no circulation in the Stommel-like model. On the other hand, the present-day state of the thermal-mode circulation in the energy-constraint model can persist for the case without gyration, if there is a certain amount of mechanical energy. This is the “pure” thermohaline circulation driven by external mechanical energy from tides, very much like the case discussed by Munk and Wunsch (1998) and H99. The thermohaline circulation in this case is pure in the sense that the circulation exists independent of the wind stress.

5. Freshwater forcing versus wind forcing to multiple states of THC

Freshwater as a force for regulating THC has been explored extensively (e.g., Marotzke 1991; Stocker and Wright 1991; NW01). On the other hand, wind works as a pump for the overturning cell (Samelson 2004). Our results show that there is a threshold of freshwater flux, which marks the boundary of the bifurcation domain of THC, and these thresholds combine into a watershed of THC bifurcation. It represents lower limits of the multiple-state region for the Stommel-like model and upper limits of the multiple-state region for the energy-constraint model. The increase of freshwater flux slows down the thermal mode and speeds up the saline mode in the Stommel-like model, but the role of freshwater to THC is reversed in the energy-constraint model. However, WDC can elevate the watershed for both models. In particular, WDC with $\omega \geq 15$ Sv can eliminate the multiple states in the Stommel-like model.

What happens to THC when freshwater and wind forcing combine? The bifurcation picture of THC in freshwater–gyration–MOR parameter space for the Stommel-like model (Fig. 4c) is more complicated than that for the energy-constraint model (Fig. 4d). Similar to the upper panels of Fig. 4, here we can find the lower limit of freshwater forcing for the existence of multiple solutions. This watershed value is about $p = 0.5$ m yr$^{-1}$ (point P in Fig. 4d) for the given parameter set in the energy-constraint model; however, these lower-limit values appear as line A–B in the Stommel-like model. Figure 3 can be obtained if we make a section cut with a fixed freshwater forcing.

6. Summary

Stommel’s model helps us to understand THC profoundly. Despite many studies based on this model, one may make exciting new discoveries through close examination. For example, salinity and freshwater play complicated roles on the stability of the finite-amplitude perturbation of THC (Mu et al. 2004). In the past decade, the dynamical roles of external mechanical energy for the maintenance of THC were investigated (Munk and Wunsch 1998; H99, and among others); however, whether the energy constraint can be used as a useful tool for understanding THC remains an issue of hot debate (Gnanadesikan et al. 2005).

In the extended Stommel’s box model, both the buoyancy and energy constraint are included. The energy constraint can offer a significant advantage on rational interpretations of the transitions and shift between two THC modes. As climate conditions change, the amount of mechanical energy sustaining the wind-driven gyre and diapycnal mixing in the world oceans is altered. As a result, the thermohaline circulation in the oceans should change in response. Our results can be summarized as follows:

1) Analysis of a two-box model under both constraints demonstrates that in parameter space some OGCMs have a single stable mode of THC only (such as Vellinga et al. 2002), but some possess multiple stable modes (e.g., Weaver et al. 1993).

2) Including wind stress as a forcing may suppress the multiple solutions of THC. This is consistent with the result of Longworth et al. (2005). Our simple two-box model also indicates that there is a threshold in the strength of WDC beyond which there are no multiple steady states of THC. This threshold depends on the choice of the constraint regulating the circulation. In the Stommel-like model, it is insensitive to the buoyancy parameter $c$. On the other hand, in the energy-constraint model, it is sensitive to the energy parameter $e$. The meaning of such a threshold and its implications for oceanic circulation and climate remain unclear, and it is left for further study.

3) Some of the most important issues in the study of THC are catastrophes and sudden changes in the circulation because these phenomena are intimately related to abrupt climate changes. The essential question is what can trigger the sudden transition/bifurcation of THC? Many previous studies indicated that freshwater can be the trigger. For given initial conditions, freshwater flux can set the threshold of THC (Tziperman 2000). Our results show that the wind forcing is another potential trigger for bifurcation/catastrophe of THC. The Stommel-like model gives rise to the smallest value of freshwater flux for the existence of multiple states of THC, and these thresholds constitute a watershed for THC bifurcation.
All in all, THC is depicted as an ocean “conveyor.” It is well known that the freshwater may work as a brake and winds may work as a “mainspring.” The two-box models studied here are certainly highly idealized; thus, results obtained from such simple models should be interpreted with caution. Because of the simplicity of such box models, they have been widely used in many applications, including textbooks. We hope that this modified version of the model, equipped with the energy constraint and the wind-driven gyre, may be useful in studies related to other applications. A question is raised from our result: what should the real ocean should be, a thermal-unstable or a haline-unstable system? The immediate challenges are to determine the energy of THC in different modes and to develop the numerical model based on the energy constraint.

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