Turbulent energy production and entrainment at a highly stratified estuarine front

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[1] Rates of turbulent kinetic energy (TKE) production and buoyancy flux in the region immediately seaward (~1 km) of a highly stratified estuarine front at the mouth of the Fraser River (British Columbia, Canada) are calculated using a control volume approach. The calculations are based on field data obtained from shipboard instrumentation, specifically velocity data from a ship mounted acoustic Doppler current profiler (ADCP), and salinity data from a towed conductivity-temperature-depth (CTD) unit. The results allow for the calculation of vertical velocities in the water column, and the total vertical transport of salt and momentum. The vertical turbulent transport quantities ($\bar{u}w$, $\bar{v}w$) can then be estimated as the difference between the total transport and the advective transport. Estimated production is on the order of $10^{4}$. This rate of TKE production is at the upper limit of reported values for ocean and coastal environments. Flux Richardson numbers in this highly energetic system generally range from 0.15 to 0.2, with most mixing occurring at gradient Richardson numbers slightly less than $\frac{1}{4}$. These values compare favorably with other values in the literature that are associated with turbulence observations from regimes characterized by scales several orders of magnitude smaller than are present in the Fraser River.


1. Introduction

[2] Stratified shear flows become turbulent when the stabilizing influence of the stratification overcomes the velocity shear [Thorpe, 1971, 1973]. These flows can be found within the pycnocline in oceans, lakes, reservoirs, and estuaries as well as stratified regions of the atmosphere. The onset of turbulence in such flows greatly enhances vertical transport, affecting distributions of nutrients, sediments, and pollutants, as well as the dynamically important quantities of momentum and buoyancy. An understanding of these turbulent processes at scales relevant to geophysical flows is an important goal both for engineering applications in the natural environment and for a fundamental understanding of regional dynamics. Most investigations concerning the nature of shear-stratified turbulence, however, are based on laboratory experiments [e.g., Ivey and Imberger, 1991] or observations from relatively low-energy geophysical regimes [e.g., Gregg, 1989], due in large part to the difficulty involved in obtaining reliable measurements from more energetic oceanic environments [Gargett and Moum, 1995].

[3] As shown by Ivey and Imberger [1991], the status of the instantaneous energy budget of a turbulent field can be described by the overturn Froude number, $Fr_T = \left(L_O/L_T\right)^{\frac{3}{2}}$, and the overturn Reynolds number, $Re_T = \left(L_I/L_K\right)^{\frac{3}{2}}$, where $L_O = \left(\varepsilon/\mathcal{N}^2\right)$ is the Ozmidov scale, $L_K = \left(\nu^3/\varepsilon\right)^{\frac{1}{2}}$ is the Kolmogorov scale, and $L_I$ is a representative turbulent length scale. Here $\varepsilon$ represents the rate of dissipation of turbulent kinetic energy (TKE), $\nu$ is the kinematic viscosity of the fluid, and $\mathcal{N} = \left(g/\rho \partial\rho/\partial z\right)^{\frac{1}{2}}$ is the buoyancy frequency, where $g$ represents gravity, $\rho$ is the density of the fluid, and $z$ is the vertical coordinate. The Ozmidov and Kolmogorov scales are fundamental length scales of the turbulent field, with $L_O$ representing the largest eddy size that can be supported by a given turbulent dissipation rate within a region of specified stratification, and $L_K$ defining the smallest turbulent scales, below which the turbulent energy is dissipated into heat. The overturn Reynolds
number can also be written as $Re_T = \frac{1}{R_f} \left( \frac{\nu}{\nu_T} \right)$, and the expression $(\nu_T)$ is commonly used as a normalized expression for dissipation.

[4] If a turbulent field can be assumed homogeneous and isotropic, then the advection of TKE can be neglected, and the flux Richardson number can be defined as $R_f = \frac{\epsilon}{\nu}$, where $\epsilon$ is the dissipation rate. This ratio represents the fraction of TKE generated from shear production, $P$, that is converted to potential energy through buoyancy flux, $B$. A clear understanding of the nature of $R_f$ is critical for developing estimates of eddy diffusivity from observations of turbulent dissipation, as discussed by Osborn [1980], who suggested that $R_f$ should be equal to 0.15 based on observations of oceanic turbulence. Ivey and Imberger [1991] similarly suggested a range between 0.15 and 0.2 based on laboratory flows characterized by values of $n$ between approximately 10 and 100, with an overturn Froude number equal to 1. An overturn Froude number equal to 1 is consistent with most oceanic thermocline measurements [Gregg, 1987; Peters et al., 1988], where Kelvin-Helmholz billows are believed to be the primary mechanism of turbulence generation. Their laboratory data also indicated that $R_f$ should fall off rapidly with increases in both parameters, $\epsilon$ and $n$. Recent observations of atmospheric boundary layer turbulence [Pardyjak et al., 2002] have indicated $R_f$ values of approximately 0.2 for flows with gradient Richardson numbers $n = n^2 / (\epsilon / \nu)$ near the critical value of $1/n$, and values of $n^2 / \epsilon$ on the order of $10^3$. Similarly, Kay and Jay [2003a] have observed values of $R_f$ between 0.18 and 0.26 along a saline interface in the highly stratified Columbia River estuary during ebb tide, characterized by $n^2 / \epsilon$ values on the order of $10^3$ to $10^5$.

[5] Some recent studies have suggested values of $R_f$ as high as 0.4 associated with shear produced turbulence at a highly energetic tidal front. Convervally, Barry et al. [2001a, 2001b] suggest that the mixing efficiency for laboratory generated grid turbulence may decrease to zero for $\epsilon / \nu_T > 1000$, coincident with an increase in the overturn Froude number. Similar results have been observed in DNS simulations performed by Ivey et al. [1998]. Field observations of the interior of two stratified lakes, where turbulence is driven mainly by internal wave breaking, found negligible mixing efficiency at $n^2 / \epsilon$ values ranging from $10^{-1}$ to $10^{-3}$ [Etemad-Shahidi and Imberger, 2001], and similar conditions have been observed in the oceanic thermocline [Moun, 1996]. Etemad-Shahidi and Imberger suggest that the differences between these results and earlier laboratory studies are due to differences in the mechanisms triggering the turbulence. Similarly, Balmforth et al. [1998] have suggested that grid-generated turbulence may not be representative of shear-produced turbulence.

[6] This brief review suggests that there is considerable variability among existing observations of $R_f$. In 1991, Imberger and Ivey [1991] recognized that a “strong debate” had been underway for several years regarding the magnitude of $R_f$. Despite significant effort over the last decade, this debate remains unresolved, particularly for highly energetic flows.

[7] The classic estuarine outflow is characterized by a stratified shear flow, providing an ideal setting for investing
River discharge is dominated by the summer freshet, during which discharge typically increases by a factor of 10 relative to low flow, and net outflow velocities reach 0.5 to 1 m s$^{-1}$. The highly energetic estuarine environment generated through the interaction of these two strong forcing mechanisms results in the formation of a salt wedge with salinity differences of greater than 20 psu across only a few meters in the vertical and less than 500 m in the horizontal. This salt wedge advances landward some 10 to 20 km into the channel on each tidal cycle, only to be flushed back to the mouth daily during the strong ebb [Geyer and Farmer, 1989]. The front location remains nearly stationary at the mouth for several hours prior to advancing again on the following flood. During this period, the front is positioned just landward of a distinct bathymetric break, which transects the channel at an angle of approximately 45°, separating channel depths on the order of 10 m from bathymetry exceeding 50 m in the Strait of Georgia. The analyses presented here focus on the frontal zone during this stationary period.

The data evaluated in this study was collected on July 25, 1999, 5 days past neap tide. River discharge on July 25 was approximately 7500 m$^3$ s$^{-1}$, as measured at the most seaward gauging station, located at Hope, approximately 150 km upstream of the mouth. Data were collected from the R/V Clifford Barnes (University of Washington), using two hull-mounted ADCPs, operating at 1200 kHz and 300 kHz, and a “tow-yoed” Ocean Sensors 200 Series CTD unit. Combined use of the two ADCPs allowed for consistent bottom tracking coupled with a vertical resolution on the order of 25 cm across the top 25 m of the water column. Vertical resolution of the CTD unit was on the order of 5 to 10 cm. Data were collected from repeated passes across the front as shown in Figure 3.

Data from the passes indicate strong stratification and convective acceleration across the front, as shown in Figure 4. The bold lines in the figure represent mean “freshwater streamlines,” and indicate the bounding surfaces for specific fractions of the original river discharge. These surfaces were estimated based on fresh water conservation, assuming a salinity of pure Georgia Strait water of $S_o = 27$ psu,

$$Q_o = \int_{z_o}^{0} u \left( \frac{S_o - S}{S_o} \right) b dz,$$

where $Q_o$ is constant and equal to the total fresh water flux in the cross section, $u$ is the velocity parallel to a mean upper layer flow direction (defined as the $x$ direction), $S$ is the local salinity, and $b = b(x, z)$ is a plume width in the cross-stream direction. Using equation (1) and some simple assumptions regarding the shape of the vertical profile of $b$, the trajectory of fresh water streamlines can be evaluated. Sensitivity to the shape of the $b$ profile was tested by performing multiple calculations using a variety of profiles, including width profiles that were constant, linear, and parabolic with depth. The angle of the freshwater streamlines relative to the isohalines shows that the initially fresh discharge becomes progressively more saline through the liftoff zone. This can only be accomplished through the diffusive effects of turbulent mixing.

### 3. Estimation of Turbulent Fluxes

The control volume analysis used for the estimation of turbulent transport quantities relies on three conservation constraints: volume, salt, and momentum. The three resolved unknowns include the mean velocity normal to the bottom surface of the control volume and the vertical turbulent transport of salt and momentum. The results of the turbulent flux calculations for salt and momentum are independent of the choice of the control volume, but the calculated velocity across the bottom surface of the control volume is reference frame dependent, based on the orientation of the surface, as described in Appendix A. Here we
will calculate two quasi-vertical velocities, the diahaline velocity, $w_h$, taken relative to isohaline surfaces, and the “jet-entrainment” velocity, $w_j$, taken relative to freshwater streamlines.

[13] Volume conservation is used to identify the mean velocity across the bottom control volume surface. Written as an integral across the control volume surface, this can be expressed as

$$\int \int \vec{U} \cdot d\vec{A} = \frac{\partial V}{\partial t},$$

(2)

where $\vec{U}$ represents the local velocity at the control volume surface, $\vec{A}$ is a unit vector perpendicular to the surface, the integral represents summation over the entire control volume surface, and $V$ is the total volume contained within the control surface. Assuming no contribution from lateral fluxes (an assumption that will be discussed in further detail below and in Appendix B), the control volume continuity equation can be rewritten in differential form as

$$\frac{\partial}{\partial x} \left[ \int \int \text{budz} \right] + \bar{w} = \frac{\partial}{\partial t} \left[ \int \int \text{bdz} \right],$$

(3)

where the integrals are taken across the limits of the control volume, and $\bar{w}$ represents the mean velocity normal to the bottom surface of the control volume, which is equal to $w_h$ if the bottom control volume surface is coincident with an isohaline, or $w_j$ if aligned with a freshwater streamline.

[14] A similar treatment of the salt balance allows for estimation of the total vertical salt flux, $\text{Sw}$, starting with an expression for the integrated salt balance,

$$\int \int \int S \vec{U} \cdot d\vec{A} = \frac{\partial}{\partial t} \left[ \int \int \int S dV \right].$$

(4)

with salinity represented by $S$. This can be written in differential form as

$$\frac{\partial}{\partial x} \left[ \int S \text{budz} \right] - \bar{S} \bar{w} = -\frac{\partial}{\partial t} \left[ \int \int S \text{bdz} \right].$$

(5)

Once an estimate of the total vertical salt flux has been generated, the turbulent part can be extracted following a Reynolds decomposition, and making use of the quasi-vertical velocity estimated in equation (2),

$$\overline{\text{Sw}}^\prime = \overline{\text{Sw}} - \bar{S} \bar{w},$$

(6)

where $\bar{S}$ is the mean salinity of the bottom control volume surface. If the bottom surface is coincident with an

**Figure 4.** Cross section through the front for the three passes, (a) A, (b) B, and (c) C. Dashed contours represent salinity, in psu. Arrows represent streamwise velocity, based on the scale in the lower left-hand corner of the figure. Bold solid lines are freshwater streamlines, calculated through the conservation of freshwater as described in the text. CTD tow-yo track is shown in the background of each figure, and ADCP profile locations are identified by the dots near the bottom of each panel. Regions used for control volume analyses are delineated by the two vertical lines.
isohaline, \( S \) is equal to the salinity associated with the
isohaline.

[15] Treatment of the momentum balance requires knowl-
edge of the net force on the control volume associated with
the local pressure, but is otherwise similar to the salt
balance. Estimation of the turbulent momentum flux can
be represented by the integrated and differential forms of the
momentum equation,

\[
F_x = \sum_{C} \int_{S} u \rho \cdot dA + \frac{\partial}{\partial t} \int_{C} \int_{V} u \rho dV .
\]  
\( \sum_{C} \int_{S} u \rho \cdot dA + \frac{\partial}{\partial t} \int_{C} \int_{V} u \rho dV . \)  
\( \sum_{C} \int_{S} u \rho \cdot dA + \frac{\partial}{\partial t} \int_{C} \int_{V} u \rho dV . \)

\[
g \frac{\partial}{\partial x} \left( \int_{b} \left[ \int_{C} \rho d\pi + \frac{\partial}{\partial z} \right] d\pi \right) + \frac{\partial}{\partial x} \left( \int_{b} \int_{C} u^2 d\pi \right) - \Pi b
\]  
\( g \frac{\partial}{\partial x} \left( \int_{b} \left[ \int_{C} \rho d\pi + \frac{\partial}{\partial z} \right] d\pi \right) + \frac{\partial}{\partial x} \left( \int_{b} \int_{C} u^2 d\pi \right) - \Pi b
\]

\[
\overline{u \cdot w} = \overline{\Pi} - \overline{\Pi w},
\]

where \( F_x \) represents force in the streamwise direction
associated with the local pressure, and \( \eta \) is the local surface
displacement.

[16] This analysis method is similar in many respects to
the “method of constrained differences” described by Kay
and Jay [2003b], but utilizes an array of spatially distributed
data instead of profiles from a single location. This distinc-
tion is important for the present case, as convective ac-
celeration plays a key role in the momentum balance due to the
rapid rise of the pycnocline through the lift-off zone. The
convective acceleration term can be adequately constrained
only through an appropriate spatial analysis.

[17] The size of the control volumes used for the
analyses was on the order of 0.5 to 1 km in the \( x \) direction
(as identified in Figure 4). The top control volume surface
was always taken coincident with the water surface, so
that the quasi-vertical fluxes would be directed only across
the lower control volume boundary. The orientation of the
lower boundary, across which turbulent fluxes and normal
velocities are calculated, varied depending on the nature
of the conservation calculation (i.e., volume, salt, or
momentum). Although the calculations outlined above
are viable for any defined volume, so that a standard set
of boundary surfaces could be used for all calculations, it
is preferable to use separate boundary surfaces associated
with each process for a variety of reasons. For the diaha-
line velocity and turbulent salt flux estimates, a lower
boundary represented by isohalines was used. Not only
was this a natural choice for calculations involving salt
flux, but the tighter spacing of isohalines near the base of
the plume provided a higher resolution in this region
where the majority of salt flux occurs. Alternatively,
freshwater streamlines were used to identify the jet-en-
trainment velocity profiles and turbulent momentum flux
calculations. The freshwater streamline concept provides a
natural reference frame for further calculations involving
momentum fluxes, particularly in the present case, where
momentum enters the system associated with the fresh
river discharge. This choice provides enhanced resolution
of momentum fluxes near the surface, where there are
strongly sheared velocities, and strong gradients in the
turbulent flux of streamwise momentum. Time-dependent
terms were estimated by comparing subsequent passes, but
were generally small.

[18] Mean velocity and salinity profiles for the three
passes, representing the two independent variables in the
shear-stratified environment, are presented in Figure 5.
The highly sheared streamwise velocity profiles are pre-
sented in the first panel, with values of \( \Delta u \) on the order
of 2 m s\(^{-1}\). Likewise, the salinity profiles identify a zone

\[ \Delta U = 2 \text{ m s}^{-1} \]

\[ \rho = g \frac{\partial}{\partial x} S - 0.15 \text{ m s}^{-1} \]

\[ g = \frac{\Delta S}{\Delta t} - 20 \text{ psu} \]

\[ g = \frac{\Delta S}{\Delta t} - 20 \text{ psu} \]

Figure 5. Vertical profiles of measured quantities, velocity and salinity, in the control volume region for
each of the three lift-off passes. Plots represent mean profiles across the control volume region, which has
a streamwise dimension of approximately 1 km for passes A and B, and approximately 400 m for pass C.
of strong stratification between roughly 3 and 8 m below the surface, with a value of $\Delta \rho$ on the order of 20 psu, and a corresponding $g' = g\Delta \rho(\rho_o)^{-1}$ value of approximately 0.15 m s$^{-2}$. Profiles of the gradient Richardson number based on the mean profiles of Figure 5 are shown in Figure 6. Much of the shear layer has values of $R_{ig}$ less than 0.25.

### 3.1. Volume Conservation and Vertical Entrainment Velocity

[19] Diahaline and jet-entrainment velocities were calculated using the method described by equations (2) and (3), above, utilizing isohalines and freshwater streamlines, respectively, as bounding surfaces for the control volumes. Mean vertical profiles of $w_h$ and $w_j$ for each of the passes are presented in the two panels of Figure 7. Error ranges shown in Figure 7 represent the standard deviation of the results associated with five different assumptions regarding the vertical shape of the plume width function, $b$. This error is negligible at depths less than approximately five meters, and generally less than 30% at depth. The profiles of $w_h$ in the panel at the left show entrainment of both surface and deep water into a developing mixed layer centered at approximately 3 to 4 m depth and approximately 10 to 14 psu. In the panel on the right, the profiles of $w_j$ indicate that ambient fluid is being entrained into all portions of the initial discharging plume, with higher velocities at depth. At the bounding limit of the plume, $w_h$ is equal to $w_j$ because the lowest streamline of the expanding discharge is coincident with the 27 psu isohaline.

### 3.2. Salt Conservation

[20] Total vertical salt transport is calculated as described above, utilizing a series of isohaline surfaces as the bottom control volume surface. The first panel of Figure 8 shows profiles of turbulent salt flux, expressed as buoyancy flux,

$$B = g\beta \Delta \rho \bar{w}^2,$$

where $\beta = (\partial \bar{w} / \partial z) = 0.77 \times 10^{-3}$ psu$^{-1}$. In the second panel, profiles of eddy diffusivity, derived from the turbulent salt flux profiles $K_e = -\rho \bar{w} \bar{w}' (\partial \Delta \rho / \partial z)^{-1}$ are plotted. The buoyancy flux profiles are similar for the three passes, which peak near $2 \times 10^{-4}$ m$^2$ s$^{-1}$ in the middle of the pycnocline near 4 m depth. The profiles of eddy diffusivity indicate values of $K_e$ on the order of 2 to $6 \times 10^{-3}$ m$^2$ s$^{-1}$ across the middle portion of the water column. The negative values of both buoyancy flux and eddy diffusivity observed below about 8 m for pass B are unrealistic, and are most likely explained by unresolved lateral influxes of salt and volume.

### 3.3. Momentum Conservation

[21] The procedure for calculation of the total vertical momentum flux requires an estimate of net streamwise force on the control volume. A hydrostatic force balance is assumed, and the baroclinic contribution is estimated from density profiles within the control volume. The barotropic contribution is determined from a surface gradient, $\partial \bar{w} / \partial x$, estimated as the gradient required to produce a layer of no motion, as observed at depths sufficiently below the plume. The reference level for this layer was typically taken at 10 m below the surface, in order to extend the control volumes landward of the bathymetric break. In most cases, the 10 m depth was sufficiently deep to satisfy the no motion assumptions, but in the case of pass C the assumptions were invalidated due to weak landward currents in the lower layer near the front. In this case, the control volume region was shortened and moved offshore, as shown in Figure 4. Alternative approaches to estimating surface slope include the assumption that there is no stress divergence at the water surface, in which case the Bernoulli equation is valid (i.e., $\eta = u^2 + \frac{1}{2} \int \Delta \rho \bar{w}^2 dx$). Another approach is to invoke zero momentum flux across the lower boundary of the plume (defined as the 100% fresh water streamline). The surface elevation profile must then force a stepwise control volume calculation of turbulent momentum flux to zero at the lower boundary. Profiles of the surface elevation, $\eta$, are shown in Figure 9, where the layer of no motion estimate is compared with the Bernoulli and zero momentum transport alternatives. Note the general consistency between the three estimates over their range of overlap.
The water surface exhibits its steepest gradient across the frontal zone, where there is a significant conversion of potential to kinetic energy associated with acceleration in the plume.

Profiles of turbulent momentum flux, $u'w'$, are shown in Figure 10. These profiles peak between approximately 2 and 5 m below the surface at values on the order of $1 \times 10^{-3}$ m$^2$ s$^{-2}$. In Figure 11 vertical profiles for two quantities derived from the turbulent momentum flux are shown. In the first panel, profiles of TKE production are plotted, calculated as the product of the turbulent momentum flux and the local shear,

$$ P = -u'w' \frac{\partial u}{\partial z} $$

(11)

These profiles are similar in shape to the turbulent momentum flux profiles, with production peaking at values...
on the order of 0.5 to 1 \times 10^{-3} \text{ m}^2 \text{ s}^{-3}. Profiles of eddy viscosity 
\[ K_z = -\overline{u'w'}(\partial h/\partial x)^{-1} \] are shown in the second panel. These profiles are consistent in shape and magnitude with the profiles of eddy diffusivity shown in Figure 8, except near the base of the pycnocline, where the momentum flux estimates are less reliable.

Figures 9. Estimates of surface elevation across the lift-off region based on three different methods of estimating the surface slope, \( \partial h/\partial x \). The solid line represents the assumption of a layer of no motion, and zero stress divergence at a depth of 10 m below the surface. These results were used for subsequent momentum budget calculations. The dotted line represents the surface profile required to force the stress to zero at depth (\( \sim 15 \text{ m} \) below the surface) using a modified form of the control volume analysis. The dashed profile was derived using a Bernoulli approach based on the extrapolated surface velocities, assuming zero stress divergence at the surface. The dashed profile is terminated in the seaward direction where surface salinities exceeded 1 psu, providing evidence of mixing and an invalidation of the zero stress divergence assumption.

3.4. Lateral Effects

[24] A potential source of error to the control volume calculations is the lateral flux of volume, salt and momentum. A separate analysis of the importance of these errors was performed using the observed cross-channel velocity structure and an estimated cross-channel length scale, as described in Appendix B. In all cases, lateral flux contributions tend to be negligible in the upper portion of the water column but may increase rapidly below approximately 5 to 6 m, toward values on the order of the contributions of vertical advective and turbulent fluxes to the momentum and salt balances. Calculations in the upper water column are relatively unaffected by lateral fluxes due to the alignment of the streamwise direction with the mean upper layer flow direction. Further discussion of the control volume results is limited to those within the upper region of the water column where errors associated with lateral fluxes are small.

4. Turbulence, Entrainment, and Closure

[25] The turbulent quantities estimated in section 3 were derived from mean flow quantities by closing budgets of volume, salt, and momentum. Owing to the nature of the calculations, individual fluctuating quantities were not estimated independently, but only as turbulent fluxes, represented by correlations (e.g., \( \overline{S'W'} \)). The estimated turbulent quantities represent means over the observation period, and across the length of the control volume, which was generally between 0.5 to 1 km. The observation period was equal to the time required for the observation vessel to transit along the length of the control volume, which ranged from 3 to 7 min.

[26] The TKE dissipation rates shown in Figure 12 represent a large amount of turbulent energy in comparison to other studies from ocean and coastal environments. For

Figures 10. Turbulent momentum flux \( \overline{u'w'} \) profiles from control volume analysis. Each estimate is bracketed by standard deviation as in Figure 7. Note that estimates below approximately 5 to 6 m are influenced by unresolved lateral fluxes as discussed in the text.
example, Gregg [1989] reports that dissipation rates within the oceanic thermocline typically fall within a range of order $10^{-10}$ to $10^{-6}$ m$^2$ s$^{-3}$. Higher rates have been observed in tidal channels and in estuaries, but rarely exceeding $10^{-4}$ m$^2$ s$^{-3}$, and typically several orders of magnitude lower [see Grant et al., 1962; Gargett and Moum, 1995; Peters, 1999]. Corresponding values of turbulent eddy diffusivities are on the order of $0.5 \times 10^{-2}$ m$^2$ s$^{-1}$, as shown in Figures 8 and 11. These values are more typical of observed $K_z$ values in ocean and coastal environments [e.g., Ledwell et al., 1993; Osborn, 1980], due to the intense stratification present. The large values of dissipation relative

Figure 11. TKE production ($P$), and eddy viscosity ($K_z^n$) profiles from control volume analyses. Each estimate is bracketed by standard deviation as in Figure 7. Note that estimates below approximately 5 to 6 m are also influenced by unresolved lateral fluxes as discussed in the text. Eddy viscosity profiles for passes B and C exceed the bounding limits of the plot at depth. The scale was constrained to provide enhanced detail in the central portion of the water column.

Figure 12. TKE dissipation ($\varepsilon$), and $\varepsilon/\nu N^2$ profiles derived from production and buoyancy flux estimates shown in Figures 11 and 8, respectively. Each estimate is bracketed by standard deviation as in Figure 7, and subject to unresolved lateral fluxes below approximately 5 to 6 m, as discussed in the text.
to diffusivity are explained by the unusually strong vertical density gradients in the frontal zone.

[27] A third expression of turbulent intensity, and one that can be corroborated with independent observations, is the outer length scale of the turbulence, represented by the Ozmidov scale, and the corresponding observed overturn scale. In general, Ozmidov scales associated with the three passes ranged from 15 to 35 cm. Profiles of the local Ozmidov scale for each of the three passes are shown in Figure 13, compared with observed overturn scales [Thorpe, 1977] for three distinct regions associated with each pass, as represented by the shaded bars. The overturn scales shown here were calculated as the RMS value of all non-zero displacements observed within each region (0–5, 5–15, and >15 psu). Representative overturn Froude numbers for each region, based on the observed Ozmidov and overturn scales, are also shown in Figure 13.

[28] Within the middle region (5–15 psu), where there is linear shear and stratification, and the control volume estimates are reliable, the Ozmidov and overturn scales suggest an overturn Froude number on the order of 1.0 to 1.4. Although the exact value of this ratio may be a function of the evolution of the turbulence field [e.g., Wijesekera and Dillon, 1997; Gibson, 1998] or the gradient Richardson number [Baumert and Peters, 2000], the overturn Froude number is generally of order 1 for all three passes.

4.1. Flux Richardson Numbers

[29] Estimates of the flux Richardson number can be generated from the independently estimated production and buoyancy flux profiles, yielding values on the order of 0.15 to 0.25. A single $Ri_f$ value was calculated for each pass as the mean of individual $Ri_f$ estimates taken at 1 psu intervals between 5 and 15 psu, where the shear and stratification are both generally linear. These values are plotted with respect to the overturn Froude number, and $\overline{N_2}$ in Figures 14 and 15, respectively. A comparison

Figure 13. Profiles of Ozmidov scale, $L_O$, for each of the three passes based on the dissipation profiles shown in Figure 12, and the stratification shown in Figure 5. Each estimate is bracketed by standard deviation as in previous figures. Shaded horizontal bars represent mean observed overturn scales, $L_T$, for three salinity ranges: 0–5, 5–15, and >15 psu. Note that the observed overturn scale for the >15 psu range for Pass A extends beyond the range of the axes. Bold numbers represent the mean value of the overturn Froude number, $Fr_T = (L_O/L_T)^{\frac{3}{2}}$ within each of the regions.

Figure 14. Mean values of the flux Richardson number (B/P) for each of the three passes, A, B, and C (represented by the circle, square, and triangle, respectively) plotted against the overturn Froude number. Points shown represent the mean of calculated values at isohalines from 5 to 15 psu. The shaded region represents the results of DNS simulations of a stratified shear flow for a variety of Prandtl numbers from 0.1 to 2.0, as described by Ivey et al. [1998, Figure 2].
Figure 15. Mean values of the flux Richardson number (B/P) for each of the three passes, A, B, and C (represented by the circle, square, and triangle, respectively) plotted against \( v/N_e^2 \). Points shown represent the mean of calculated values at isalines from 5 to 15 psu, as in Figure 14. Shaded region defines extent of data compiled by Barry et al. [2001b, Figure 7] representing the results of laboratory experiments using grid-generated turbulence. Data represented by the shaded region are drawn from Stillenger et al. [1983], Rohr [1985], Itsweire et al. [1987], and Barry et al. [2001b].

with Figure 6 also suggests that \( R_i \approx R_f \), indicating that vertical eddy diffusivities for mass and momentum are equal [Ellison, 1957], consistent with a comparison of the vertical profiles of \( K_i^e \) and \( K_e^e \) in Figures 8 and 11.

[30] In order to determine whether the turbulence in the pycnocline might be explained by advection from the bottom boundary layer in the river channel, the magnitude of turbulent production in the bottom boundary layer and its advective contribution to the lift-off zone was assessed. Boundary layer shear production in the unstratified channel can be estimated assuming a log-layer shear profile as \( P = u^* \frac{\nu}{z} \), where \( u^* \) is a local shear velocity (\( u^2 \approx C_d \bar{u}^2 \), \( C_d \) is a bottom drag coefficient, taken as \( 3 \times 10^{-3} \)), \( \kappa \) is von Kármans constant (equal to approximately 0.4), and \( z \) is the distance above the bed. This boundary layer production scale, on the order of \( 0.5 \times 10^{-3} \) m\(^2\) s\(^{-3}\), is comparable to that observed within the stratified shear flow of the lift-off region.

[31] A length scale for the advection of this boundary layer turbulence can also be estimated by making an assumption that the observed production is balanced completely by advection, so that \( P = u^* \frac{\nu}{z} \). This relationships yields an extinction length scale, \( L_x = \frac{u^*}{P} \), which, for a representative height above the bed of \( 2.5 \) m is on the order of \( 20 \) m. This is nearly 2 orders of magnitude smaller than the streamwise length scale of the lift-off region, indicating that the observed turbulent production could not have been advected seaward from the unstratified boundary layer. Thus the turbulence observed in the stratified shear layer must be produced locally.

4.2. Local Production Versus Adverted TKE

[32] In order to determine whether the turbulence in the pycnocline might be explained by advection from the bottom boundary layer in the river channel, the magnitude of turbulent production in the bottom boundary layer and its advective contribution to the lift-off zone was assessed. Boundary layer shear production in the unstratified channel can be estimated assuming a log-layer shear profile as \( P = u^* \frac{\nu}{z} \), where \( u^* \) is a local shear velocity (\( u^2 \approx C_d \bar{u}^2 \), \( C_d \) is a bottom drag coefficient, taken as \( 3 \times 10^{-3} \)), \( \kappa \) is von Kármans constant (equal to approximately 0.4), and \( z \) is the distance above the bed. This boundary layer production scale, on the order of \( 0.5 \times 10^{-3} \) m\(^2\) s\(^{-3}\), is comparable to that observed within the stratified shear flow of the lift-off region.

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4.3. Vertical Velocities

[34] An entrainment coefficient can be defined as the ratio of an entrainment velocity to a representative streamwise velocity. As defined by Morton et al. [1956] and Ellison and Turner [1959], the term entrainment refers to the rate of increase of streamwise volume flux within a turbulent region. With respect to the Fraser lift-off, the most appropriate depth at which to evaluate an entrainment velocity would be at the base of the plume, represented in this study by the 100% freshwater streamline, which is coincident with the 27 psu isohaline. Although the calculated values of \( w_h \) and \( w_l \) at the plume base are likely to be overestimated due to unresolved lateral flows, as described in Appendix B, they can be used to identify an appropriate scale for the laboratory and numerical data occupy a different portion of the \( Fr_T - \frac{1}{\sqrt{N}} \) parameter space than the present data set, and, ultimately, no independent comparison to either parameter can adequately characterize the turbulent field. This reemphasizes the importance of the generating mechanism on the nature of the resulting turbulence, specifically with respect to the establishment of turbulent length scales.

[35] It should also be noted that high values of \( \frac{1}{\sqrt{N}} \) in the DNS simulations [i.e., Ivey et al., 1998] and many of the laboratory grid experiments [i.e., Barry et al., 2001a, 2001b] were driven by vanishingly small values of \( N^2 \). This results in Ozmidov scales that are larger than the length scales imposed on the problem by the numerical domain or tank size, which may limit the functional scale of the turbulence, effectively increasing the overturn Froude number, and driving \( R_i \) values down. In the Fraser liftoff region, and in many other geophysical examples of stratified shear turbulence [e.g., Gargett and Moum, 1995], \( N^2 \) is high, and high values of \( \frac{1}{\sqrt{N}} \) are driven by large dissipation rates.
entainment velocity on the order of 1 to 4 x 10^{-3} m s^{-1}. It should be noted that this entainment velocity can be directly related to the maximum turbulent salt flux, $\bar{S}w'$, which is observed higher in the water column, as a function of the vertical structure of the shear and density profiles. Assuming linear profiles of shear and density, it can be shown that the ratio $\bar{S}w' = \frac{S_r}{\rho w}$ should equal to a value of approximately 0.4, where $S_r$ represents the salinity of the ambient seawater, and $w$ is the entainment velocity at the base of the plume. Using this relationship, and an observed maximum value of $B = g\bar{S}w'$ equal to approximately $2 \times 10^{-4}$ m$^2$ s$^{-3}$ (see Figure 8) a corresponding entainment velocity would be equal to approximately $2.5 \times 10^{-3}$, which is consistent with the range suggested above.

[35] Given mean streamwise velocities in the upper layer of approximately 2 m $s^{-1}$, an appropriate entainment coefficient, $E$, is of order $10^{-3}$. **Ellison and Turner** [1959] suggested a dependence of the entainment coefficient on a bulk Richardson number, $\frac{E}{\sqrt{gH}}$, where $H$ is the plume depth and $U$ is a representative plume velocity. Bulk Richardson numbers in the Fraser lift-off are on the order of 0.5 to 1, with an average value on the order of 0.7. Although the results of Ellison and Turner only represent flows with bulk Richardson numbers less than about 0.3, their data suggest a decreasing trend in $E$ with increasing $Rig$, yielding entainment coefficient values of order $10^{-2}$ at the upper end of their $Rig$ range. Other data, compiled by **Christodoulou** [1986], extend the range of observations to values of the bulk Richardson number as high as $10^2$, and suggest a value of the entainment coefficient on the order of $10^{-2}$ for $Rig \sim 1$, which is consistent with the present observations.

[36] An earlier study of entainment in the Fraser River plume **Cordes et al.**, 1980] focused on a region of the plume extending approximately 40 km from Sand Heads (as opposed to 1 to 2 km in the present study). Their results suggested an entainment rate on the order of 1 to $5 \times 10^{-3}$, roughly an order of magnitude lower than the value estimated in the present study. The data sets utilized by **Cordes et al.** [1980] could not be used to determine an accurate estimate of $Rig$, although later experiments by **Stronach** [1977] have suggested values of order 1 in similar regions of the plume. The entainment coefficients of Cordes et al. suggest a value of $Rig$ of approximately 2 or 3, based on the data compiled by **Christodoulou** [1986], which appears reasonable given the observed values of $Rig$ in the first kilometer of the plume, and the associated deceleration of the plume at more significant distances from Sand Heads. The rapid transition and high momentum fluxes associated with plume lift-off in the first kilometer of the plume are responsible for driving more intense mixing processes, and lower values of $Rig$, than would be associated with the far-field, buoyancy driven plume.

[37] **Dallimore et al.** [2001] report entainment coefficients for a saline underflow into a freshwater lake that range from $10^{-4}$ to $10^{-3}$ for $Rig$ values on the order of 1. These are somewhat lower than the values observed in the Fraser, probably due to their calculation of $E$ based on the turbulent transport of salt (i.e., $\bar{S}w'$) rather than volume considerations as done here, and by **Ellison and Turner** [1959]. The results of **Dallimore et al.** [2001] are more directly comparable with the turbulent salt flux profiles generated in this study, which is undertaken in the following section.

[38] **Strang and Fernando** [2001] investigated entainment rates across a sharp interface, with values of $E$ approximately an order of magnitude larger at $Rig \sim 1$, than observed in this study. This discrepancy can be explained by the natural adjustment of the turbulence field in the Fraser liftoff toward uniformly critical gradient Richardson numbers through a broadening of the mixed layer. Within this mixed layer the density profile exhibits a departure from the two-layer case (sharp interface) toward a more linearly stratified region, resulting in a significant decrease in $Rig$. For critical values of $Rig$, **Strang and Fernando** [2001] observed a value of $E$ on the order of $10^{-3}$, similar to the present study. In situations where flow conditions continuously adjust to maintain a broadening region of critical $Rig$, which appears to be the case in the Fraser lift-off, the entainment coefficient would be approximately constant at the observed value.

### 4.4. Mechanisms of Turbulence Generation

[39] The most likely mechanism for the input of energy from the mean horizontal flow into turbulence is the generation of Kelvin-Helmholz billows through the propagation of shear instabilities. This is consistent with the observation of overturn Froude numbers close to unity, which is similar to several other large-scale field experiments [e.g., **Ferron et al.**, 1998]. The development and collapse of these instabilities in a laboratory flow similar to the Fraser lift-off is well described by **Pawlak and Armi** [2000].

[40] **Imberger and Ivey** [1991] indicate that the only two independent variables in a stratified shear flow are the shear, represented by $\Delta u$, and the stratification, which can be represented by $g'$. On the basis of the velocity and salinity observations shown in Figure 5, representative values of $\Delta u$ and $g'$ for the Fraser lift-off are approximately $2$ m s$^{-1}$, and $0.15$ m s$^{-2}$, respectively. Using these variables, and the definition of the gradient Richardson number, the gradient zone thickness, $\delta$, can be expressed as $\delta = Rig \frac{\Delta u^2}{g'}$. The observed thickness of the gradient zone is consistent with this relationship, given the representative values of $\Delta u$ and $g'$, and a gradient Richardson number on the order of $10^{-3}$.

[41] The ratio of the observed dissipation rate to a dissipation rate scaled from the independent variables, $(\Delta u'g')$, is on the order of $2 \times 10^{-3}$. This ratio is proportional to an interfacial drag coefficient, $C_{Di}$, divided by the gradient Richardson number,

$$\frac{\varepsilon}{(\Delta u'g')} = \left(1 - Rig \right) \frac{C_{Di}}{Rig} = 2 \times 10^{-3}, \quad (12)$$

suggesting an interfacial drag coefficient on the order of $5 \times 10^{-4}$. Assuming a Prandtl number of 1, with $Rig \sim Rig$, $C_{Di}$ is also equivalent to an alternative definition for the entrainment coefficient, $C_{Di} = \frac{\varepsilon}{5 (\Delta u'g')}$. [e.g., **Dallimore et al.**, 2001]. The value of $5 \times 10^{-4}$ is in good agreement with similar observations made by **Dallimore et al.** [2001] of a saline underflow into a freshwater lake.

[42] These values represent the efficiency with which energy is extracted from the mean flow and converted into...
turbulent energy, indicating that the turbulent energy is approximately 3 orders of magnitude lower than the mean flow energy. The values presented here may be representative of the processes involved in the generation of stratified-shear turbulence, but may not be constant. There may be a dependence of the drag coefficient on the bulk or gradient Richardson numbers, similar to the relationship that appears to hold for the entrainment coefficient [e.g., Ellison and Turner, 1959; Christodoulou, 1986]. Also, similarly to the flux Richardson number observations, these values may only be valid for overturn Froude numbers of order one, indicating turbulence generation through the formation of Kelvin-Helmholz instabilities.

The stratified mixing generated within an estuarine outflow may also be dependent on larger-scale aspects of the plume structure, particularly limitations on lateral plume expansion due to local bathymetry or ambient conditions in receiving waters. Imposed limits on expansion would affect turbulent buoyancy flux through their influence on velocities within the plume. Under conditions where plume expansion is restricted, streamwise gradients of velocity within the plume will be reduced relative to an unconstrained plume. The rate of mixing within the plume may depend not only on the velocity difference within the plume, but also on the rate at which the velocity changes. Thus the geometry of the plume may influence the entrainment rate. The less of a lateral constraint, the more rapidly the plume accelerates, and the more intense the mixing would be within the pycnocline. The Fraser plume is partially constrained, both by the shoals to the south and the ambient southward flow. This may reduce the rate of mixing slightly from the value found in a plume with no lateral constraints.

The net effect of these dynamic considerations may complicate predictions of TKE fluxes based solely on known river discharge and density differences. However, at local scales, it is likely that the formulation and value of the interfacial drag coefficient presented in equation (12) may well be representative of many similar stratified shear flows.

5. Conclusions

This paper has presented a robust control volume approach for estimating TKE quantities in a stratified-shear flow. The resulting estimates of turbulent dissipation, on the order of $10^{-3}$ m$^2$ s$^{-3}$, with values of $\overline{u'^2}$ on the order of $10^4$, are high compared to most other observations of dissipation in oceanic and coastal environments, and provide a good setting for the evaluation of turbulence energetics in a high-energy shear-stratified system. Observed flux Richardson numbers between 0.15 and 0.25 are consistent with less energetic laboratory and field observations where the overturn Froude number is of order one, and several other field studies at similar values of $\overline{u'^2}$. A scaling analysis based on the two independent variables in the system, $\Delta u$ and $g'$, suggests that entrainment and turbulent production may be well described through the use of entrainment and interfacial drag coefficients.

Appendix A: Reference Frames

In a stratified flow, a natural reference frame for quantifying vertical transport is perpendicular to isopycnal surfaces [e.g., Dallimore et al., 2001]. Similar motion can also be observed in unstratified conditions, such as entrainment into a turbulent jet [e.g., Tennekes and Lumley, 1972, pp. 127–132]. In this case the typical reference frame is a mean streamline bounding the initial fluid contained within the jet, or some volumetric subset of the initial jet. Assuming the receiving waters of a horizontally discharged buoyant plume are of constant density, the bounding edge of the plume must lie along an isopycnal, so that the entrainment velocity into the plume is independent of the reference frame imposed on the calculations. Within the interior of the plume, significant differences between the two reference frames would be observed depending on their respective orientation.

Generically, a normal velocity can be defined as

$$\overline{w_e} = \overline{u} \cdot \overline{n_e},$$

where $\overline{w_e}$ represents the velocity across the surface defined by the normal unit vector $\overline{n_e}$, and $\overline{u}$ is the local velocity vector. We refer to two velocities, the diahaline velocity, $\overline{w_d}$, which represents entrainment across isohalines, and the “jet-entrainment” velocity, $\overline{w_j}$. Diahaline velocities can be of either sign depending on position in the water column, as a growing mixed layer must incorporate fluid from both directions if the layer is to grow. In order to conserve salt on an isohaline surface, all fluid crossing an isohaline must be diluted through mixing, indicating that the turbulent component of salt flux and the diahaline velocity are related processes [McDougall, 1984].

The jet entrainment velocity represents entrainment into a region defined by a specified fraction of the original discharging fluid. In the case of a buoyant plume discharged at the surface, this implies that all jet entrainment velocities must be of the same sign, and directed upward. The simple diagrams in Figure A1 show the distinction between these two reference frames.

Appendix B: Lateral Effects

In addition to measurement errors associated with the raw variables, a potential source of error to the control volume calculations is the lateral import or export of volume, salt and momentum. A separate analysis of data from the 2000 freshet, as discussed by MacDonald [2003], has suggested that the flow in this region of the channel is nearly two-dimensional, particularly above the 20 psu isohaline, based on a three-dimensional salt balance and the relative contribution of lateral (southward-directed) salt flux to the overall salt budget in the region. However, due to the complexities of the interactions between entrainment and turbulence in generating the vertical transport, there are multiple pathways through which lateral fluxes may impact the calculations. An estimate of the magnitude of the contributions of lateral flux to the overall volume, momentum, and salt budgets is, therefore, an important goal.

These estimates can be obtained using the observed cross-stream velocity profile, and estimating a cross-channel length scale, $l_C$, which represents the distance to the channel boundary across which the lateral flow must decrease to...
A profile of lateral volume influx can be estimated by integrating the cross-stream velocity profile,

\[ \hat{V}(h) = \int_{s_1}^{s_2} v \left( \frac{h}{T} \right) dz dx \]  

where \( \hat{V} \) represents the lateral volume influx, and \( v \) is the cross-stream velocity.

The first panel of Figure B1 shows profiles of cross-stream velocity averaged across the control volume for each of the passes. In the second panel, the ratio of lateral volume influx to the maximum volume flux divergence across the control volume in the streamwise direction (in this case representing the lowest calculation point in the water column) is plotted. The curves in the second panel were generated using a depth dependent value of \( l_c \) based on local bathymetry. The cross-stream velocity
profiles in the first panel show relatively small and balanced velocities in the top 4 m of the water column, with more intense southward-directed velocities, approaching 30 cm s\(^{-1}\), in the lower portion of the water column. It is the flux in this lower region that supplies most of the salt to the lift-off zone [MacDonald, 2003] and is most likely to influence the salt balance and thus impact the turbulence calculations.

As expected, based on the velocity profiles in the first panel, the ratio in the second panel of Figure B1 increases rapidly below about 5 m, with errors that are generally negligible in the upper half of the water column.

**Figure B1.** (a) Vertical profiles of the cross stream velocity, positive values directed to the right of the discharging flow (northward). (b) Profile of the estimated lateral volume flux to the maximum observed streamwise volume flux divergence, as described in Appendix B.

Significant errors are identifiable in both cases in the lower portion of the water column below approximately 6 m depth, with the salt flux calculations more susceptible to lateral errors than the momentum calculations.

**Figure B2.** Lateral influx ratios for both (a) salt and (b) momentum. The ratios represent the cumulative effect of lateral influx (including both the direct lateral flux divergence of salt and streamwise momentum, and the indirect effect of lateral volume flux affecting entrainment estimates) divided by the maximum calculated values of \(\overline{S'w'}\) (shown in Figure 8 as buoyancy flux) and \(\overline{u'w'}\) (shown in Figure 10). Significant errors are identifiable in both cases in the lower portion of the water column below approximately 6 m depth, with the salt flux calculations more susceptible to lateral errors than the momentum calculations.
The negative sign of the ratio indicates that the calculated values of $\nu_1$ and $\nu_2$ are likely to be overestimated, and that velocities may begin to decrease below about 5 m rather than the monotonic increase suggested by the curves in Figure 7.

[53] Uncertainty in the value of the vertical entrainment velocity propagates into the turbulence flux estimates directly through the mean transport terms of equations (5) and (8), where it is amplified by the mean value of the transported quantity. Hence, potential errors in the lower portion of the water column due to errors in the vertical velocity are greatly amplified in the turbulent salt flux calculations due to the high value of salinity at depth. Similarly, errors are reduced in the momentum flux calculations due to the small along-channel velocities at depth.

[54] Additional errors are introduced to the turbulence calculations through the direct lateral flux of momentum and salt. These errors are typically of opposite sign to the entrainment related errors, reducing the overall impact of the lateral influx. Figure B2 presents plots similar to those shown in the second panel of Figure B1, showing the combined effect of both of these mechanisms (entrainment velocity errors induced by the lateral influx of volume, and the direct lateral flux of salt and momentum) on the salt and momentum budget calculations. These plots, combined with the curves presented in Figure B1, clearly indicate that a high degree of uncertainty is associated with all calculations at depths below approximately 5 to 6 m. Discussion in the main text is therefore limited to the upper region of the water column where errors associated with lateral fluxes are small.

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