ON THE INFLUENCE OF HORIZONTAL TEMPERATURE LAYERS IN SEA WATER ON THE RANGE OF UNDERWATER SOUND SIGNALS

H. LICHTE

TRANSLATED BY A.F. WITTENBORN

FOREWORD BY R.J. URICK

Tracor Sciences & Systems

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FOREWORD

by R. J. Urick

Any branch of science evenually reaches such a state of maturity that scientific historians, with their perspectives mellowed and distorted by subsequent technological advances, can write histories about the beginnings and subsequent developments of their subject. Underwater sound has for some time reached such a state. A number of historical summaries have appeared. Most notable are the works of T. G. Bell,(1) A. B. Wood,(2) E. Klein,(3) M. Lasky,(4)(5) all of whom have described what was done in the United States and in England during World War I and during the intervening years between Wars I and II.

During a recent visit to this country, Prof. Dr. Ing. Günter H. Ziem (of the Forschungsanstalt des Bundesweehr für Wasserschall and Geophysik, Kiel, Germany) called the attention of the writer to an article on underwater sound that appeared in the Physikalische Zeitschrift for 1919*. This paper is particularly noteworthy in that it appears to antedate any other scientific paper on underwater sound. It is remarkable for its scientific perception of the underlying phenomena of sound propagation in the sea. For example, the author correctly deduces the effects of temperature, salinity and depth on sound velocity and uses the existing static measurements on sea water, as did Kuwahara some twenty years later, to compute the velocity of sound in terms of its determining variables. He concludes that, like sound propagation up-wind and down-wind in air, sound ranges should be better in winter than in summer because of refraction,

*The WWI armistice was signed on 11 Nov. 1918.

and backs up this conclusion by field data obtained in a variety of shallow water areas. He says that in deep water the upward refraction produced by pressure should produce extraordinarily long sound listening ranges -- a fact that we have only recently been able to exploit in sonar systems. All in all, the paper is an astonishing scientific achievement on its subject for its time, and is doubtless the outgrowth of the tremendous capability of early twentieth century German physics applied to the defense of the Kaiser's U-boat fleet during World War I.

The paper has been most ably translated by an eminent underwater acoustician, Dr. A. F. Wittenborn, of Tracor. It is presented by both of us with a sense of awe and humility of the great understanding of the physics of the sea reached by German physicists during the difficult, troublesome years of World War I.

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(1) T. G. Bell, Sonar and Submarine Detection, USN Underwater Sound Lab. Report 545, 1962.

(2) A. B. Wood, From the Board of Invention and Research to the Royal Naval Scientific Service, Jour. Roy. Nav. Sci. Serv. 20, 16, 1965.

(3) E. Klein, Underwater Sound Research and Applications Before 1939, J. Acous. Sci. Am. 43, 931, 1968.

(4) M. Lasky, Review of World War I Acoustic Technology USN J. Underwater Acous. 24, 363, 1973.

(5) M. Lasky, A Historical Review of Underwater Acoustic Technology 1916-1939, J. Underwater Acous. 24, 597, 1974.

On the Influence of Horizontal Temperature Layers in Seawater on the Range of Underwater Sound Signals

By H. Lichte

With the resumption of ship traffic to and from German harbors, the providing of mine-free shipping lanes has become of increased importance. One of the more important aids to this process is the use of underwater sound signals. Therefore it is of interest to explore the propagation of sound in water.

There is a general opinion that water is significantly better for the transmission of sound signals than air because it is more homogeneous. This, however, is not the case. On the contrary, for a variety of reasons, water is acoustically inhomogeneous in horizontal layers. As a result, a deviation of sound rays from straight lines, i.e., a bending of the sound rays takes place. The various causes for this bending will be examined in the following. For the sake of clarity, numerical examples will be given for specific cases.

The velocity of sound, v, in a medium is dependent on the density, ρ, and the compressibility, χ, of the medium according to the relationship

$$v_o = \frac{1}{\sqrt{\chi_o \rho_o}} \qquad (1)$$

For water, the compressibility has been measured to be 0.000049 atm^{-1}, or, in CGS units[1],

$$\frac{(980.66)(1033.3)}{0.000049} \frac{cm\ sec^2}{gm} \qquad (2)$$

Thus, the velocity of sound is

$$v_o = \sqrt{\frac{1013300}{0.000049}} \text{ cm/sec} \tag{3}$$

$$= 1439 \text{ m/sec.} \tag{4}$$

The quantities which determine the sound velocity, compressibility and density, are dependent on various factors, but most importantly on temperature, salinity and pressure.

We will now first determine the effect of these factors, and then examine the path of sound rays in a heterogeneous water layer. The effect of temperature is the most important. It will therefore be considered first.

Let us assume that the temperature of the water decreases uniformly with depth from the surface to the bottom. We place the x axis (fig. 1) at the water surface with the y axis perpendicular, pointing downward. Since we can consider the temperature as constant and a sound ray as linear in a very thin layer, we get, from the refraction law,

$$\frac{v}{\sin \alpha} = \frac{v_o}{\sin \alpha_o} = \frac{v_o}{\cos \delta} \tag{5}$$

Here, v is the sound velocity and α is the angle of incidence in an arbitrary layer Δy. The quantities with the index o correspond to the water surface. δ is the initial angle from the horizontal for the ray under consideration. The ray begins at the coordinate origin. Then from fig. 1,

$$\frac{\Delta x}{\Delta y} = \tan \alpha = \frac{\sin \alpha}{\sqrt{1-\sin^2 \alpha}}$$

$$= \frac{\frac{v}{v_o} \cos \delta}{\sqrt{1 - \left(\frac{v}{v_o}\right)^2 \cos^2 \delta}} \tag{6}$$

Since

$$\frac{v}{v_o} = \sqrt{\frac{\chi_o \ \rho_o}{\chi \rho}} \tag{7}$$

where the depth dependence for χ and ρ is given by

$$\chi = \chi_o \ (1 + \varepsilon \ y) \tag{8}$$

and

$$\rho = \rho_o \ (1 + \gamma \ y), \tag{9}$$

then

$$\frac{\Delta x}{\Delta y} = \frac{\frac{\cos \delta}{\sqrt{(1 + \varepsilon \ y)(1 + \gamma \ y)}}}{\sqrt{1 - \frac{\cos^2 \delta}{(1 + \varepsilon \ y)(1 + \gamma \ y)}}} \tag{10}$$

and, by integration,

$$x = \int \frac{\cos \delta}{\sqrt{\sin^2\delta + \varepsilon\, y + \gamma\, y}}\, dy \tag{11}$$

$$= \frac{2 \cos \delta}{(\varepsilon + \gamma)} \sqrt{\sin^2\delta + \varepsilon\, y + \gamma\, y} + C. \tag{12}$$

Since for x = 0 we also have y = 0, the constant is determined to be

$$C = - \frac{2 \sin \delta \cos}{\varepsilon + \gamma} = - \frac{\sin 2\delta}{\varepsilon + \gamma} \tag{13}$$

Thus

$$\left(x + \frac{\sin 2\delta}{\varepsilon + \gamma}\right)^2 = \frac{4 \cos^2\delta \sin^2\delta}{(\varepsilon + \gamma)^2} + \frac{4 \cos^2\delta \cdot y}{(\varepsilon + \gamma)}, \tag{14}$$

from which

$$y = \left(\frac{\varepsilon + \gamma}{4 \cos^2\delta}\right) x^2 + \tan \delta \cdot x. \tag{15}$$

This equation is the equation for our sound ray. For small δ, i.e. for rays initially almost horizontal,

$$y = \frac{\varepsilon + \gamma}{4} x^2 \quad . \qquad (16)$$

At this point, equation (15) is completely general, since nothing has been said about the significance of ε and γ. But, if changes in the compressibility and the density with water depth are due to a change in the temperature, T, as we assumed above, then

$$\varepsilon = \frac{1}{\chi_o} \cdot \frac{\partial x}{\partial T} \cdot \frac{\partial T}{\partial y} \qquad (17)$$

and

$$\gamma = \frac{1}{\rho_o} \cdot \frac{\partial \rho}{\rho T} \cdot \frac{\partial T}{\partial y} \quad . \qquad (18)$$

But

$$\chi = 49 \text{ X } 10^{-6} \qquad (19)$$

and[1]

$$\frac{\partial \chi}{\partial T} = -2 \text{ X } 10^{-7} \qquad (20)$$

so that

$$\varepsilon = -4 \text{ X } 10^{-3} \cdot \frac{\partial T}{\partial y} \quad . \qquad (21)$$

Futhermore[2], since $\rho_o = 1$ and $\frac{\partial \rho}{\partial T}$ is of order 10^{-4}

$$\gamma = -10^{-4} \frac{\partial T}{\partial y} \quad . \tag{22}$$

One can, therefore, neglect γ relative to ε with negligible error, i.e., the change in sound velocity in water as a result of temperature changes is due to a change in the compressibility; changes in density as a function of temperature are not of practical significance.

The equation for a sound ray for horizontally stratified temperature layers is thus

$$y = -\frac{1}{\cos^2\delta} \times 10^{-3} \frac{\partial T}{\partial y} \cdot x^2 + \tan \delta \cdot x \quad , \tag{23}$$

or, for an initially horizontal ray,

$$y = -10^{-3} \frac{\partial T}{\partial y} \cdot x^2 \tag{24}$$

For example, if the water depth is 30 m and the temperature difference between the surface and bottom is 1^o - the higher temperature to be at the surface - then

$$\frac{\partial T}{\partial y} = -\frac{1}{30} \tag{25}$$

and

$$y = 3.33 \times 10^{-5} \cdot x^2 \tag{26}$$

A horizontal sound ray originating from a point in the vicinity of the surface therefore reaches the bottom after traveling about 1000 m, where the sound energy is essentially absorbed. All other sound rays which have an initial angle δ to the horizontal, reach the bottom earlier.

In acoustics we cannot speak generally of sharply bounded sound rays. One will still hear a little bit outside of the geometric bounds of the sound rays due to diffraction.

If the temperature difference between the surface and the bottom is greater, the curvature of the sound rays is increased, and the distance at which the sound can be heard will become even smaller than in our example.

If the temperature gradient is in the opposite direction, i.e., the temperature on the bottom is higher than at the surface, then the sound rays will curve upward instead of downward. But since the water surface is practically a perfect reflector[1], the sound rays will first be reflected downward, at an angle equal to the angle of incidence, and then, as a result of the temperature layers, again bent toward the surface, if, as might be possible for a rough surface, a significant part of the sound energy is not reflected into the bottom, where it would be absorbed.

Similarly to a change in temperature with water depth, a change in salinity will also produce a bending of sound waves in water, and thus a deviation from straight line paths. The compressibility decreases with increasing salinity, as the following table, taken from Krümmel's Handbook of Oceanography[4], shows.

Salinity in PPT:	0	5	10	15	20
Compressibility X 10^7:	490	484	478	472	466
Salinity in PPT:	25	30	35	40	
Compressibility X 10^7:	461	455	450	422	

If the salinity varies by 1 PPT, the compressibility varies by about 1.2 X 10^{-7}. A corresponding change in compressibility is produced by a temperature change of 0.6°C, which corresponds to a change in sound velocity of 1.8 m/sec.

The dependence of the density on salinity is given with sufficient accuracy by the formula[5]

$$\rho = 1 + 0.0008 \cdot S$$

where S is the salinity in PPT.

An absolute increase in salinity of 1 PPT thus corresponds to a relative increase in density of 0.8 PPT and a decrease in sound velocity of 0.4 PPT, i.e., 0.58 m/sec. The effect of density changes produced by salinity changes on sound velocity is about 1/3 of that produced by changes in compressibility.

A further cause for the bending of sound rays, which very likely plays a role only at very great water depths, is the change in sound velocity as a function of pressure. Density and compressibility, which determine sound velocity, are dependent on water pressure, with density increasing with increasing pressure[6] while the compressibility, on the other hand, decreases[7].

The percentage change in density with increasing pressure can, however, be neglected compared to the change in compressibility, as can be seen from the tables. It is thus necessary to consider only the influence of the compressibility. From the measurements of Amagat[1] above, the change in the compressibility of water at 0° caused by an increase in pressure from 1 to 200 atm is the same as that which occurs for a water temperature increase from 0° to 20°. In water 2000 m deep, a temperature difference of 20° between surface and bottom is completely compensated for. In still deeper water, a bending of the sound rays toward the surface takes place.

Finally, it is mentioned that natural currents in water, with respect to the audibility of sound, play a role similar to that of the wind in the propagation of sound in air. It is well known that one can hear better with the wind than one can against it. The explanation for this[8] can be found in the change of sound velocity with height above the earth. Thus, a sound ray traveling against the wind will generally be bent upward, and will, at some distance, travel over the head of an observer. A sound ray which travels with the wind will, on the other hand, be bent downward so that, as a result, one achieves greater ranges in this direction.

The results are exactly like this in water. Naturally, however, they are not as directly observable as they are in air.

Now, what do these results mean for underwater sound ranges?

We are primarily interested in the waters of the North and East Seas. From the tables in Krümmel's Handbook[9] we find that in general anothermal layering occurs in summer, i.e., the water is warmer at the surface than below, while in the winter the opposite (cathothermal) layering occurs.

Salinity, on the other hand, does not vary much with depth. The water is either isohaline or there is a slight increase with depth both in summer and in winter.

In any case, the magnitudes are such that the influence of changes of salinity on the compressibility is small as compared to the effects of changes of temperature. Therefore, since as a rule the source ships are not in deep water, we need, as a practical matter, to concern ourselves only about the temperature effects. The annual course of underwater sound ranges must therefore be the same as the annual course of the temperature profile. One would therefore expect, for anothermal layering of the water, i.e., in summer, shorter ranges than for cathothermal layering of the water, i.e., in winter.

The observations agree with this very well. In the curves of Fig. 2, several measurements of underwater sound ranges are shown. These were made from various source ships[10]. The abscissa represents months (Roman Numerals) and the ordinate represents ranges in nautical miles. The numbers in parentheses near the individual points are the number of observations from which the mean value was calculated for that month. The numbers below each of the curves are the smallest and those above are the largest ranges observed for that month. One can see that, corresponding to the theory given above, the ranges are greatest in winter and shortest in summer. Large variations occur within the individual months, which can be explained by coincidental currents as well as by differences in the receiving equipment.

There are no available ranges for the deep ocean. However, because of the effect of pressure on the compressibility, and hence on the velocity of sound - compressibility decreasing from top to bottom to produce increasing sound velocity and thus upward bending of the sound rays - one can expect significantly greater ranges than in shallow waters.

The exact transmission loss formula, i.e., the dependence of signal strength on distance, cannot be obtained from these curves. Extensive experiments on this subject were conducted in 1915/16. We will report on these in the future.*

Kiel, April 1919

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* A search of Physikalische Zeitschrift for a number of subsequent years has revealed no trace of this report - R. J. Urick.

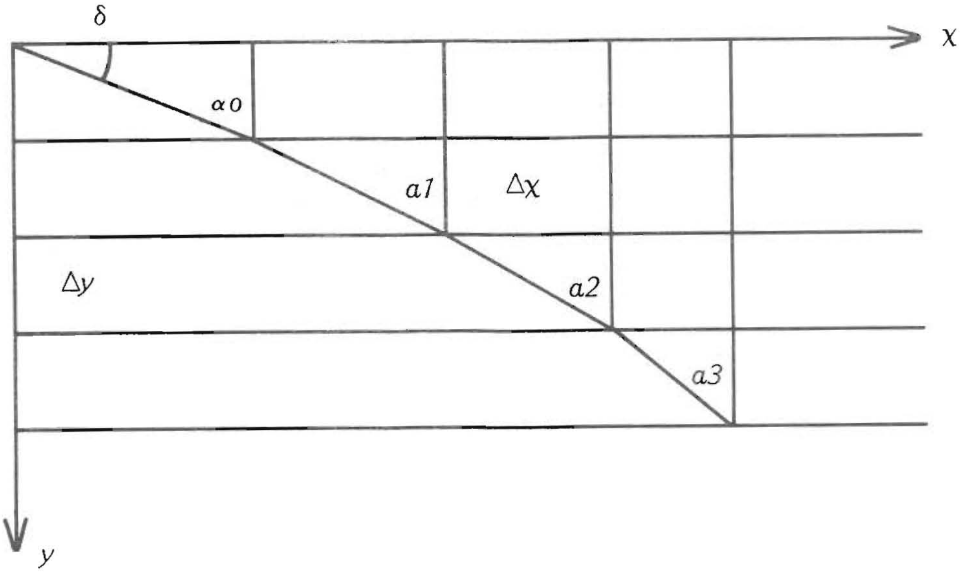

Figure 1

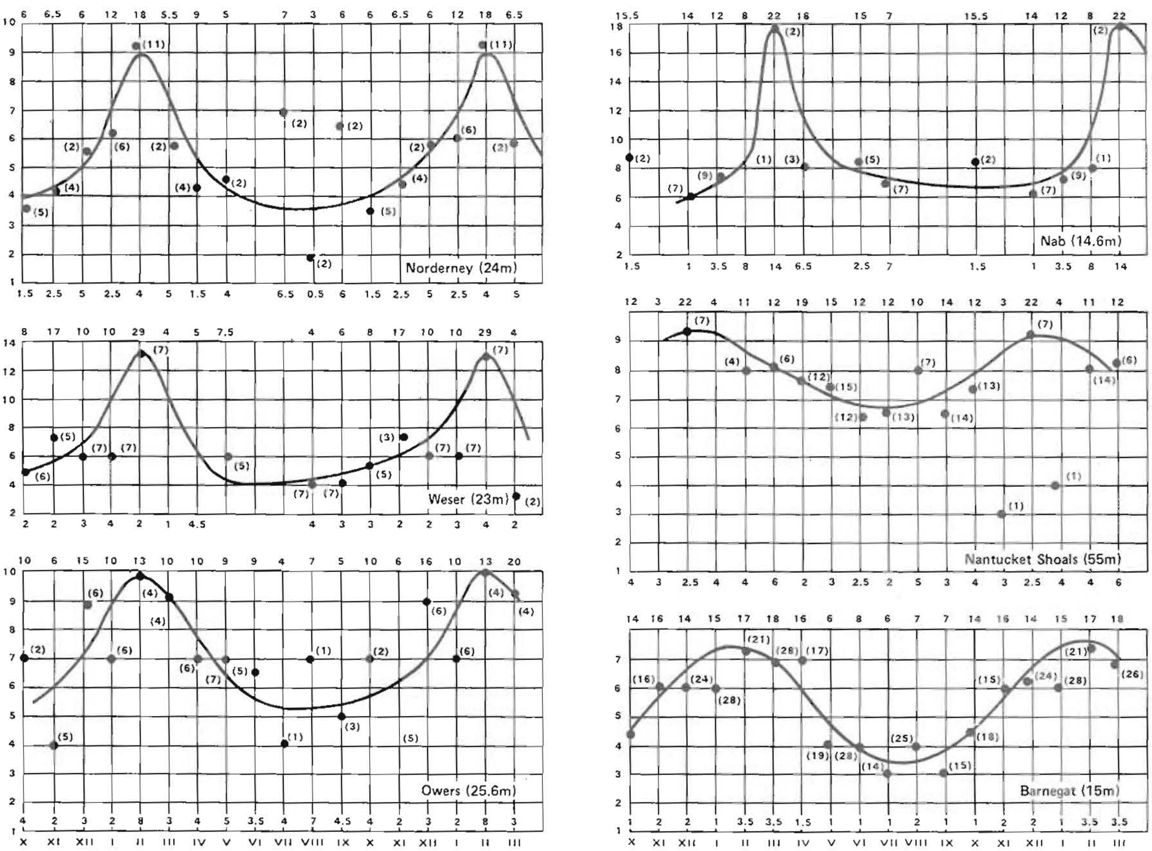

Figure 2

1. Kohlrausch, Lehrbuch der praktischen Physik, 2nd Edition, p. 708, Table XIXa. For water a minimum occurs in the compressibility (41.2 X 10^{-6}) at 62°. For practical considerations, $\frac{\partial \chi}{\partial T}$ can be considered as constant.

2. Kohlrausch, l.c., p. 694, Table IV. The density reaches a maximum at 4°. $\partial \rho / \partial T$ is negative only for temperatures above 4°.

3. Rayleigh, Theorie des Schalles, Deutsch von Neesen 1880, II, 98.

4. Krümmel, Handbuch der Ozeanographie, Bd. I, 1907, p. 285. This also contains additional literature.

5. The formula corresponds to the formula of Knudsen given in Krümmel, l.c., Bd. I., 237 Column II.

6. Krümmel, l.c., Bd. I, p. 288.

7. Kohlrausch, l.c., Bd. I, p. 708, Table XIXb.

8. Rayleigh, l.c., p. 155 ff.

9. Krummel, l.c., Bd. I, p. 468 ff., 480 ff., 349, 250.

10. This material was gathered before the war by tugs equipped with underwater sound receivers and was analyzed by myself from the point of view of the above theory. It was graciously made available to me by Chief Engineer Wolf, for which I thank him here.