

WOODS HOLE OCEANOGRAPHIC INSTITUTION

Woods Hole, Massachusetts

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The 1960 Summer Program of Theoretical Studies in
Geophysical Fluid Dynamics

by

Willem V. R. Malkus

Final Report

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APPROVED FOR DISTRIBUTION

Paul M. Fye, Director

1. Description of the program:

This ten-week work-study-discussion program is centered about a formal course called Geophysical Fluid Dynamics. Eight participants are selected from graduate and postgraduate applicants. In the discussions emphasis is placed on the formulation of tractable research problems in geophysics. The participants are encouraged to work on satisfactory problems thus formulated and to continue with their research after returning to their respective institutions.

2. Participants supported by the National Science Foundation grant:

Dr. Louis Howard, Ph.D. in Physics, Princeton University
Associate Professor of Mathematics, M.I.T.

Dr. Robert Kraichnan, Ph.D. in Physics, M.I.T.
Senior Staff Member, Institute of Mathematical
Sciences, N.Y.U.

Mr. R. R. Blandford, graduate student in Physics-Geophysics, California Institute of Technology

Mr. W. Blumen, graduate student in Meteorology, M.I.T.

Mr. R. L. Duty, graduate student in Mathematics, Brown University

Mr. R. Ellis, graduate student in Mathematics-Physics, Miami University

Mr. A. S. Furumoto, graduate student in Geophysics-Physics, St. Louis University

Mr. K. I. Gross, graduate student in Mathematics-Physics, Brandeis University

Mr. W. R. Holland, graduate student in Oceanography-Physics, University of California at Los Angeles

Mr. J. Pedlosky, graduate student Aeronautics, M. I. T.

3. Course Topics for 1960:

The first four weeks of the course were presented by the Principal Invited Lecturer, Professor Louis N. Howard. Dr. Howard's purpose was to provide a sound background in advanced fluid dynamics to student fellows from the variety of disciplines listed in paragraph 2. His lectures covered the basic equations, the special geophysical approximations, wave theory, the theory of hydrodynamic stability and boundary layer theory.

The remainder of the course was devoted to special topics presented by members of the Woods Hole staff. Dr. W. Malkus devoted two weeks to the statistical explorations for a useful theory of turbulence. Dr. H. Stommel discussed the present view of the oceanic circulation in the seventh and eighth week of the course. His purpose was to raise the important questions in a mathematical framework which could suggest research problems to the students. In the ninth week Dr. Melvin Stern outlined his studies of baroclinic instability. These large scale meteorological and oceanographic motions still pose many questions to the researcher, even as linearized problems. The last week of the course was given by Dr. J. Malkus who discussed the energy transports in the tropical atmosphere.

4. Student lectures in 1960:

An important part of this program is the requirement that graduate participants prepare a one-hour lecture on original material. Their work can represent an extension of some topic covered in the course or an application of the techniques evolved to some geophysical problem. They are assisted by the staff both in the choice of their topic and as their work progresses. However, it is made clear that the originality of their effort will be the measure of its success.

Six National Science Foundation supported graduates and one Harvard University fellowship holder spoke at the end of the 1960 course.

Mr. R. Blandford discussed Ocean current models using potential vorticity.

Mr. B. Holland discussed a comparison of steady fluid motion maintained by a non-uniform wind-stress distribution and steady motion maintained by a non-uniform temperature distribution.

Mr. W. Blumen discussed a simplified model of flow over an obstacle - with application to the atmosphere.

Mr. R. Duty discussed the propagation of shallow water waves in a viscous fluid in a rotating system.

Mr. A. Furumoto discussed convection of water maintained by cooling from below.

Mr. J. Pedlosky discussed stability of salt fingers.

Mr. R. Lindzen discussed stability of thermally stratified sheer flows.

All these students had made a considerable effort to prepare a stimulating and original lecture.

5. Publication of the lectures and student research studies:

The notes of the summer course were taken by students assigned to each lecture. The edited version of these notes, together with student studies, proved to be both a valuable record of the summer effort and a real contribution to geophysical fluid dynamics. They have been reproduced in three volumes, a total of 460 pages, and are available to interested persons. The introduction and table of contents are included as an appendix to this report.

6. Changes contemplated for 1961:

Our experience in the past two summers suggests that even well-prepared participants benefit from a review period at the beginning of the course. Hence, we have planned to devote the first few weeks of the course to a reading program and a formal study of advanced fluid dynamics. The following weeks will deal with selected problems at the frontier of the field, but with less formality than this summer. In the first period graduate students will be required to exhibit their mastery of the material covered. In the second period the graduate student will be primarily concerned with the selection and preparation of his lecture. Interaction of post-graduate participants with the Oceanographic staff was most rewarding last summer. We hope to further encourage joint research efforts, continuing into the academic year, between staff members and the senior participants.

Appendix
of a final report to the National Science Foundation

Notes on the 1960
Summer Study Program
in
GEOPHYSICAL FLUID DYNAMICS
at
The WOODS HOLE OCEANOGRAPHIC INSTITUTION

Edited by
E. A. Spiegel
Institute for Mathematical Sciences

The notes may be obtained by writing the Fellowship Committee,
Woods Hole Oceanographic Institution, Woods Hole, Massachusetts

Contents of the Volumes

- Volume I. Lectures on Fluid Dynamics - L.N. Howard
- Volume II. Special Topics
- Turbulence - W.V.R. Malkus
- Oceanic Circulation - H. Stommel
- Baroclinic Instability - M. Stern
- Energy Transports
- in the Tropical Atmosphere - J.S. Malkus
- Volume III. Student Lectures.

LIST OF PARTICIPANTS

Regular WHOI Staff Members

K. Bryan
J. Malkus
W. Malkus
K. Rooth
M. Stern
H. Stommel

Visiting Staff Members and Post-doctoral Participants

A. Arons (Amherst College)
F. Bisshopp (Brown University)
L. Howard (M.I.T.)
R. Kraichnan (Inst. for Math. Sciences)
D. Lilly (U.S. Weather Bureau)
E. Spiegel (Inst. for Math Sciences)
H. Wexler (U.S. Weather Bureau)

Student Fellows:

R.R. Blandford	Phys.-Geophys.	California Inst. of Tech.
W. Blumen	Meteorology	Mass. Inst. of Tech.
R. L. Duty	Mathematics	Brown University
R. Ellis	Math.-Phys.	Miami University
A.S. Furumoto	Geo.-Phys.	St. Louis University
K. I. Gross	Math.-Phys.	Brandeis University
W. R. Holland	Ocean.-Phys.	Univ. of California at L.A.
R. S. Lindzen	Phys.	(NSF Harvard Fellow)
J. Pedlosky	Aero.	Mass. Inst. of Tech.

Editor's Preface

The past decade has brought an exciting upsurge of interest and research in geophysical fluid dynamics. This development has been particularly manifested by the activities and enthusiasms of a growing number on the staff of the Woods Hole Oceanographic Institution, with the result that many scientists interested in fluid dynamics have become frequent visitors there. In summer especially, the regular staff of the Institution has provided a nucleus for lively gatherings of oceanographers, meteorologists, physicists, mathematicians, and even astrophysicists.

Against this background of meeting and discussion students, sponsored by the Institution's summer fellowship program, have come to take part in the various research programs that develop. As the size of the summer group has increased the danger has arisen that the summer fellowship students might become lost in milieu of high level discussion, and not profit adequately from their efforts. Accordingly, the possibility of providing tangible opportunities for the training of summer fellowship students was explored, and it was decided to institute a summer course in geophysical fluid dynamics.

The first course in geophysical fluid dynamics at Woods Hole was given in the summer of 1959 by staff members of the Institution and some of their summer colleagues. The participants numbered about twenty and included four graduate-student and two postdoctoral fellowship holders provided for by funds from the National Science Foundation. At the time

the dragon which adorns the cover of this volume was born. He was created by Prof. Henry Stommel in recognition of the efforts of Prof. Willem Malkus in organizing the course. The success which this first course enjoyed accounts for the reappearance of our dragon in its present position of prominence. For in the summer of 1960, a second course was given whose contents are outlined in the present notes.

These notes were prepared by the students, whose names are given above, with the capable assistance of Mrs. Mary Thayer. They were designed as working notes to be of use during the course. For each series of lectures, two students accepted the responsibility of preparing the notes and it was attempted (with surprising success) to have the notes typed, duplicated and distributed within four days after each lecture. Naturally, such a project could be completed only with rough edges, but the final collection of notes has succeeded very well in presenting the essential content and spirit of the course. They have therefore been assembled in limited number for use by interested persons.

It has seemed worthwhile to divide the notes into three volumes to avoid making them too cumbersome for easy reference. The division of material reflects the structure of the course. In Volume I we have an introduction to the subject as given by the invited lecturer, Professor L.N. Howard. The second volume contains notes on the more specialized lectures given by various staff members of the Institution. Finally, the manuscripts summarizing the student research lectures are reproduced in

Volume III. The topics discussed by the students were either selected by them or suggested by staff members.

Those of us from other institutions who have participated in this course have been treated to an abundant bill of fare, as a look at these notes will attest. For this, we can but express our gratitude to Dr. Willem Malkus and the other staff members of the Oceanographic Institution for their extensive efforts. We are also indebted to the Institution itself for its hospitalities and facilities. Finally, we should like to thank the National Science Foundation for providing funds for student fellowships and the support of an invited lecturer.

E. A. Spiegel

Inst. for Mathematical Sciences

Sept. 1960.

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