A CONTINENTAL SHELF BOTTOM BOUNDARY LAYER MODEL: THE EFFECTS OF WAVES, CURRENTS, AND A MOVEABLE BED

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SCOTT MICHAEL GLENN

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Signature of Author ____________________________
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Certified by ____________________________
William D. Grant
Thesis Supervisor

Accepted by ____________________________
Arthur B. Baggeroer
Chairman, Joint Committee for Oceanographic Engineering

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ABSTRACT

A simple model for the bottom boundary layer on the continental shelf is presented. The governing equations are developed for a stratified, turbulent Ekman layer in a combined wave and current flow over a moveable sediment bed. An eddy diffusivity closure scheme that includes the effect of suspended sediment, temperature, and salinity induced stratification on the vertical turbulent diffusion of mass and momentum couples the resulting unsteady conservation equations for fluid momentum, fluid mass, and suspended sediment mass. The wave velocity, current velocity, and suspended sediment concentration profiles predicted by the simultaneous solution of the conservation equations require the physical bottom roughness and a sediment reference concentration to be specified as boundary conditions. The physical bottom roughness associated with biologically generated bedforms, wave generated ripples, and near bed sediment transport are calculated as functions of the flow and sediment conditions. Using expressions for the height of sediment transporting layer and the sediment velocity, an expression for the sediment reference concentration is developed by matching laboratory measurements of sediment transport rates in oscillatory flow. The model predicts that the bottom flow field is highly dependent on (1) the nonlinear wave and current interaction, which increases the boundary shear stress and enhances vertical turbulent diffusion, (2) the effect of the boundary shear stress on a moveable sediment bed, which determines the physical bottom roughness and the amount of sediment in suspension, and (3) the effect of stable stratification, which inhibits vertical turbulent transport and couples the flow to the suspended sediment and fluid density profiles. The validity of the theoretical approach is supported by model predictions that are in excellent agreement with high quality data collected during two continental shelf bottom boundary layer experiments for a wide range of flow and bottom conditions.

Thesis Supervisor: Dr. William D. Grant

Title: Associate Scientist
Woods Hole Oceanographic Institution

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a	=	wave amplitude
A_b	=	wave bottom excursion amplitude
C	=	instantaneous concentration, fluid
C_m	=	mean concentration, fluid
C_p	=	periodic concentration, fluid
C'	=	turbulent concentration, fluid
C_n	=	instantaneous concentration, sediment class n
C_{nm}	=	mean concentration, sediment class n
C_{np}	=	periodic concentration, sediment class n
C'_n	=	turbulent concentration, sediment class n
C_{nb}	=	bed concentration, sediment class n
$C_n(z_o)$	=	instantaneous reference concentration, sediment class n
$C_{nm}(z_o)$	=	mean reference concentration, sediment class n
$C_{np}(z_o)$	=	periodic reference concentration, sediment class n
C_T	=	instantaneous sediment concentration in near bed transport layer
C_D	=	drag coefficient
c_w	=	wave phase velocity
d	=	sediment grain diameter

LIST OF PRINCIPAL SYMBOLS (cont.)

F	=	function that modulates linear eddy viscosity profile in the Ekman layer
f	=	Coriolis parameter
f_{cw}	=	combined wave and current friction factor
f'_{cw}	=	combined wave and current skin friction factor
g	=	acceleration of gravity
h	=	water depth
h_T	=	instantaneous height of near bed transport layer
h_{Tm}	=	maximum height of near bed transport layer
i	=	$(-1)^{1/2}$
k	=	wave number
k_x	=	x-component of wave number
k_y	=	y-component of wave number
$\hat{k}$	=	vertical unit vector
k_b	=	physical bottom roughness
k_{bN}	=	Nikuradse roughness
k_{bB}	=	roughness associated with bedforms
k_{bT}	=	roughness associated with near bed transport
k_{bc}	=	apparent bottom roughness
k_h	=	bedform height
k_w	=	bedform cross-stream width

LIST OF PRINCIPAL SYMBOLS (cont.)

k_l	=	bedform streamwise length
k_W	=	cross-stream width of bed associated with bedform
k_L	=	streamwise length of bed associated with bedform
N	=	Brunt-Vaisala frequency
L	=	Monin-Obukov length
L_c	=	Monin-Obukov length, $z > \delta_w$
L_{cw}	=	Monin-Obukov length, $z < \delta_w$
l_c	=	Ekman layer scale height
l_{cw}	=	wave boundary layer scale height
p	=	instantaneous pressure
p_c	=	current pressure
p_w	=	wave pressure
p'	=	turbulent pressure
Q_s	=	total dry weight of sediment transported
q_s	=	average volumetric sediment transport rate
R	=	parameter in stability parameter definition
r	=	regression coefficient
R_f	=	Richardson flux number
s	=	relative sediment density
S_*	=	dimensionless sediment parameter

LIST OF PRINCIPAL SYMBOLS (cont.)

S'_n	=	normalized excess skin friction, sediment class n
T	=	wave period
t	=	time
u	=	x-component of velocity
u_g	=	x-component of geostrophic velocity
u_{100}	=	current speed 1 meter above the bottom
$\vec{u}$	=	instantaneous horizontal velocity
$\vec{u}_c$	=	current horizontal velocity
$\vec{u}_w$	=	wave horizontal velocity
$\vec{u}'$	=	turbulent horizontal velocity
$\vec{u}_a$	=	current velocity used to define boundary shear stress
$\vec{u}_b$	=	maximum wave bottom velocity from linear wave theory
$\vec{u}_n$	=	instantaneous horizontal velocity, sediment class n
$\vec{u}_r$	=	specified reference current velocity
$\vec{u}_*$	=	shear velocity
$\vec{u}_{*c}$	=	shear velocity derived from average boundary shear stress

LIST OF PRINCIPAL SYMBOLS (cont.)

$\vec{u}_{*cw}$	=	shear velocity derived from maximum boundary shear stress
u_T	=	instantaneous particle velocity in near bed transport layer
u_{Tm}	=	maximum particle velocity in near bed transport layer
v	=	y-component of velocity
v_g	=	y-component of geostrophic velocity
V_2	=	parameter in average boundary shear stress definition
w	=	z-component of velocity
w_n	=	z-component of velocity, sediment class n
w_{fn}	=	particle fall velocity, sediment class n
w_w	=	wave vertical velocity
x	=	horizontal Cartesian coordinate
y	=	horizontal Cartesian coordinate
z	=	vertical Cartesian coordinate
z_r	=	height of specified reference current velocity
z/L	=	constant stress layer stability parameter
z_o	=	physical bottom
z_{oc}	=	apparent bottom
α	=	parameter in maximum shear stress definition

LIST OF PRINCIPAL SYMBOLS (cont.)

β	=	constant in nondimensional velocity gradient
γ	=	constant in nondimensional sediment gradient
γ_0	=	reference concentration constant
δ_c	=	current boundary layer height
δ_w	=	wave boundary layer height
δ_D	=	displacement thickness
ζ	=	dimensionless vertical coordinate for the wave boundary layer
ζ_0	=	dimensionless z_0
ζ_c	=	generalized stability parameter, $z > \delta_w$
ζ_{cw}	=	generalized stability parameter, $z < \delta_w$
η	=	ripple height
θ	=	wave phase angle
θ_g	=	geostrophic velocity turning angle
κ	=	von Karman's constant
λ	=	ripple length
λ_w	=	surface wave length
ν	=	kinematic viscosity
ν_t	=	neutral eddy viscosity
ν_{tc}	=	neutral eddy viscosity, $z > \delta_w$
ν_{tcw}	=	neutral eddy viscosity, $z < \delta_w$
ν_{tm}	=	stratified eddy diffusivity of momentum

LIST OF PRINCIPAL SYMBOLS (cont.)

ν_{ts}	=	stratified eddy diffusivity of sediment mass
ν_{tf}	=	stratified eddy diffusivity of fluid mass
ξ	=	dimensionless vertical coordinate for the Ekman layer
ξ_c	=	dimensionless height of Ekman layer
ξ_w	=	dimensionless height of wave boundary layer
ρ	=	fluid density
ρ_b	=	bottom fluid density
ρ_m	=	mean fluid density
ρ_p	=	periodic fluid density
ρ'	=	turbulent fluid density
ρ_s	=	sediment density
ρ_T	=	total density of fluid-sediment suspension
$\overline{\rho_T}$	=	Reynolds averaged total density
ρ'_T	=	turbulent total density
$\vec{\tau}$	=	shear stress
$\vec{\tau}_b$	=	instantaneous boundary shear stress
$\vec{\tau}_c$	=	average boundary shear stress (accounting for direction)
$\vec{\tau}_{cw}$	=	maximum boundary shear stress
$\vec{\tau}_b{}'$	=	instantaneous skin friction
$\vec{\tau}'_{bm}$	=	maximum skin friction

LIST OF PRINCIPAL SYMBOLS (cont.)

ϕ	=	dimensionless sediment transport function
ϕ_c	=	angle between $\vec{u}_a$ and $\vec{u}_b$
$\bar{\phi}_c$	=	angle between $\vec{u}_r$ and $\vec{u}_b$ for near bottom model, $0^\circ \leq \bar{\phi}_c \leq 90^\circ$
ϕ_r	=	angle between $\vec{u}_r$ and $\vec{u}_b$ for Ekman layer model, $-90^\circ \leq \phi_r \leq 90^\circ$
ϕ_m	=	nondimensional velocity gradient
ϕ_s	=	nondimensional sediment gradient
χ	=	logarithmic vertical coordinate in Ekman layer
ψ_β	=	breakoff Shields parameter
ψ_c	=	critical Shields parameter
ψ'	=	instantaneous Shields parameter based on skin friction
ψ'_m	=	maximum Shields parameter based on skin friction
$\vec{\Omega}_H$	=	horizontal component of planetary angular velocity
Ω_V	=	vertical component of planetary angular velocity
ω	=	radian wave frequency

1. INTRODUCTION

A quantitative description of the bottom boundary layer on the continental shelf is necessary to understand a wide range of coastal oceanographic problems. The design of submarine structures, such as pipelines, requires detailed knowledge of the near bottom flow field to estimate realistically the resulting hydrodynamic forces on the structures. Calculation of sediment transport rates require flow field descriptions that are most accurate near the bottom where sediment concentrations are highest. Prediction of shelf circulation patterns require specification of the magnitude, direction, and spatial variability of the boundary shear stress associated with the near bottom flow field. Estimates of wave attenuation rates due to bottom friction requires the instantaneous boundary shear stress to be known. An accurate model of the bottom boundary layer on the continental shelf will significantly improve our ability to understand these problems.

A simple model for the bottom boundary layer on the continental shelf is developed here. The model is designed to predict the vertical structure of the bottom boundary layer at a point on the continental shelf from a minimum number of input parameters. Important physical processes influencing the bottom boundary layer structure include: (1) the simultaneous presence of both surface waves and low frequency currents, (2) moveable bed effects (ripple formation, near bed sediment transport, and suspended sediment transport), (3) the stabilizing or destabilizing effect of bioturbation, (4) self induced stratification of the flow field by suspended sediment, (5) planetary rotation, (6) temperature and salinity stratification, (7) internal waves, and (8) the effect of bottom topography. With the exception of internal waves and bottom topography, which are often second order or site specific problems, the above physical processes are incorporated into a continental shelf, bottom boundary layer model. Given the wave conditions (period, amplitude, direction, and water depth), the current conditions (current speed and direction at a known height above the bottom), bottom conditions (type of sediment, type of bedforms), and the temperature and salinity profiles, the bottom boundary layer model can be used to calculate vertical velocity profiles for the wave and the current, vertical concentration profiles of suspended sediment, suspended sediment transport rates, and the boundary shear stress.

Development of the bottom boundary layer model proceeds as follows. A physical description of the important processes affecting the bottom boundary layer is presented in Chapter 2. In Chapter 3, a theoretical model for the near bottom flow in the absence of stratification is discussed. The Grant and Madsen (1979) combined wave and current model and the Grant and Madsen (1982) moveable bed roughness in oscillatory flow model are reviewed. A theoretical model for the near bottom flow that includes suspended sediment induced stratification is developed in Chapter 4. The effect of stratification is incorporated in the Grant and Madsen (1979) combined wave and current model and a model is developed to predict sediment reference concentrations. In Chapter 5, a theoretical model for the full Ekman layer is developed. The near bottom, combined wave and current model is extended by including the effects of planetary rotation and stratification associated with suspended sediment, temperature and salinity. Finally, velocity profiles predicted by the near bottom model are compared in Chapter 6 to near bottom velocity profiles measured during two bottom boundary layer experiments on the continental shelf.

2. PHYSICAL MODEL FOR THE BOTTOM BOUNDARY LAYER ON THE CONTINENTAL SHELF

The major physical processes affecting the bottom boundary layer will be discussed in this chapter. The processes involved are the interaction of a wave boundary layer and a current boundary layer, the physical bottom roughness associated with a bioturbated moveable bed, stratification of the near bottom flow field by suspended sediment, and stratification of the outer Ekman layer by temperature and salinity. In subsequent chapters, mathematical models for these physical processes will be developed.

2.1 Wave and Current Interaction

The near bottom flow field on the continental shelf typically has velocity components associated with surface gravity waves and with tidal, wind generated, or density driven, low frequency currents. Because of the contrasting time and length scales associated with each component, two distinct boundary layers develop, one associated with the current and one associated with the wave. The current boundary layer, or Ekman layer, is relatively steady with a thickness limited by water depth or Ekman layer height. For fully rough turbulent flow, the current boundary layer can be divided into an inner, near bottom, constant stress or logarithmic region and an outer log deficit region. Planetary rotation causes the current velocity vector in the Ekman layer to rotate in a clockwise direction with distance from the bottom in the Northern Hemisphere. Ekman turning of the velocity vector, however, is significant only in the outer log deficit region. Above the current boundary layer, the turbulent shear stress associated with the current is negligible. In this geostrophic core region, the current is adequately described by inviscid theory.

Nested within the current boundary layer is a wave boundary layer. In comparison, the wave boundary layer is oscillatory with a typical thickness of 2 to 20 centimeters. For fully rough turbulent flow, the wave boundary layer also can be divided into an inner constant stress region and an outer log deficit region. Above the wave boundary layer, the turbulent shear stress associated with the wave is negligible. The wave above the wave boundary layer is also adequately described by inviscid theory. A typical bottom boundary layer on the continental shelf is illustrated in Figure 2.1.

Maximum near bottom wave velocities on the continental shelf are typically the same order of magnitude or greater than near bottom current velocities. The small scale of the wave boundary layer compared to the current boundary layer causes the boundary shear stress associated with the wave to be much greater than that associated with the current. Boundary shear stress, however, is a nonlinear function of the instantaneous wave plus current velocity,

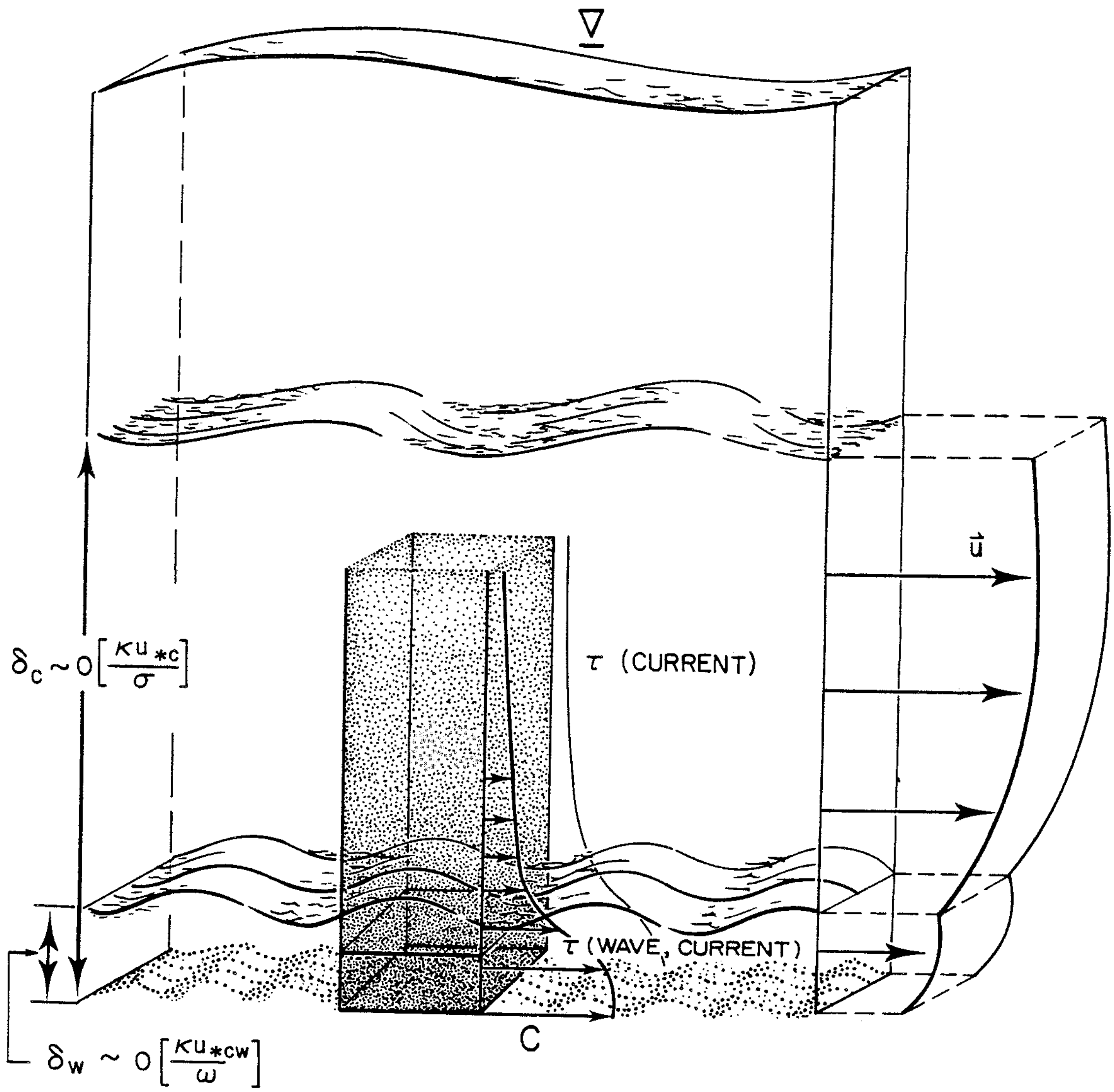

FIGURE 2.1

Skematic of the continental shelf bottom boundary layer illustrating the nested wave and current boundary layer structure.

which varies in magnitude and direction over a wave period. The wave and current therefore interact to generate a shear stress that also varies in magnitude and direction and is different from the shear stress associated with each component individually. For example, the constant stress experienced by the time average current in the inner region of the current boundary layer must be the time average of the instantaneous boundary shear stress, which is enhanced by the presence of the waves.

In the constant stress region for the current, a semi-logarithmic velocity profile is generated in the absence of stratification by turbulent diffusion of low momentum fluid away from the boundary. Above the wave boundary layer, in the potential flow region for the wave, turbulent diffusion is associated with the time average current only. Within the wave boundary layer, however, turbulent diffusion is associated with the combined wave and current flow. Because the diffusion of low momentum fluid within the wave boundary layer is enhanced by the wave, current velocities in this region are reduced. The resulting reduction in the current shear within the wave boundary layer is similar to that caused by enhanced turbulent diffusion in the boundary layers behind upstream bumps. Just as dissipation is increased behind stream bumps, the increased dissipation in the wave boundary layer causes the current above the wave boundary layer to appear to feel a much larger bottom roughness than physically exists.

2.2 Physical Bottom Roughness

The boundary shear stress in a fully rough turbulent flow also depends on the physical bottom roughness; a larger bottom roughness results in a larger shear stress. Because the moveable sediment bed commonly found on the continental shelf can be modified by marine organisms or changing flow conditions, the physical bottom roughness is highly dependent on the sediment characteristics and the boundary shear stress. The physical bottom roughness of a moveable bed is often partitioned into components associated with three physical processes; (1) drag on sediment grains in the bed, (2) form drag on bedforms, and (3) drag on sediment grains in the near bed transport layer. Since the shear stress associated with the wave is usually much greater than that associated with the current, moveable bed effects are expected to be wave dominated.

If the boundary shear stress is below that required to initiate sediment motion, the constant bottom roughness is associated with the sediment grains in the bed and with existing bedforms. Existing bedforms could be ripples formed during previous high flow events or bedforms generated by benthic organisms. If the boundary shear stress is increased above that required to initiate sediment motion, the bottom roughness becomes a function of the boundary shear stress and is no longer a constant. Pre-existing bedforms begin to erode as near bed sediment transport begins. As the boundary shear stress continues to increase, near bed sediment transport also increases. If the sediment is sand or silt, a flat bed is unstable and ripples will form. Ripples remain in equilibrium with the flow as the boundary shear stress increases until a breakoff point is reached. As the boundary shear stress increases past the breakoff point, ripple heights decay until the bed is flat but covered by an intense near bed sediment transport layer. When sediment transport occurs, the bottom roughness is associated with all three components; sediment grains, bedforms, and near bed transport. The bedform component, which is important for low flow situations when there is little near bed transport, is found to depend on bedform geometry. The transport component, which is important for high flow situations when bedforms have been washed out, is found to depend on the height of the near bed transport layer.

2.3 Stratification

The bottom boundary layer on the continental shelf can also be affected by stable stratification. Because stable stratification inhibits the vertical turbulent transport of mass and momentum, less low momentum fluid is transported up through the water column in a stratified flow. A stratified flow will experience an increased shear in the velocity profile compared to a neutral flow with the same boundary shear stress. In the bottom boundary layer stratification can be caused by vertical gradients of suspended sediment, temperature, or salinity.

2.3.1 Suspended sediment induced stratification

Self induced stratification of the near bottom flow field can occur when the boundary shear stress is large enough to suspend significant amounts of sediment in the water column. Since the boundary shear stress associated with the wave is greater than that associated with the current, waves are more effective at suspending sediment than currents. The effect of suspended sediment induced stratification therefore is expected to be greater for a combined wave and current flow than for a pure current flow.

In the bottom boundary layer, the upward turbulent diffusion of sediment is balanced by the sediment's tendency to fall out of suspension, resulting in a concentration profile that decreases with distance from the bottom. The vertical density gradient caused by the decreasing sediment concentration can stably stratify the flow field if the sediment has the appropriate fall velocity. If the fall velocity is too large, very little sediment will be suspended and the resulting density gradient will be insignificant. If the fall velocity is too small, large amounts of sediment will be mixed uniformly throughout the water column, again resulting in an insignificant density gradient. For the flow to be stratified by suspended sediment, the sediment fall velocity must be in an intermediate range that causes large density gradients.

For stratification to influence vertical turbulent diffusion, there must be a significant density difference over the length scale of the momentum transporting eddies. This suggests that suspended sediment induced stratification will not affect the entire bottom boundary layer. In the constant stress layer, the momentum transporting eddies scale with distance from the bottom. Close to the bottom, the momentum transporting eddies are so small that the density differences over the eddy length scale are also small. Away from the bottom the eddy length scale is large, but there is so little sediment in suspension that again there is little density difference. Stratification induced by suspended sediment therefore is only expected to be significant in some regions of the bottom boundary layer.

2.3.2 Temperature and salinity induced stratification

Vertical turbulent transport in the bottom boundary layer can also be inhibited by stratification due to temperature and salinity gradients. Unlike the atmosphere, temperature and salinity gradients are not maintained by input from the bottom, but are caused by advection or surface input. High turbulence levels and mixing rates near the bottom will tend to smooth out any density differences due to temperature or salinity. Typical temperature and salinity profiles on the continental shelf consist of a bottom mixed layer of uniform density with significant density gradients above. If the Ekman layer height exceeds the bottom mixed layer height, temperature and salinity stratification will tend to cap the Ekman layer, inhibiting its vertical growth. In contrast to suspended sediment induced stratification, which is important to the near bottom flow, temperature and salinity stratification is usually expected to be most important in the upper Ekman layer.

3. THEORETICAL MODEL FOR A NEUTRAL NEAR BOTTOM FLOW IN COMBINED WAVES AND CURRENTS OVER A MOVEABLE BED

From the physical model presented in the preceding chapter, it is clear that a bottom boundary layer model must account for the effects of combined waves and currents over a bioturbated moveable bed in a possibly stratified Ekman layer. Existing theoretical models address several aspects of this problem. Grant and Madsen (1979) and Smith (1977) have developed a combined wave and current model for the near bottom flow that neglects the effect of stratification. Grant and Madsen (1982) have modeled the roughness of a moveable bed in oscillatory flow. Rhoads et al. (1978), Nowell et al (1981) and Grant et al. (1982) have examined the effect of bioturbation on the initiation of sediment motion. Grant and Glenn (in press) have modeled the effect of biologically induced bedforms on the near bottom flow. Smith and McLean (1977; 1977) have included the effect of suspended sediment induced stratification in a pure current model. Long (1981) developed a temperature and salinity stratification correction that applies throughout a pure current Ekman layer. Elements of these models will be used in the bottom boundary layer model developed here.

In this chapter, a model for the near bottom flow field on the continental shelf in the absence of stratification is described. The Grant and Madsen (1979) combined wave and current model, which is the fundamental component of the bottom boundary layer model, is reviewed in Section 3.1. A boundary condition required by the combined wave and current model is that the velocity is zero at a height determined by the physical bottom roughness. A review of the Grant and Madsen (1982) moveable bed roughness in oscillatory flow model is included in the discussion of the physical bottom roughness model in Section 3.2. Together, these models can be used to calculate near bottom velocity profiles for the wave and the current when stratification is negligible.

3.1 Neutral Near Bottom Model

Grant and Madsen (1979) have developed a model that predicts the near bottom flow field over a rough bottom in an unstratified fluid due to the combined presence of a wave and a current at arbitrary angles. This model is reviewed here. The governing equations for the problem are set up, a turbulent closure scheme is described, and the resulting equations are solved for the wave and the current velocity profiles.

3.1.1 Governing equations

The equation governing the near bottom flow when suspended sediment transport is negligible is the usual horizontal conservation of fluid momentum equation

$$\frac{D\vec{u}}{Dt} + 2\,\Omega_V\,\hat{k} \times \vec{u} + 2\,\vec{\Omega}_H \times w\,\hat{k} = -\frac{1}{\rho}\vec{\nabla}\,p + \nu\left(\vec{\nabla}^2\vec{u} + \frac{\partial^2\vec{u}}{\partial z^2}\right) \tag{3.1}$$

where x, y, and z are the components of a Cartesian coordinate system with z measured positive upward from the bottom, t is time, $\vec{u} = (u,v)$ is the horizontal fluid velocity vector with components u and v, w is the vertical fluid velocity, $\vec{\Omega}_H$ and Ω_V are the horizontal and