bwavesc.m

Barotropic coastal trapped wave modes with complex frequency: edge, shelf and Kelvin waves

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This set of Matlab mfiles (all with names beginning with "bwavesc") can be used to calculate barotropic coastal wave properties in the absence of density stratification. The wave frequency is complex so that unstable or strongly damped modes can be treated. You are allowed to have a mean alongshore flow, if desired, and you can apply the rigid lid approximation. The model can be run in the non-rotating limit if desired. Once a wave's frequency is found, the modal structure is displayed. The code can use an exact open boundary condition or a closed condition at either side of the domain.

Although there are several mfiles in this package, but there are really only three that the user is likely ever to use directly:

bwavescsetuc.m This file asks the user a sequence of questions, and creates an array which summarizes the answers and is then used to drive the main code.

It is called as:

```
>> arrayin = bwavescsetup;
```

where arrayin is the resulting driver array.

bwavescfinch.m This file is used to change the input array without having to re-enter all the material. When you call it, it will first give a menu asking what sort of thing needs to be changed, and will then ask more specific questions.

It is called as:

```
>> newarrayin = bwavescfinch(arrayin);
```

where "newarrayin" is the revised input array, and "arrayin" is the original input array.

bwavesc.m This is the central file called in order to carry out the wave calculations. It calls several other mfiles for specific tasks.

It is called as:

>>bwavesc(arrayin,name);

where "arrayin" is the input array from bwavescsetup.m or bwavescfinch.m. The second input, name, is a string variable that would be used to name an output file. Outputs are displayed on your screen, and you are given the option of saving your results to a mat file "name".

A good deal more detail, including examples, is provided at the end of this document.

The Problem:

We seek coastal-trapped wave solutions (both sub-and superinertial) for an ocean without stratification. Specifically, we consider a straight boundary at $x = 0$, where the depth $h(x)$ varies offshore (toward larger x). All isobaths are parallel to the shoreline. Either open or closed boundary conditions are possible at either boundary. The effects of friction are confined to infinitesmally thin boundary layers. Frictional effects do not have to be small. Further, there is a steady mean alongshore flow, $v_0(x)$.

The linearized depth-integrated equations of motion are

$$U_t + v_0 U_y \qquad - fV = -\frac{1}{\rho_0} p_x - \frac{r}{h} U \tag{1a}$$

$$V_t + v_0 V_y + V v_{0x} \qquad + fU = -\frac{1}{\rho_0} p_y - \frac{r}{h} V \tag{1b}$$

$$\delta \frac{1}{g\rho_0}\left(p_t + v_0 p_y\right) + U_x + V_y = 0 \quad . \tag{1c}$$

where (U, V) is the vector of depth-integrated (offshore, alongshore) velocity and p is the pressure. Constants f, g and ρ_0 are the Coriolis parameter, the acceleration due to gravity and the uniform water density. The bottom resistance coefficient is r. Specifically, the bottom stress $(\tau_B^x, \tau_B^y) = \rho_0\, r\, (u, v)$ where (τ_B^x, τ_B^y) is the bottom stress vector, and (u, v) is the interior velocity near the bottom. It is assumed that $(u, v) = (U, V)/h$. Subscripts with regard to independent variables (x, y, t) represent partial differentiation.

When $\delta = 0$, the rigid lid approximation is enforced. This assumption is valid when

$$\frac{gH}{f^2 L^2} \gg 1 \quad , \tag{2}$$

where H is a representative depth. Otherwise, $\delta = 1$ (free surface boundary condition).

All dependent variables are taken to have a form like

$$p(x,y,t) = P(x) \exp[i(\omega t + ly)] \quad . \tag{3}$$

Using this simplification, a single equation for pressure is obtained:

$$0 = \left(\frac{h}{\gamma}P\right)_x - i\left(\frac{r}{h}\right)_x \frac{h}{\gamma\omega''} P_x + P\left[-\frac{\delta\omega''}{g\omega'} - \varepsilon\frac{l^2}{\gamma}h + \frac{fl}{\omega''}\left(\frac{h}{\gamma}\right)_x\right] \tag{4}$$

where

$$\gamma = ff' - \omega''^2 \tag{5a}$$

$$f' = f + v_{0x} \tag{5b}$$

$$\omega'' = \omega + lv_{ox} \quad . \tag{5c}$$

$$\omega' = \omega + lv_0 - i\frac{r}{h} \tag{5d}$$

This problem can be solved with either open or closed boundary conditions at $x = 0$ and $x = x_{Max}$. The closed condition is

$$U = 0 \tag{6a}$$

so that

$$0 = \omega' P_x + flP \quad . \tag{6b}$$

The open boundary condition is exact, but it requires that, at the boundary,

$$\begin{aligned} v_{0x} &= 0 \\ v_{0xx} &= 0 \\ h_x &= 0 \\ r_x &= 0 \end{aligned} \tag{7}$$

When this is true, equation (4) has constant coefficients, and the solution P_0 for outside the boundaries of the grid is

$$P_0(x) = A \exp[-\beta_B |x - x_B|] \tag{8a}$$

where

$$\beta_B = \sqrt{\frac{\delta\gamma}{gh_B} + l^2} \tag{8b}$$

and h_B is the water depth at the open boundary, $x = x_B$ ($x_B = 0$ and /or x_{Max}), and A is a constant. Thus, the open boundary condition becomes

$$Px = \beta_B P(0) \quad , h_B = h(0) \tag{8c}$$

when applied at $x = 0$ and

$$Px = -\beta_B P(x_{Max}) \quad , h_B = h(x_{Max}) \tag{8d}$$

when applied at $x = x_{Max}$.

One caution is required. When

$$\omega^2 > f^2 + c_0^2 l^2 \quad , \tag{9}$$

(where $c_0^2 = gh_M$ and h_M is the maximum depth), there is a continuum of onshore-offshore propagating solutions, and no trapped modes exist.

The sorts of waves that can be treated with this software are summarized in a schematic diagram of the stable modes' dispersion curves (Figure 1). See Huthnance (1975) for a thorough discussion. For $\omega_R < f$, there can only be trapped waves, and these fall into two categories. First, there is a rapidly propagating (speed often of order c_0) Kelvin wave, which has dynamics similar to a long gravity wave and yet has alongshore flow in essentially geostrophic balance. Further, there is an infinite (as long as topography is smooth) set of trapped topographic Rossby waves which propagate more slowly and are dispersive for larger wavenumbers. These are called barotropic continental shelf waves, or simply shelf waves. Higher shelf wave modes propagate more slowly and have an increasing number of zero crossings in the pressure modal structure $P(x)$. All of these waves propagate phase in the $-y$ direction when $f > 0$ (i.e., to the south off the east coast of the United States, or to the North off the west coast). The sense of phase propagation reverses in the southern hemisphere. All of these waves can be damped and modified by bottom friction. Further, there is potential for barotropic instabilities throughout the space.

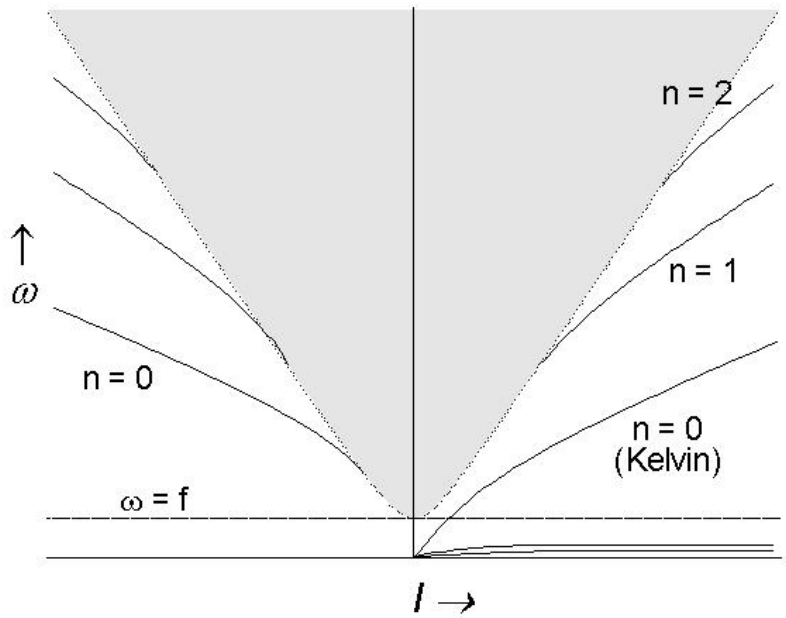

Figure 1: Schematic real dispersion curves for the sorts of waves that can be resolved with this software. This plot is for $f > 0$, a closed boundary at the coast and an open boundary offshore where the water is deepest. The shaded area represents the continuum where there are no trapped waves. The first two shelf waves are plotted in the region $\omega_R < f$, $l > 0$.

For $\omega_R > f$, there are infinite sets of edge waves, which propagate in both the positive and negative y directions. These waves are essentially long gravity waves trapped in shallow water because, for a given l, the wave cannot propagate once the water becomes deep enough. Higher order edge waves have higher frequencies and an increasing number of zero crossings in their modal structure. One interesting aspect of this collection involves the near-Kelvin wave (the only dispersion curve that crosses $\omega_R = f$ in Figure 1). Regardless of the frequency, the modal structure has no zero crossings in x, but its nature does change with increasing frequency or wavenumber. Specifically, for smaller ω_R, including for $\omega_R = O(f)$, the wave has very large offshore scales as is the case for a traditional barotropic Kelvin wave in a flat-bottom ocean. However, for larger ω or l, the cross-shelf scale decreases and the wave becomes an $n = 0$ edge wave. I take n to be the number of zero crossings in the pressure modal function.

Bottom friction can be included at lowest order. Even moderate bottom stress values can substantially change wave modal structures and propagation rates, especially for $\omega_R << f$ (Power et al., 1989, Brink, 2006). Also, with bottom friction, or a spatially variable mean flow, the coastal long wave modes no longer form an orthogonal set. For this reason, there is no "coastal long wave" option with this code.

This software can be used to study linearly unstable solutions, including those in the surf zone where Earth's rotation is unlikely to be important (e.g., Bowen and Holman, 1989, Dodd et al., 1992, Slinn et al., 1998).

How the software works:

For given f, h, v_0, alongshore wavenumber l and boundary conditions, equation (4) is solved, and the complex ω is found via resonance iteration. Specifically, an arbitrary forcing is applied to the left hand side of (4), and the frequency is varied in search of a maximum for P response. The process is complete once ω has converged to a fractional accuracy set by "acc" in the input array. The responses is measured by

$$R = \int h|P|^2 dx \quad . \tag{10}$$

The actual search is carried out using the Matlab function "fminsearch", searching for a minimum of $1/R$.

Initially, the program plots out v_0, r and h and outputs statements describing boundary conditions, f, number of grid points, etc. The code also checks the fields of $h(x)$, $v_0(x)$ and $r(x)$ to make sure that they are consistent with the chosen boundary conditions (eqns. 7). The code also checks to see if the necessary condition for barotropic instability is met. Once the solution (optimal ω and P) is found, the pressure field is normalized (arbitrarily) so that

$$1 = \int h|P|^2 dx \quad . \tag{11}$$

When a dispersion curve is calculated, the curve is plotted as it develops, along with the evolving complex modal structure.

Finally, when all calculations are completed, the user is given the option of saving results to a file with the name provided in the call statement.

Using the software:

Examples of usage are attached as an appendix to this document. All of the mfiles used by this package have names beginning with "bwavesc….". In practice, one first calls "bwavecsetup" in order to create an input array.

```
>>test = bwavescsetup;
```

This setup code asks a number of questions to obtain needed information about model assumptions, grid size and resolution, topography, mean flow and bottom friction. It asks for a first guess complex frequency corresponding to the first wavenumber to be considered. In the end, an array ("test" in this example) is then generated.

Next, one calls the main routine.

```
>> bwavesc(test,'title');
```

where 'title' is a string that (if desired) is used to create an output file like "title.mat". At the beginning of the run, a plot is given of v_0, r and h and some information is output regarding assumptions, f and grid. Also, if the mean flow is non-zero, the code checks for whether the necessary condition for barotropic instability is met. Then the code begins to search for the complex resonant frequency corresponding to the first wavenumber. Information about the search is provided on the screen. Once a resonance is found, the results are plotted, and an estimate is made for the first-guess frequency for the next wavenumber.

Some notes on using the software:

Searching in the complex plane can be rather finicky and it is sometimes not very obvious how to make a good initial guess at complex frequency or how to identify modes. It is strongly advised that the user learns to use the real frequency code (bwavesp.m or bigr*.m) before starting to work with this. At a minimum, one can start by using bwavesc with v_0 = 0 and r = 0 to see how it works with a stable, undamped configuration.

If you are searching for a solution where the frequency is real, do not make an initial guess of $\omega_I = 0$. This leads to a very inefficient search because fminsearch will not know the correct order of magnitude of ω_I. It is much more effective to use a small but nonzero estimate, e.g. $\omega = (2.2\times10^{-5} + i1\times10^{-10})$ 1/sec rather than $\omega = (2.2\times10^{-5} + i0)$ 1/sec. Similarly, in looking at results, if (e.g.) $|\omega_I/\omega_R|$ < acc, you should interpret ω_I as zero.

A good deal of caution is required near the inertial frequency, where (when v_0 = 0) a spurious solution exists (section 3.9 of Pedlosky, 1979; Dale, Sherwin and Huthnance, 2001). The solution obeys equations 4, 6 and 8, but does not satisfy the original equations of motion (equations 1). Further, the presence of this spurious mode deflects nearby dispersion curves. The frequency range over which this distortion occurs was shown by Dale et al. to depend on the grid resolution: the finer the resolution, the less problem you will have. I have found that for reasonable x grid resolution (say, 1 km for a realistic shelf), the zone of influence is less than about $0.01f$ wide in frequency. However, sometimes a computed dispersion curve will find the spurious mode, and then follow it rather than a physically correct mode. When this happens, the sudden turn of the dispersion curve to follow $\omega = f$ is very obvious, and the offending solution can be rejected. One can usually keep the computed dispersion curves from "jumping" by using finer resolution in alongshore wavenumber and/or demanding more accurate calculations.

If you are interested in waves having $|\omega/f|$ > 1 (e.g., edge waves and higher frequency Kelvin waves), you really should use the free surface boundary condition.

If the $x = 0$ or $x = x_{Max}$ boundary is open, the bottom should be flat at the boundary. Also $v_{0x} = r_x = 0$ at the open boundary. The user will get an error message if there is a problem. In the case of an open boundary condition, the water does not have to be shallow.

Note that, for depth, mean velocity and bottom friction, the requested input (in bwavescsetup or bwavescfinch) can call for an array of x locations. The array x array does not have to start with zero, and it can be as short as only one element. The software will fill in any gaps you may leave near the boundaries. However, the array of x locations must have monotonically increasing values, e.g. [10 20 30 40]. It cannot be decreasing (e.g. [40 30 20 10]), and it has to be monotonic (e.g., it cannot be [10 30 20 40]).

If you give the algorithm an initial estimate for frequency that has few places of accuracy (say 1e-5), the Matlab search code will look around at fairly large increments (perhaps ± 20%), and the search can miss nearby solutions. If you provide more places of accuracy (say 1.01e-5), the search will be initially confined to more nearby locations (perhaps ± 3%), and it becomes much more likely that nearby solutions will be found.

The search can sometime miss a frequency.

If the model obtains a good solution, the inverse resonance parameter in the output (rni) should be several orders of magnitude lower than neighboring values. If this is not true, you may either have a bad solution or there could be some other problem. Using a finer accuracy tolerance (acc) will tend to eliminate false solutions.

For an accuracy estimate (“e” option in bwavescfinch.m), I find that 0.0001 works out fairly well. This means frequency accuracy to 0.01% of the absolute value of the initial frequency guess.

The bottom stress is assumed to be proportional to the bottom velocity. For the errors associated with this assumption in the presence of a mean flow, see Brink (1997).

All frequencies are in radians/sec and all wavenumbers are in radians/cm. Because radians are nondimensional, this could also be written as “1/sec” etc.

If the code is searching for a trapped wave in the continuum frequency range (gray area in Figure 1), trouble could result. If no reasonable resonance is found, the best strategy is to increase the magnitude of the wavenumber (so as to escape the continuum: see Figure 1) and try again.

In some cases, the structure of the forcing (a spike at mid-grid) will be obvious in the modal structure. This is a failing that typically happens when the forcing does not occur near (in x) to where the peak amplitude of the wave occurs. Improving grid resolution does not seem to help. Increasing the required fractional accuracy for frequency does help some. Alternatively, one can go into the bwavescsol.m file, near line 40, and change nf so that the forcing occurs closer (in *x*) to where the wave's modal structure has large amplitude.

The water depth *h* must always be > 0, including at the boundaries. The depth at a coastal wall can be made extremely small, but it cannot vanish.

When the mean alongshore flow is nonzero, it is possible to have critical layer (where ω'' passes through zero and ω is real) "solutions". When frequency is real to within the given precision (acc), a warning is given when a critical layer occurs.

Disclaimer:

Although considerable effort has been made to make sure this software is correct and easy to use, there is no guarantee of perfection. If errors are found, or if documentation could be improved, please contact kbrink@whoi.edu .

References:

Bowen, A.J. and R.A. Holman, 1989: Shear instabilities of the mean longshore current 1. Theory. J. Geophys Res., 94, 18,023-18,030.

Brink, K.H., 1990: On the damping of free coastal-trapped waves. *J. Phys. Oceanogr.*, 20, 1219-1225.

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Dale, A.C., J.M. Huthnance, and T.J. Sherwin, 2001: Coastal-Trapped Waves and Tides at Near-Inertial Frequencies. , J. Phys. Oceanogr., **31**, 2958–2970.

Dodd, N., J. Oltman-Shay, and E.B. Thornton, 1992: Shear instabilities in the longshore current: a comparison of observation and theory. J. Phys. Oceanogr., 22, 62-82.

Huthnance, J.M., 1975: On trapped waves over a continental shelf. J. Fluid Mech., 69. 689-704.

Pedlosky, J., 1979: Geophysical Fluid Dynamics, Springer-Verlag, New York, 624pp.

Power, S. B., J. H. Middleton, and R. H. J. Grimshaw, 1989: Frictionally modified continental shelf waves and the subinertial response to wind and deep-sea forcing. J. Phys. Oceanogr., 19, 1486–1506, https://doi.org/10.1175/1520-0485(1989)019,1486: FMCSWA.2.0.CO;2.

Slinn, D.N., J.S. Allen, P.A. Newberger and R.A. Holman, 1998: Nonlinear shear instabilities of alongshore currents over barred beaches. J. Geophys. Res., 103,18,357-18,379.

Sample: Running bwavescsetup.m

The following is a copy of what you see on the screen as you run bwavescsetup.m.

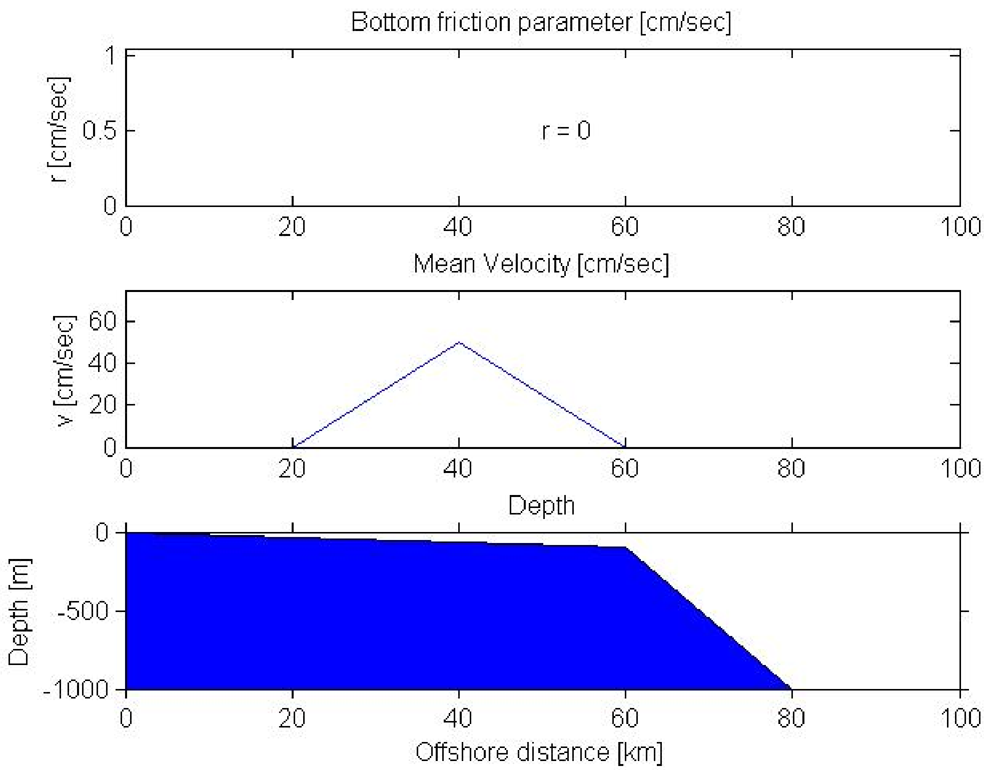
```
>>
>> acdemo = bwavescsetup;

    Barotropic coastal-trapped waves with complex frequency

This mfile will ask you a sequence of questions
    that are used to build the input array.

How many total gridpoints do you want in the cross shelf direction? (nn) 100
Enter the domain width (W) (km) 100
Enter the nominal fractional accuracy for the solution (acc) 1e-4
Enter 0 for a rigid lid, 1 for a free surface (del) 1
Enter 0 for closed boundary at x = 0, 1 for open boundary (icbc) 0
Enter 0 for closed boundary at x = x_max, 1 for open boundary (iobc) 1
Enter the Coriolis parameter (f) (1/sec) 1e-4
Enter the number of frequencies to be computed (nw) 6
Enter the first alongshore wavenumber to use (rlz) (1/cm) 3.7e-7
Enter the wavenumber increment to use after rlz (drl) (1/cm) 1e-8
First guess at real part of frequency (1/sec)? 1.4e-4
First guess at imaginary part of frequency (1/sec)? -5.4e-5
How many distance, depth pairs will you provide (ndep >=1) 3
Array of offshore distances for depth values (xdep in km) (dimension ndep) [0 60 80]
Array of depths corresponding to xdep (depr in m) [10 100 1000]
How many distance, mean v pairs will you provide (nv >= 0 ) 3
Array of offshore distances for mean flow values (xv in km) (dimension nv) [20 40 60]
Array of mean velocities corresponding to xv (vbar in cm/sec) [0 50 0]
Number of distance, bottom friction pairs to read (nr) 0
>>
```

At this point, the following plot appears in a plot window:



Sample: running bwavesc.m

The following is a copy of what you see on the screen as you run bwavesc.m

```
>>
>> bwavesc(acdemo,'sample')
f = 0.0001 1/sec
Free surface
Closed boundary at x = 0
Open boundary at x = x_Max
100 grid points in x

   Potentially barotropically unstable!
        ((f + v_x)/h)_x changes sign




rl, w, rni = 3.7e-07, 0.00014-5.4e-05i,  2.9686e-43
rl, w, rni = 3.7e-07, 0.000147-5.4e-05i,  2.4412e-42
```

rl, w, rni = 3.7e-07, 0.00014-5.67e-05i, 1.928e-43
rl, w, rni = 3.7e-07, 0.000133-5.67e-05i, 8.2206e-43
rl, w, rni = 3.7e-07, 0.0001365-5.6025e-05i, 4.1007e-44
rl, w, rni = 3.7e-07, 0.0001365-5.8725e-05i, 2.9385e-43
rl, w, rni = 3.7e-07, 0.00013737-5.7544e-05i, 7.276e-44
rl, w, rni = 3.7e-07, 0.00013387-5.6869e-05i, 5.3983e-43
rl, w, rni = 3.7e-07, 0.00013847-5.6742e-05i, 3.7786e-44
rl, w, rni = 3.7e-07, 0.00013759-5.5223e-05i, 2.4898e-44
rl, w, rni = 3.7e-07, 0.0001377-5.4063e-05i, 1.2797e-43
rl, w, rni = 3.7e-07, 0.00013956-5.5941e-05i, 1.208e-43
rl, w, rni = 3.7e-07, 0.00013727-5.6004e-05i, 4.0715e-45
rl, w, rni = 3.7e-07, 0.00013639-5.4485e-05i, 1.3097e-43
rl, w, rni = 3.7e-07, 0.00013795-5.6178e-05i, 4.0963e-45
rl, w, rni = 3.7e-07, 0.00013762-5.6958e-05i, 2.4353e-44
rl, w, rni = 3.7e-07, 0.00013761-5.6525e-05i, 5.7806e-45
rl, w, rni = 3.7e-07, 0.0001376-5.5657e-05i, 6.5506e-45
rl, w, rni = 3.7e-07, 0.00013761-5.6308e-05i, 1.3254e-45
rl, w, rni = 3.7e-07, 0.00013693-5.6134e-05i, 1.5191e-44
rl, w, rni = 3.7e-07, 0.00013769-5.6167e-05i, 3.9336e-46
rl, w, rni = 3.7e-07, 0.00013804-5.6471e-05i, 1.0598e-44
rl, w, rni = 3.7e-07, 0.00013746-5.6121e-05i, 6.7542e-46
rl, w, rni = 3.7e-07, 0.00013754-5.598e-05i, 6.5043e-46
rl, w, rni = 3.7e-07, 0.00013778-5.6026e-05i, 1.2127e-45
rl, w, rni = 3.7e-07, 0.00013754-5.6097e-05i, 1.3497e-46
rl, w, rni = 3.7e-07, 0.00013769-5.6284e-05i, 1.2841e-45
rl, w, rni = 3.7e-07, 0.00013758-5.6056e-05i, 1.0423e-46
rl, w, rni = 3.7e-07, 0.00013742-5.5986e-05i, 1.5241e-45
rl, w, rni = 3.7e-07, 0.00013763-5.6122e-05i, 2.6313e-47
rl, w, rni = 3.7e-07, 0.00013767-5.6081e-05i, 1.6247e-46
rl, w, rni = 3.7e-07, 0.00013757-5.6093e-05i, 3.8875e-47
rl, w, rni = 3.7e-07, 0.00013762-5.6159e-05i, 9.4408e-47
rl, w, rni = 3.7e-07, 0.00013761-5.6133e-05i, 2.2602e-47
rl, w, rni = 3.7e-07, 0.00013766-5.6162e-05i, 2.2292e-46
rl, w, rni = 3.7e-07, 0.00013759-5.611e-05i, 2.2395e-48
rl, w, rni = 3.7e-07, 0.00013758-5.6121e-05i, 2.8546e-47
rl, w, rni = 3.7e-07, 0.00013761-5.6122e-05i, 1.1154e-47
rl, w, rni = 3.7e-07, 0.0001376-5.6099e-05i, 2.7102e-48
rl, w, rni = 3.7e-07, 0.00013758-5.6087e-05i, 2.9483e-47
rl, w, rni = 3.7e-07, 0.00013761-5.6113e-05i, 1.3698e-48

rl (cm^-1), w (complex: sec^-1) = 3.7e-07, 0.00013761-5.6113e-05i

rl, w, rni = 3.8e-07, 0.00014132-5.763e-05i, 6.9591e-43
rl, w, rni = 3.8e-07, 0.00014839-5.763e-05i, 4.0689e-42
rl, w, rni = 3.8e-07, 0.00014132-6.0511e-05i, 1.4164e-42

rl, w, rni = 3.8e-07, 0.00013426-6.0511e-05i, 1.3715e-42
rl, w, rni = 3.8e-07, 0.00013426-5.763e-05i, 5.0037e-43
rl, w, rni = 3.8e-07, 0.00013072-5.6189e-05i, 2.0184e-42
rl, w, rni = 3.8e-07, 0.00014132-5.4748e-05i, 6.1371e-43
rl, w, rni = 3.8e-07, 0.00013426-5.4748e-05i, 4.1692e-43
rl, w, rni = 3.8e-07, 0.00013072-5.3307e-05i, 2.0839e-42
rl, w, rni = 3.8e-07, 0.00012719-5.763e-05i, 6.1657e-42
rl, w, rni = 3.8e-07, 0.00013779-5.5468e-05i, 2.3178e-44
rl, w, rni = 3.8e-07, 0.00013779-5.2587e-05i, 4.0695e-43
rl, w, rni = 3.8e-07, 0.00014132-5.3307e-05i, 7.7233e-43
rl, w, rni = 3.8e-07, 0.00013602-5.4388e-05i, 1.5522e-43
rl, w, rni = 3.8e-07, 0.00013602-5.7269e-05i, 1.2258e-43
rl, w, rni = 3.8e-07, 0.00013779-5.835e-05i, 2.2763e-43
rl, w, rni = 3.8e-07, 0.00013647-5.5378e-05i, 3.8816e-44
rl, w, rni = 3.8e-07, 0.00013823-5.3577e-05i, 2.4082e-43
rl, w, rni = 3.8e-07, 0.00013658-5.6346e-05i, 2.0812e-44
rl, w, rni = 3.8e-07, 0.0001379-5.6436e-05i, 2.3002e-44
rl, w, rni = 3.8e-07, 0.00013669-5.7314e-05i, 7.8071e-44
rl, w, rni = 3.8e-07, 0.00013751-5.593e-05i, 3.3619e-45
rl, w, rni = 3.8e-07, 0.00013619-5.584e-05i, 4.3971e-44
rl, w, rni = 3.8e-07, 0.00013747-5.6287e-05i, 4.6376e-45
rl, w, rni = 3.8e-07, 0.00013841-5.5871e-05i, 5.1947e-44
rl, w, rni = 3.8e-07, 0.00013704-5.6228e-05i, 2.9851e-45
rl, w, rni = 3.8e-07, 0.00013708-5.587e-05i, 1.9624e-45
rl, w, rni = 3.8e-07, 0.00013688-5.5662e-05i, 1.0076e-44
rl, w, rni = 3.8e-07, 0.0001366-5.6168e-05i, 1.6525e-44
rl, w, rni = 3.8e-07, 0.00013729-5.5989e-05i, 1.7113e-46
rl, w, rni = 3.8e-07, 0.00013733-5.5632e-05i, 6.3595e-45
rl, w, rni = 3.8e-07, 0.00013711-5.6079e-05i, 6.9107e-46
rl, w, rni = 3.8e-07, 0.00013732-5.6198e-05i, 1.2935e-45
rl, w, rni = 3.8e-07, 0.00013726-5.6116e-05i, 2.7961e-46
rl, w, rni = 3.8e-07, 0.00013743-5.6027e-05i, 1.5088e-45
rl, w, rni = 3.8e-07, 0.00013719-5.6066e-05i, 1.173e-46
rl, w, rni = 3.8e-07, 0.00013722-5.5939e-05i, 3.4734e-46
rl, w, rni = 3.8e-07, 0.00013725-5.6072e-05i, 6.2359e-47
rl, w, rni = 3.8e-07, 0.00013715-5.6148e-05i, 7.6959e-46
rl, w, rni = 3.8e-07, 0.00013725-5.6029e-05i, 1.2044e-47
rl, w, rni = 3.8e-07, 0.00013731-5.6035e-05i, 2.1293e-46
rl, w, rni = 3.8e-07, 0.00013722-5.6058e-05i, 3.1965e-47
rl, w, rni = 3.8e-07, 0.00013722-5.6015e-05i, 1.6333e-47
rl, w, rni = 3.8e-07, 0.00013726-5.5986e-05i, 1.0215e-46
rl, w, rni = 3.8e-07, 0.00013723-5.604e-05i, 3.0452e-48
rl, w, rni = 3.8e-07, 0.00013726-5.6054e-05i, 3.417e-47
rl, w, rni = 3.8e-07, 0.00013723-5.6025e-05i, 2.8453e-48
rl, w, rni = 3.8e-07, 0.00013721-5.6036e-05i, 2.4764e-47
rl, w, rni = 3.8e-07, 0.00013724-5.6031e-05i, 1.9071e-48

rl (cm^-1), w (complex: sec^-1) = 3.8e-07, 0.00013724-5.6031e-05i

rl, w, rni = 3.9e-07, 0.00013688-5.5949e-05i, 3.8777e-47
rl, w, rni = 3.9e-07, 0.00014372-5.5949e-05i, 1.7672e-42
rl, w, rni = 3.9e-07, 0.00013688-5.8746e-05i, 3.8313e-43
rl, w, rni = 3.9e-07, 0.00013003-5.8746e-05i, 3.4619e-42
rl, w, rni = 3.9e-07, 0.0001403-5.6648e-05i, 5.0516e-43
rl, w, rni = 3.9e-07, 0.00013346-5.8047e-05i, 8.623e-43
rl, w, rni = 3.9e-07, 0.00013859-5.6998e-05i, 1.7793e-43
rl, w, rni = 3.9e-07, 0.00013859-5.42e-05i, 2.3512e-43
rl, w, rni = 3.9e-07, 0.00013816-5.5337e-05i, 8.3155e-44
rl, w, rni = 3.9e-07, 0.00013645-5.4288e-05i, 1.2057e-43
rl, w, rni = 3.9e-07, 0.00013698-5.4965e-05i, 3.9378e-44
rl, w, rni = 3.9e-07, 0.0001357-5.5577e-05i, 6.8089e-44
rl, w, rni = 3.9e-07, 0.00013632-5.5517e-05i, 2.1212e-44
rl, w, rni = 3.9e-07, 0.00013621-5.65e-05i, 3.6128e-44
rl, w, rni = 3.9e-07, 0.0001364-5.6117e-05i, 1.1969e-44
rl, w, rni = 3.9e-07, 0.00013696-5.6548e-05i, 1.8144e-44
rl, w, rni = 3.9e-07, 0.0001368-5.629e-05i, 6.4468e-45
rl, w, rni = 3.9e-07, 0.00013728-5.6122e-05i, 8.8043e-45
rl, w, rni = 3.9e-07, 0.00013706-5.6121e-05i, 3.239e-45
rl, w, rni = 3.9e-07, 0.00013713-5.5779e-05i, 3.7e-45
rl, w, rni = 3.9e-07, 0.00013705-5.5907e-05i, 1.3122e-45
rl, w, rni = 3.9e-07, 0.00013687-5.5735e-05i, 1.4993e-45
rl, w, rni = 3.9e-07, 0.00013692-5.5831e-05i, 4.0531e-46
rl, w, rni = 3.9e-07, 0.00013674-5.5873e-05i, 8.9942e-46
rl, w, rni = 3.9e-07, 0.00013682-5.5881e-05i, 2.115e-46
rl, w, rni = 3.9e-07, 0.00013678-5.5999e-05i, 7.0536e-46
rl, w, rni = 3.9e-07, 0.00013688-5.5873e-05i, 9.4098e-47
rl, w, rni = 3.9e-07, 0.00013694-5.594e-05i, 1.873e-46
rl, w, rni = 3.9e-07, 0.00013691-5.5926e-05i, 4.618e-47
rl, w, rni = 3.9e-07, 0.0001369-5.6001e-05i, 3.2882e-46
rl, w, rni = 3.9e-07, 0.00013689-5.5905e-05i, 1.2941e-47
rl, w, rni = 3.9e-07, 0.00013686-5.5928e-05i, 2.6453e-47
rl, w, rni = 3.9e-07, 0.00013687-5.5885e-05i, 5.8532e-47
rl, w, rni = 3.9e-07, 0.00013688-5.5933e-05i, 8.756e-48
rl, w, rni = 3.9e-07, 0.00013691-5.591e-05i, 4.096e-47
rl, w, rni = 3.9e-07, 0.00013687-5.5923e-05i, 5.1125e-48
rl, w, rni = 3.9e-07, 0.00013686-5.5951e-05i, 6.9511e-47
rl, w, rni = 3.9e-07, 0.00013688-5.5917e-05i, 3.8089e-49
rl, w, rni = 3.9e-07, 0.00013687-5.5907e-05i, 6.8314e-48
rl, w, rni = 3.9e-07, 0.00013687-5.5914e-05i, 2.0566e-48

rl (cm^-1), w (complex: sec^-1) = 3.9e-07, 0.00013688-5.5917e-05i

rl, w, rni = 4e-07, 0.00013652-5.5802e-05i, 7.1412e-47
rl, w, rni = 4e-07, 0.00014335-5.5802e-05i, 2.0046e-42
rl, w, rni = 4e-07, 0.00013652-5.8592e-05i, 4.4399e-43
rl, w, rni = 4e-07, 0.00012969-5.8592e-05i, 4.0925e-42
rl, w, rni = 4e-07, 0.00013993-5.65e-05i, 5.7491e-43
rl, w, rni = 4e-07, 0.00013311-5.7895e-05i, 1.012e-42
rl, w, rni = 4e-07, 0.00013823-5.6849e-05i, 2.0287e-43
rl, w, rni = 4e-07, 0.00013823-5.4058e-05i, 2.6744e-43
rl, w, rni = 4e-07, 0.0001378-5.5192e-05i, 9.3773e-44
rl, w, rni = 4e-07, 0.00013609-5.4146e-05i, 1.3909e-43
rl, w, rni = 4e-07, 0.00013663-5.4821e-05i, 4.486e-44
rl, w, rni = 4e-07, 0.00013535-5.5432e-05i, 8.0361e-44
rl, w, rni = 4e-07, 0.00013596-5.5372e-05i, 2.5127e-44
rl, w, rni = 4e-07, 0.00013585-5.6353e-05i, 4.3072e-44
rl, w, rni = 4e-07, 0.00013605-5.597e-05i, 1.4627e-44
rl, w, rni = 4e-07, 0.00013661-5.64e-05i, 2.1151e-44
rl, w, rni = 4e-07, 0.00013644-5.6143e-05i, 7.7494e-45
rl, w, rni = 4e-07, 0.00013692-5.5976e-05i, 9.712e-45
rl, w, rni = 4e-07, 0.0001367-5.5974e-05i, 3.5959e-45
rl, w, rni = 4e-07, 0.00013677-5.5633e-05i, 3.8596e-45
rl, w, rni = 4e-07, 0.00013669-5.5761e-05i, 1.2793e-45
rl, w, rni = 4e-07, 0.00013651-5.5589e-05i, 1.6642e-45
rl, w, rni = 4e-07, 0.00013656-5.5685e-05i, 3.8215e-46
rl, w, rni = 4e-07, 0.00013639-5.5727e-05i, 1.2164e-45
rl, w, rni = 4e-07, 0.00013646-5.5735e-05i, 3.1793e-46
rl, w, rni = 4e-07, 0.00013642-5.5852e-05i, 1.0021e-45
rl, w, rni = 4e-07, 0.00013652-5.5727e-05i, 9.0669e-47
rl, w, rni = 4e-07, 0.00013658-5.5794e-05i, 1.5128e-46
rl, w, rni = 4e-07, 0.00013655-5.5779e-05i, 2.2974e-47
rl, w, rni = 4e-07, 0.00013655-5.5855e-05i, 3.9395e-46
rl, w, rni = 4e-07, 0.00013653-5.5759e-05i, 5.0049e-48
rl, w, rni = 4e-07, 0.00013656-5.5736e-05i, 9.5071e-47
rl, w, rni = 4e-07, 0.00013653-5.5786e-05i, 1.6802e-47
rl, w, rni = 4e-07, 0.00013651-5.5765e-05i, 3.369e-47
rl, w, rni = 4e-07, 0.00013654-5.5776e-05i, 5.6671e-48
rl, w, rni = 4e-07, 0.00013654-5.5749e-05i, 2.1092e-47
rl, w, rni = 4e-07, 0.00013653-5.5777e-05i, 3.7783e-48
rl, w, rni = 4e-07, 0.00013652-5.576e-05i, 1.0912e-47
rl, w, rni = 4e-07, 0.00013654-5.5772e-05i, 9.9273e-49

rl (cm^-1), w (complex: sec^-1) = 4e-07, 0.00013654-5.5772e-05i

rl, w, rni = 4.1e-07, 0.00013619-5.5627e-05i, 1.1584e-46

rl, w, rni = 4.1e-07, 0.000143-5.5627e-05i, 2.2854e-42
rl, w, rni = 4.1e-07, 0.00013619-5.8408e-05i, 5.1488e-43
rl, w, rni = 4.1e-07, 0.00012938-5.8408e-05i, 4.8076e-42
rl, w, rni = 4.1e-07, 0.0001396-5.6322e-05i, 6.6139e-43
rl, w, rni = 4.1e-07, 0.00013279-5.7713e-05i, 1.1773e-42
rl, w, rni = 4.1e-07, 0.00013789-5.667e-05i, 2.3543e-43
rl, w, rni = 4.1e-07, 0.00013789-5.3889e-05i, 3.0639e-43
rl, w, rni = 4.1e-07, 0.00013747-5.5019e-05i, 1.0811e-43
rl, w, rni = 4.1e-07, 0.00013577-5.3976e-05i, 1.5824e-43
rl, w, rni = 4.1e-07, 0.0001363-5.4649e-05i, 5.0716e-44
rl, w, rni = 4.1e-07, 0.00013502-5.5258e-05i, 9.164e-44
rl, w, rni = 4.1e-07, 0.00013563-5.5198e-05i, 2.8148e-44
rl, w, rni = 4.1e-07, 0.00013553-5.6176e-05i, 4.9877e-44
rl, w, rni = 4.1e-07, 0.00013572-5.5794e-05i, 1.6749e-44
rl, w, rni = 4.1e-07, 0.00013628-5.6223e-05i, 2.5118e-44
rl, w, rni = 4.1e-07, 0.00013612-5.5967e-05i, 9.2806e-45
rl, w, rni = 4.1e-07, 0.00013659-5.58e-05i, 1.1701e-44
rl, w, rni = 4.1e-07, 0.00013637-5.5798e-05i, 4.4954e-45
rl, w, rni = 4.1e-07, 0.00013645-5.5458e-05i, 4.4861e-45
rl, w, rni = 4.1e-07, 0.00013627-5.5287e-05i, 5.4268e-45
rl, w, rni = 4.1e-07, 0.00013635-5.5671e-05i, 1.7e-45
rl, w, rni = 4.1e-07, 0.00013609-5.5839e-05i, 4.6452e-45
rl, w, rni = 4.1e-07, 0.00013636-5.5554e-05i, 1.5175e-45
rl, w, rni = 4.1e-07, 0.0001362-5.551e-05i, 3.2234e-46
rl, w, rni = 4.1e-07, 0.00013604-5.5583e-05i, 1.5706e-45
rl, w, rni = 4.1e-07, 0.00013628-5.5561e-05i, 3.8658e-46
rl, w, rni = 4.1e-07, 0.00013612-5.5576e-05i, 4.0643e-46
rl, w, rni = 4.1e-07, 0.00013624-5.5565e-05i, 1.0508e-46
rl, w, rni = 4.1e-07, 0.00013623-5.5682e-05i, 6.1137e-46
rl, w, rni = 4.1e-07, 0.00013621-5.5553e-05i, 6.1001e-47
rl, w, rni = 4.1e-07, 0.00013626-5.5491e-05i, 6.907e-46
rl, w, rni = 4.1e-07, 0.00013621-5.5593e-05i, 8.418e-48
rl, w, rni = 4.1e-07, 0.00013618-5.5581e-05i, 2.65e-47
rl, w, rni = 4.1e-07, 0.00013618-5.5621e-05i, 1.1374e-46
rl, w, rni = 4.1e-07, 0.0001362-5.557e-05i, 1.1226e-47
rl, w, rni = 4.1e-07, 0.00013623-5.5582e-05i, 5.2437e-47
rl, w, rni = 4.1e-07, 0.00013619-5.5581e-05i, 4.436e-48
rl, w, rni = 4.1e-07, 0.0001362-5.5604e-05i, 2.5336e-47
rl, w, rni = 4.1e-07, 0.0001362-5.5579e-05i, 1.5764e-48

rl (cm^-1), w (complex: sec^-1) = 4.1e-07, 0.0001362-5.5579e-05i

rl, w, rni = 4.2e-07, 0.00013587-5.5385e-05i, 2.9135e-47
rl, w, rni = 4.2e-07, 0.00014266-5.5385e-05i, 2.5929e-42
rl, w, rni = 4.2e-07, 0.00013587-5.8155e-05i, 5.7967e-43

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rl, w, rni = 4.2e-07, 0.00012907-5.8155e-05i,  5.6268e-42
rl, w, rni = 4.2e-07, 0.00013926-5.6078e-05i,  7.5166e-43
rl, w, rni = 4.2e-07, 0.00013247-5.7462e-05i,  1.3593e-42
rl, w, rni = 4.2e-07, 0.00013756-5.6424e-05i,  2.6494e-43
rl, w, rni = 4.2e-07, 0.00013756-5.3655e-05i,  3.5515e-43
rl, w, rni = 4.2e-07, 0.00013714-5.478e-05i,  1.2528e-43
rl, w, rni = 4.2e-07, 0.00013544-5.3741e-05i,  1.8745e-43
rl, w, rni = 4.2e-07, 0.00013597-5.4412e-05i,  6.1436e-44
rl, w, rni = 4.2e-07, 0.0001347-5.5018e-05i,  1.0774e-43
rl, w, rni = 4.2e-07, 0.00013531-5.4958e-05i,  3.4231e-44
rl, w, rni = 4.2e-07, 0.0001352-5.5932e-05i,  5.561e-44
rl, w, rni = 4.2e-07, 0.00013539-5.5552e-05i,  1.8844e-44
rl, w, rni = 4.2e-07, 0.00013595-5.5979e-05i,  2.6399e-44
rl, w, rni = 4.2e-07, 0.00013579-5.5724e-05i,  9.3098e-45
rl, w, rni = 4.2e-07, 0.00013626-5.5557e-05i,  1.2466e-44
rl, w, rni = 4.2e-07, 0.00013605-5.5556e-05i,  4.3119e-45
rl, w, rni = 4.2e-07, 0.00013612-5.5218e-05i,  5.5163e-45
rl, w, rni = 4.2e-07, 0.00013604-5.5344e-05i,  1.7747e-45
rl, w, rni = 4.2e-07, 0.00013586-5.5173e-05i,  2.6512e-45
rl, w, rni = 4.2e-07, 0.0001359-5.5269e-05i,  7.3518e-46
rl, w, rni = 4.2e-07, 0.00013573-5.531e-05i,  1.7036e-45
rl, w, rni = 4.2e-07, 0.00013581-5.5319e-05i,  5.075e-46
rl, w, rni = 4.2e-07, 0.00013577-5.5435e-05i,  1.1162e-45
rl, w, rni = 4.2e-07, 0.00013587-5.5311e-05i,  2.4039e-46
rl, w, rni = 4.2e-07, 0.00013593-5.5377e-05i,  1.7061e-46
rl, w, rni = 4.2e-07, 0.00013592-5.5452e-05i,  6.0226e-46
rl, w, rni = 4.2e-07, 0.00013588-5.5346e-05i,  3.9524e-47
rl, w, rni = 4.2e-07, 0.00013582-5.5354e-05i,  2.3694e-46
rl, w, rni = 4.2e-07, 0.0001359-5.5371e-05i,  3.5876e-47
rl, w, rni = 4.2e-07, 0.00013588-5.5411e-05i,  1.203e-46
rl, w, rni = 4.2e-07, 0.00013588-5.5362e-05i,  5.346e-48
rl, w, rni = 4.2e-07, 0.00013585-5.5376e-05i,  6.6894e-47
rl, w, rni = 4.2e-07, 0.00013589-5.5373e-05i,  6.8978e-48
rl, w, rni = 4.2e-07, 0.00013591-5.5349e-05i,  7.9091e-47
rl, w, rni = 4.2e-07, 0.00013588-5.5376e-05i,  3.9678e-48

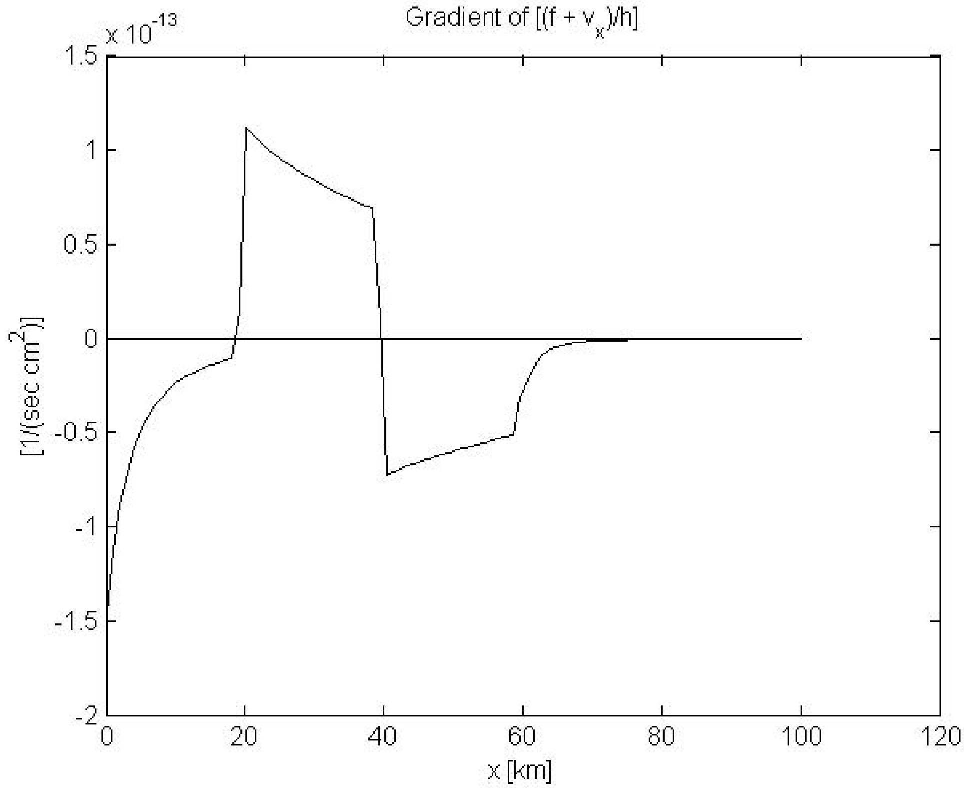
rl (cm^-1), w (complex: sec^-1) = 4.2e-07,  0.00013588-5.5376e-05i

Do you want to save this curve? yes = 1, no = 0 1
Saved as sample.mat
File includes complex dispersion curve, x, h, vbar, r, pressure (last frequency only),
             f, and del
>>
```

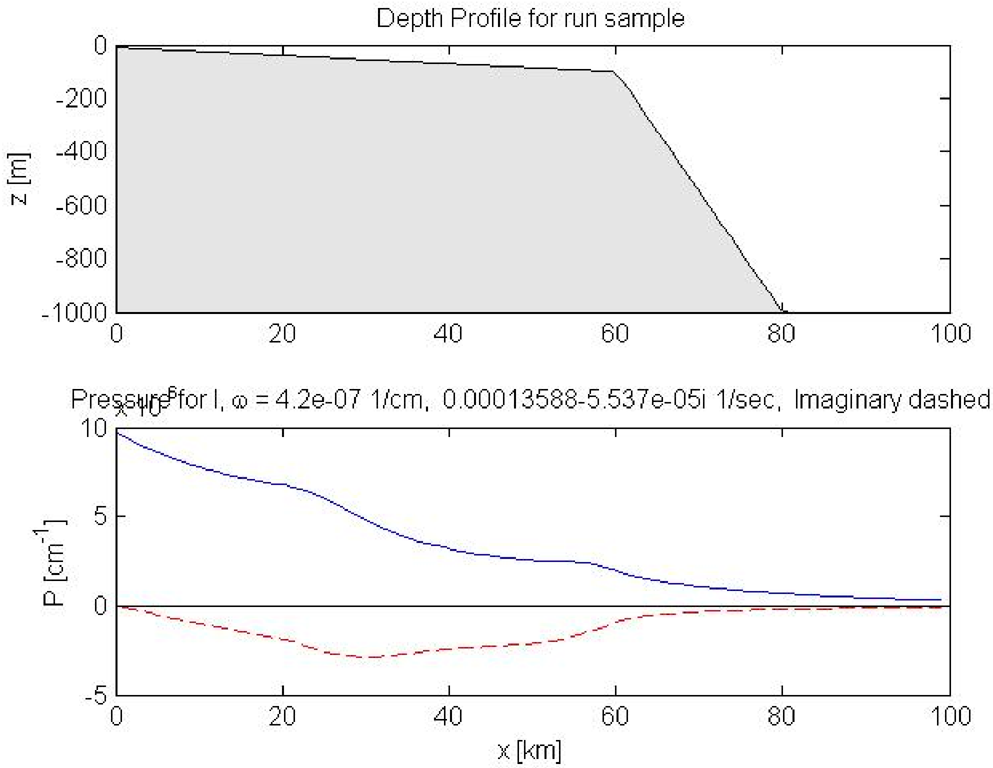
Note that the user is asked at the end whether she wants to save the information. Answered "1" here to save file.

Four plots are produced in this example. One is a repeat of the figure from bwavescsetup.m. *The other plots follow.*

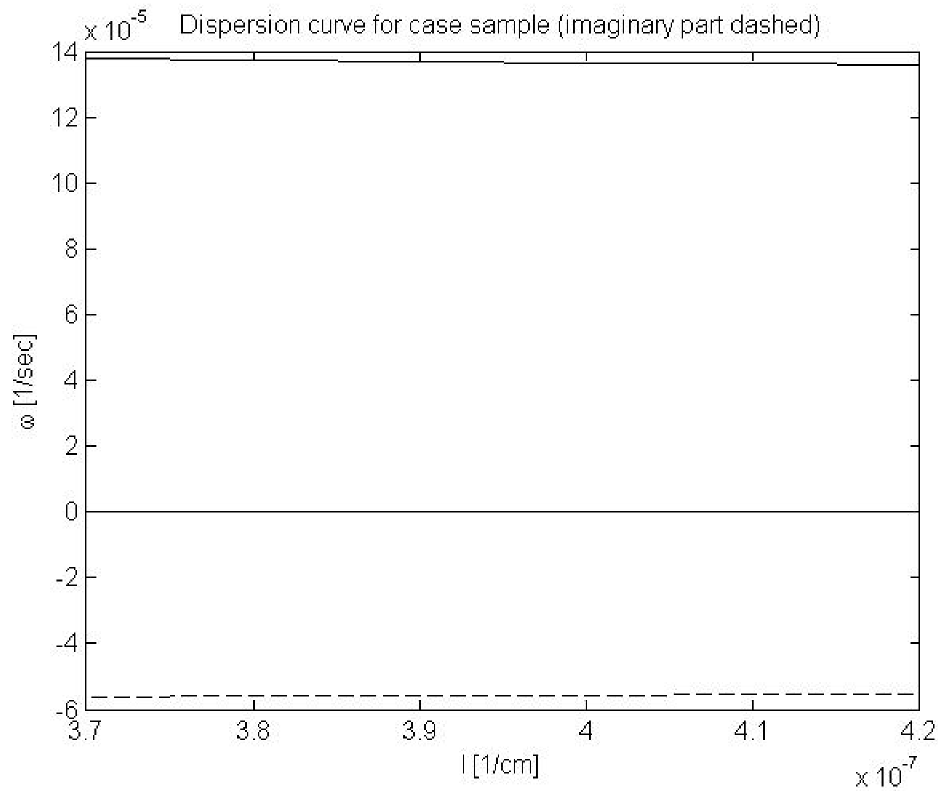
One plot shows the vorticity gradient. This plot will only appear when the gradient changes sign.



One plot shows pressure modal structure (real part as a solid line and imaginary as a dashed line):



and another plot shows the dispersion curve (real part as a solid line and imaginary as a dashed line). This example is pretty dull…..:



Sample: Running bwavescfinch.m

The follow is what appears on the screen. This run changes the range of wavenumbers to compute.

```
>>
>> acdemo = bwavescfinch(acdemo);
```

First you need to select what you want to change.

Options are:
  Grid size: enter "g"
  Initial Frequency guess: w
  Coriolis parameter: f
  Domain size: x
  Model assumptions: a
  Dispersion curve definition: d
  Nominal accuracy: e
  Depth profile: h

Bottom friction: r
Mean flow field: v

Any arrays are row arrays, not column arrays

Select an option d

Previous number of dispersion curve points = 6
Enter number of points on the dispersion curve 30
Old first wavenumber, increment along curve (cm^-1) = 3.7e-07   1e-08
Enter new first alongshore wavelength (cm-1) 3e-7
Enter new wavenumber increment (cm-1) 1e-8
>>

Note that first the user selects a category ("d" in this case). Then more specific information is required.

Description of the relevant mfiles:

bwavesc.m

This is the main mfile that drives all of the others. It uses the input array to define the key variables, and calls routines to fill out depth, velocity and bottom friction arrays. It then steps through the required wavenumbers to obtain and store modal complex frequencies. Finally, it gives the user an option to save results to a file.

bwavesccal.m

This file is essentially the interface between the fminsearch function and the code that sets up and solves the governing equation. This is also the place where the response magnitude is calculated.

bwavescdep.m

This file takes the input information about water depth, interpolates it onto the model grid, and computes the depth gradient. If information has not been provided about depth near the boundaries (e.g., if xh(1) > 0), the topography is filled out by assuming that depth is constant over the gap. The file also tests to make sure the depth profile is consistent with assumptions.

bwavescfinch.m

This file (described above, and shown as a sample) allows the user to change aspects of the input array without having to create a new file from scratch. The user is first asked what category is to be changed, and then more specific questions are asked.

bwavescpl.m

This file plots out the modal structure each time a new modal frequency is calculated. For reference, the depth profile is also plotted.

bwavescr.m

This file carries out two functions. First, it takes input information about the friction coefficient r and interpolates it onto the grid. If information is not provided for the whole grid, existing values are extended out to the boundaries. For example, if nr = 1, xrr = 0 and rrr = 0.01, the result will be to set r = 0.01 cm/sec over the whole domain. Secondly, this file plots out the profiles of friction coefficient r mean alongshore velocity v_0 and depth h.

bwavescsetup.m

This file (discussed above) is used to create an input array to drive wave calculations. It functions by asking a sequence of questions. It plots out the profiles of friction coefficient r, mean alongshore velocity v_0 and depth h. It also executes a few consistency checks to make sure nothing foolish gets done. The user will only see the results of these checks if there is a problem.

bwavescsol.m

This file creates a matrix equation representing equation (4) and applies the boundary conditions set by the user. Given a value for alongshore wavenumber and a guess at frequency, it then solves for pressure, and this array is returned to bwavesccal.m to evaluate for resonance. The arbitrary forcing is a spike at about the middle of the model grid.

bwavescvel.m

This file takes the input information about mean velocity and interpolates it onto the model grid. It also calculates the first and second derivatives of this quantity. It checks whether the necessary condition for barotropic instability is met, i.e., whether

$$Q_x = \left(\frac{f + v_{0x}}{h}\right)_x \tag{12}$$

changes sign. If there is a sign change, a warning is given and a relevant plot is presented.

bwavesc2p.m

This file converts an input array suitable for use with bwavesc.m (complex frequency) into one suitable to use with bwavesp.m (real frequency).