

1 Here we describe the model with equations. Parameter values and nomenclature are in Table S1, S2,
 2 respectively. The model code is freely available in Zenodo/GitHub at <https://zenodo.org/record/3265448>
 3 (doi: 10.5281/zenodo.3265448).

4

5 **Computation of C, N and O₂ metabolism**

6 **Light period.** The rate of photosynthesis depends on the abundance of chlorophyll ([eq. 5] in the main
 7 text), which is dynamic and related to Fe allocation ([eq. 12] in the main text). We also compute an
 8 “ideal” abundance of Chlorophyll; that would support sufficient C fixation to match the demands of N
 9 limited biosynthesis, based on the stored N. The biomass production rate which determines the ideal
 10 chlorophyll concentration (mol C cell⁻¹ s⁻¹) is thus equal to the N limited biomass production rate,
 11 described as follows by using Michaelis-Menten form:

$$\lambda^{Chl-ideal} = \lambda^{max} \frac{N_S}{N_S + K_{N_S} Y_{bio-all}^{N:C} / Y_{bio}^{N:C}} \quad [\text{eq. S1}]$$

12 where λ^{max} (mol C cell⁻¹ s⁻¹) is the maximum biomass production rate, N (mol N cell⁻¹) is the N storage,
 13 K_{N_S} (mol N cell⁻¹) is the half-saturation constant of N storage for biomass production, and $Y_{bio-all}^{N:C}$ and
 14 $Y_{bio}^{N:C}$ (mol N mol C⁻¹) are the N:C ratio of biomass with and without the nutrient storage, respectively.

15 Photosynthesis must meet the demands for storage to support night time activity. We compute
 16 the rate of C storage production based on the current magnitude of C storage:

$$P_{C_S}^{Chl-ideal} = (C_S^{max} - C_S) R_{C_S} \quad [\text{eq. S2}]$$

17 where $P_{C_S}^{Chl-ideal}$ (mol C cell⁻¹ s⁻¹) is the ideal C storage production rate, C_S^{max} (mol C cell⁻¹) is the
 18 maximum C storage, C_S (mol C cell⁻¹) is the C storage, and R_{C_S} (s⁻¹) is the rate constant for the C storage
 19 production. If the C store is not at maximum capacity, photosynthesis is required to provide a supply at a
 20 rate sufficient to keep up with the prescribed storage rate, R_{C_S} .

21 By plugging [eq. S1] and [eq. S2] into [eq. 5] ($\lambda^{Chl-ideal}$ into λ and $P_{CS}^{Chl-ideal}$ into dC_S/dt) and
 22 assuming $Exc = N_{2fix} = P_{CO2}^{N2fix} = 0$ and solving the equation for Chl , we obtain the “ideal” chlorophyll
 23 concentration Chl^{ideal} (mol C cell⁻¹):

$$Chl^{ideal} = \frac{\lambda^{Chl-ideal}(1 + E) + P_I^{Chl-ideal}}{P_I} \quad [\text{eq. S3}]$$

24 Here, E (dimensionless) is the ratio of the carbohydrate production rate to the biomass production rate
 25 for providing energy for biosynthesis (1–3), and P_I (s⁻¹) is the photosynthesis rate per chlorophyll. For
 26 obtaining P_I , we use a “target theory” based equation used in many other studies (4–7) with the addition
 27 of a sigmoidal photo-inhibition term $\Omega(I)$:

$$P_I = P_I^{max} \left(1 - e^{-A_I I} - \Omega(I) \right) \quad [\text{eq. S4}]$$

$$\Omega(I) = \frac{B_I P_I^{max}}{1 + \exp(-C_I I + D_I)} \quad [\text{eq. S5}]$$

28 where P_I^{max} (s⁻¹) is the maximum photosynthesis rate per chlorophyll, and A_I , B_I , C_I and D_I are constant
 29 parameters.

30 Since the chlorophyll concentration is typically not at this ideal state, we parameterize the
 31 synthesis terms accordingly. When $Chl^{ideal} > Chl$ where Chl is the actual chlorophyll amount (mol C
 32 cell⁻¹), the growth is limited by the amount of chlorophyll and C storage and biomass synthesis are
 33 accordingly reduced from their ideal rates by a factor M_{Chl} (dimensionless):

$$M_{Chl} = \frac{Chl}{Chl^{ideal}}. \quad [\text{eq. S6}]$$

34 Alternatively, when $Chl^{ideal} \leq Chl$,

$$M_{Chl} = 1. \quad [\text{eq. S7}]$$

35 Hence biomass production rate λ (mol C cell⁻¹ s⁻¹) is evaluated as follows:

$$\lambda = \lambda^{Chl-ideal} M_{Chl}. \quad [\text{eq. S8}]$$

36 When $Chl_{ideal} < Chl$, the extra C fixed that exceeds the potential rate of C storage production is assumed
 37 to be excreted to the environment at a rate given by:

$$Exc = P_I Chl - \lambda(1 + E) - (C_S^{max} - C_S) R_{CS} \quad [\text{eq. S9}]$$

38 but otherwise, $Exc = 0$. Once biomass production rate λ is computed, we can energetically and
 39 stoichiometrically relate λ to the biomass production balanced respiration rate R_{Bio} (mol O₂ cell⁻¹ s⁻¹) (1–
 40 3), and to consumption of the N storage dN_S/dt (mol N cell⁻¹ s⁻¹) respectively:

$$R_{Bio} = \lambda Y_{synth}^{O_2:bio} \quad [\text{eq. S10}]$$

41 and from [eq. 7],

$$\frac{dN_S}{dt} = -\lambda Y_{bio-all}^{N:C} \quad [\text{eq. S11}]$$

42 where $Y_{synth}^{O_2:bio}$ (mol O₂ mol C⁻¹) is the ratio of O₂ production and biomass synthesis, and $Y_{bio}^{N:C}$ (mol N
 43 mol C⁻¹) is the N:C ratio in total biomass. Also, we can stoichiometrically relate C fixation rate to the
 44 photosynthetic O₂ production rate P_{O_2} (mol O₂ cell⁻¹ s⁻¹):

$$P_{O_2} = P_I Chl Y_{photo}^{O_2:CH} \quad [\text{eq. S12}]$$

45 where $Y_{photo}^{O_2:CH}$ (mol O₂ mol C⁻¹) is the O₂/C ratio in the photosynthesis. Using these rates (P_I , λ , P_{Cstore} ,
 46 Exc) in [eq. 5] ~ [eq. 7] and step forward these equations, we obtain values for the next time step.

47 During the light period, N_{2fix} , $P_{CO_2}^{N_2fix}$ and $P_{CO_2}^{RP}$ are assumed zero.

48 We assume that the intracellular O₂ rapidly equilibrates (as is reasonable for small cells). From
 49 the respiration rate R_{O_2} and the O₂ production rate P_{O_2} and [eq. 8] with a pseudo-steady state
 50 assumption, V_{O_2} (mol O₂ cell⁻¹ s⁻¹) is calculated as follows:

$$V_{O_2} = R_{O_2} - P_{O_2}. \quad [\text{eq. S13}]$$

51 This pseudo-steady state assumption is valid in this study since the amount of intracellular O₂ is
 52 significantly smaller than that of cellular C despite the magnitude of fluxes affecting these pools is
 53 similar. With [eq. 2], by assuming a spherical shape of the cells, we obtain the intracellular O₂
 54 concentration $[O_2^{cell}]$ as follows:

$$[O_2^{cell}] = [O_2] - \frac{V_{O_2}}{4\pi r \kappa_{O_2}}. \quad [\text{eq. S14}]$$

55
 56 **Dark period.** During the dark period, the rates of N₂ fixation and respiration are influenced by the
 57 amount of stored C, the electron and energy source for both N₂ fixation and respiration. The potential C
 58 storage decomposition rate (mol C cell⁻¹ s⁻¹) is based on the availability of the C storage and represented
 59 by the following Michaelis-Menten form:

$$D_{C_S}^{potential} = -D_{C_S}^{max} \frac{C_S}{C_S + K_{C_S}^{dec}} \quad [\text{eq. S15}]$$

60 where $D_{C_S}^{max}$ (mol C cell⁻¹ s⁻¹) is the maximum rate of C storage decomposition, and $K_{C_S}^{dec}$ (mol C cell⁻¹)
 61 is the half saturation constant of the C storage for $D_{C_S}^{potential}$. This $D_{C_S}^{potential}$ imposes the limit of N₂
 62 fixation and respiration (see below). In addition, respiration is limited by the potential O₂ uptake rate
 63 (mol O₂ cell⁻¹ s⁻¹) and the enzymatically constrained respiratory potential. The potential O₂ uptake is
 64 obtained when the intracellular O₂ concentration is zero:

$$V_{O_2}^{potential} = 4\pi r \kappa_{O_2} [O_2] \quad [\text{eq. S16}]$$

65 The enzymatically constrained respiratory potential (mol O₂ cell⁻¹ s⁻¹) is assumed to increase with time
66 after the initiation of the dark period and is represented as follows:

$$R_{enzyme}^{potential} = (t_{dark})^{PI} C_{O_2}^{potential} \quad [\text{eq. S17}]$$

67 where t_{dark} (s) is the time passed since the initiation of the dark period, PI (dimensionless) and $C_{O_2}^{potential}$
68 (unit depends on PI) are the power factor and the respiratory coefficient for $R_{enzyme}^{potential}$. We have used
69 this power factor in order to express the observed non-linear time dependence of respiration during the
70 dark period (8). If $V_{O_2}^{potential} > R_{enzyme}^{potential}$, the potential respiration rate $R_{O_2}^{potential}$ (mol O₂ cell⁻¹ s⁻¹) is
71 equal to $R_{enzyme}^{potential}$, whereas $V_{O_2}^{potential} \leq R_{enzyme}^{potential}$, it is equal to $V_{O_2}^{potential}$ (mol O₂ cell⁻¹ s⁻¹).

72 Once we obtain the potential respiration rate, the next step is to compute the potential N₂ fixation
73 rate. The potential N₂ fixation rate (mol N cell⁻¹ s⁻¹) is constrained by the availability of nitrogenase
74 enzyme, as well as intracellular O₂:

$$N_{2fix}^{potential} = Fe_N C_{Fe}^{N2fix} \frac{[O_2^{cell}]_{cri} - [O_2^{cell}]}{[O_2^{cell}]_{cri}} \quad [\text{eq. S18}]$$

75 where Fe_N (mol Fe cell⁻¹) is the amount of Fe in nitrogenase (see below), C_{Fe}^{N2fix} (mol N mol Fe⁻¹ s⁻¹) is
76 the N₂-fixing capacity per nitrogenase Fe, and $[O_2^{cell}]_{cri}$ (mol O₂ m⁻³) is a critical O₂ concentration
77 above which N₂ fixation cannot occur (thus, when $[O_2^{cell}] > [O_2^{cell}]_{cri}$, $N_{2fix}^{potential} = 0$).

78 We now can ask whether the cell is N or C limited: i.e. whether there is enough C stored to meet
79 the demands of potential N₂ fixation. To do that, we first define the ideal C storage decomposition rate
80 D_{Cs}^{ideal} (mol C cell⁻¹ s⁻¹) based on those N₂ fixation and respiration rates:

$$D_{Cs}^{ideal} = \frac{N_{2fix}^{potential} Y_{N2fix}^{C:N}}{R_{H2}} + \frac{R_{O2}^{potential}}{Y_{non-synth}^{O2:CH}} \quad [\text{eq. S19}]$$

81 where $Y_{N2fix}^{C:N}$ (mol C mol N⁻¹) is the ratio of carbohydrate consumption (for electron donation for N₂
 82 fixation) to N₂ fixation, R_{H2} (dimensionless) is a coefficient for electron recycling from hydrogen
 83 molecules, and $Y_{non-synth}^{O2:CH}$ (mol O₂ mol C⁻¹) is the ratio of O₂ consumption and carbohydrate
 84 consumption in non-synthesis respiration (2, 3). If $D_{Cs}^{ideal} < D_{Cs}^{potential}$, the decomposition rate of C
 85 storage D_{Cs} is controlled by D_{Cs}^{ideal} and $N_{2fix} = N_{2fix}^{potential}$. If $D_{Cs}^{ideal} > D_{Cs}^{potential}$, $D_{Cs}^{potential}$ controls the
 86 rate D_{Cs} , and the N₂ fixation rate is down regulated by $D_{Cs}^{potential}$:

$$N_{2fix} = \left(\frac{D_{Cs}^{potential} Y_{non-synth}^{O2:CH} - R_{O2}^{potential}}{Y_{N2fix}^{C:N} Y_{non-synth}^{O2:CH}} \right) R_{H2} \quad [\text{eq. S20}]$$

87 If $R_{O2}^{potential} > D_{Cs}^{potential}$, there is no N₂ fixation and the respiration rate equals $D_{Cs}^{potential}$.

88 Finally, we test if there is enough respiration for producing energy for N₂ fixation R_{N2fix} (mol
 89 O₂ cell⁻¹ s⁻¹). The respiratory requirement is proportional to N₂ fixation:

$$R_{N2fix} = N_{2fix} Y_{N2fix}^{O2:N} \quad [\text{eq. S21}]$$

90 where $Y_{N2fix}^{O2:N}$ is a conversion factor (mol O₂ mol N⁻¹). When $R_{O2}^{potential} > R_{N2fix}$, $R_{O2} = R_{O2}^{potential}$, and
 91 otherwise, we impose $R_{O2} = R_{N2fix}$ as long as C store is available, which we have confirmed in this study.
 92 Under the pseudo-steady state of cellular O₂ and zero photosynthesis, from [eq. 8]:

$$V_{O2} = R_{O2} \quad [\text{eq. S22}]$$

93 And from [eq. S14], intracellular O₂ concentration is obtained. CO₂ production rate is computed as
 94 follows:

$$P_{CO_2}^{N_2fix} + P_{CO_2}^{RP} = \frac{R_{O_2}}{Y_{non-synth}^{O_2:CH}} \quad [\text{eq. S23}]$$

95 Equating D_{C_s} to $-dC_s/dt$ and plugging these obtained values (N_{2fix} , R_{O_2} , $P_{CO_2}^{N_2fix} + P_{CO_2}^{RP}$) into [eq. 5] and
 96 [eq. 7], and applying a finite-difference method, we can compute the values for each time step. Here P_I ,
 97 λ , Exc are assumed zero. When we turn off respiratory protection (thus $P_{CO_2}^{RP} = 0$), $P_{CO_2}^{N_2fix} =$
 98 $R_{N_2fix}/Y_{non-synth}^{O_2:CH}$.

99

100 **Computation of Fe metabolism**

101 **Light period.** We have simulated the translocation of Fe among different cellular components as
 102 indicated in a proteomic study (9); photosystem Fe, buffer Fe, and nitrogenase Fe. Computing Fe flux
 103 and Fe content in each component is essential in predicting photosynthesis and N_2 fixation. The rate of
 104 Fe translocation is influenced by the amount of Fe in each location.

105 When $Chl^{ideal} > Chl$ there is translocation of Fe from buffer to the photosystem, whose flux F_B^P
 106 (mol Fe cell⁻¹ s⁻¹) is represented as follows:

$$F_B^P = (F_{e_p}^{ideal} - F_{e_p}) R_B^P \frac{F_{e_B}}{F_{e_B} + K_{Fe}} \quad [\text{eq. S24}]$$

107 where $F_{e_p}^{ideal}$ (mol Fe cell⁻¹) is the ideal Fe mass in the photosystem ($=Chl^{ideal} Y_{photo}^{Chl:Fe}$), F_{e_p} (mol Fe
 108 cell⁻¹) is the actual Fe mass in the photosystem, R_B^P (s⁻¹) is the photosystem production efficiency, F_{e_B}
 109 (mol Fe cell⁻¹) is the Fe mass in the buffer, and K_{Fe} (mol Fe cell⁻¹) is the half saturation constant of Fe
 110 metabolisms. We assume that the amount of the photosystem Fe per chlorophyll is constant so that
 111 $F_{e_p}^{ideal}$ (mol Fe cell⁻¹) is obtained from Chl^{ideal} calculated in equation [eq. S3]. R_B^P is a function of time
 112 in the light period:

$$R_B^P = t_{light} C_B^P \quad [\text{eq. S25}]$$

113 where C_B^P (s^{-2}) is a constant value. This time function is to represent the observed gradual increases in
 114 Fe^P (9).

115 When $Chl_{ideal} \leq Chl$, there is a flux of Fe from the photosystem to the buffer F_P^B (mol Fe cell^{-1}
 116 s^{-1}), represented as follows:

$$F_P^B = (Fe_P - Fe_P^{ideal}) R_P^B \frac{Fe_P}{Fe_P + K_{Fe}} \quad [\text{eq. S26}]$$

117 where R_P^B (s^{-1}) is the rate constant for F_P^B . In addition to the Fe transfer between the photosystem and the
 118 buffer, there can be a flux of Fe from nitrogenase to the buffer F_N^B ($\text{mol Fe cell}^{-1} \text{ s}^{-1}$) if there is still some
 119 remaining Fe in nitrogenase (this flux tends to occur during the early light period):

$$F_N^B = Fe_N R_N^B \frac{Fe_N}{Fe_N + K_{Fe}} \quad [\text{eq. S27}]$$

120 where Fe_N (mol Fe cell^{-1}) is the amount of Fe in nitrogenase, R_N^B (s^{-1}) is the rate constant for F_N^B , and
 121 K_{Fe}^N (mol Fe cell^{-1}) is the nitrogenase Fe half saturation constant for the nitrogenase-buffer Fe transfer.
 122 Finally, F_B^N is assumed to be zero. With these fluxes, we predict the mass of Fe in each cell component.
 123 Fe_P , Fe_B , Fe_N and Chl ($= Fe_P Y_{photo}^{Chl:Fe}$) are computed based on the balances of the above Fe fluxes for
 124 each time step [eq. 9]~[eq. 11].

125

126 **Dark period.** During the early dark period, Fe atoms move from the photosystem to the buffer, and the
 127 buffer to nitrogenase. The flux of Fe from the photosystem to the buffer ($\text{mol Fe cell}^{-1} \text{ s}^{-1}$) is based on
 128 the following equation:

$$F_P^B = (F_{e_P} - F_{e_P}^{min}) R_P^B \frac{F_{e_P}}{F_{e_P} + K_{Fe}} \quad [\text{eq. S28}]$$

129 where $F_{e_P}^{min}$ (mol Fe cell⁻¹) is the minimum cellular Fe in the photosystem and other variables are the
 130 same as in equation [eq. S26]. The flux of Fe from the buffer to nitrogenase (mol Fe cell⁻¹ s⁻¹) is
 131 represented as follows in a similar form:

$$F_B^N = (F_{e_N}^{ideal} - F_{e_N}) R_B^N \frac{F_{e_B}}{F_{e_B} + K_{Fe}} \quad [\text{eq. S29}]$$

132 where $F_{e_N}^{ideal}$ (mol Fe cell⁻¹) is the ideal Fe mass in the nitrogenase that fulfills the potential N₂ fixation
 133 rate based on C storage availability, R_B^N (s⁻¹) is the rate constant for nitrogenase production. R_B^N is
 134 influenced by the O₂ concentration, dark period time t_{dark} (s), and C storage:

$$R_B^N = C_B^N (t_{dark})^{P2} ([O_2^{cell}]_{crit}^{nitroge} - [O_2^{cell}]) \left(\frac{C_S}{C_S + K_{C_S}^{nitroge}} \right)^{P3} \quad [\text{eq. S30}]$$

135 where C_B^N (unit varies depending on $P2$ and $P3$) is a constant term for R_B^N , $P2$ and $P3$ (dimensionless)
 136 are power factors, necessary to accurately represent repeatedly observed non-linear time dependence of
 137 N₂ fixation (8–10), $[O_2^{cell}]_{crit}^{nitroge}$ (mol O₂ m⁻³) is a critical O₂ concentration above which nitrogenase
 138 can be synthesized (thus, when $[O_2^{cell}] > [O_2^{cell}]_{crit}^{nitroge}$, $R_B^N = 0$), $K_{C_S}^{nitroge}$ (mol C cell⁻¹) is a half
 139 saturation constant of C_S for nitrogenase production.

140 $F_{e_N}^{ideal}$ (mol Fe cell⁻¹) is computed based on the ideal N₂ fixation rate. First, we compute the
 141 ideal N₂ fixation rate based on the concentration of the N storage and C storage (mol N cell⁻¹ s⁻¹):

$$N_{2fix}^{ideal-store} = \frac{N_S^{max} - N_S}{N_S^{max}} N_{2fix}^{max} \frac{C_S}{C_S + K_{C_S}^{N2fix}} \quad [\text{eq. S31}]$$

142 where N_S^{max} (mol N cell⁻¹) is the N storage capacity, N_{2fix}^{max} (mol N cell⁻¹ s⁻¹) is the maximum possible
 143 N₂ fixation rate, and K_{CS}^{N2fix} (mol C cell⁻¹) is the Half saturation constant of carbohydrate storage for N₂
 144 fixation. Then, we compare this value to another ideal N₂ fixation rate (mol N cell⁻¹), which, this time, is
 145 based on the balance between maximum C storage decomposition and maximum respiration:

$$N_{2fix}^{ideal-balance} = \left(\frac{D_{CS}^{max}}{Y_{N2fix}^{CH:N}} - \frac{V_{O2}^{potential}}{Y_{non-synth}^{O2:CH} Y_{N2fix}^{CH:N}} \right) R_{H2} \quad [\text{eq. S32}]$$

146 We then take the lower value between $N_{2fix}^{ideal-store}$ and $N_{2fix}^{ideal-balance}$ for the ideal N₂ fixation rate
 147 N_{2fix}^{ideal} (mol N cell⁻¹ s⁻¹), based on which, we compute the ideal Fe amount in nitrogenase:

$$Fe_N^{ideal} = \frac{N_{2fix}^{ideal}}{C_{Fe}^{N2fix}} \quad [\text{eq. S33}]$$

148 Based on the obtained fluxes through the above computations, Fe_P , Fe_B and Fe_N (mol Fe cell⁻¹),
 149 (cellular Fe in photosystems, Fe-buffer, and nitrogenase, respectively) are computed for each time step.
 150 F_N^B is assumed to be zero, but the flux of Fe from nitrogenase to the buffer can occur with a negative
 151 value of F_B^N . In each time step, we compute these fluxes and use them to predict the mass of Fe (and *Chl*
 152 $= Fe_P Y_{photo}^{Chl:Fe}$) for each time step through finite-difference method with [eq. 9]~[eq. 11].

153

154 **Late dark period.** During the late dark period, experimental data indicate that the cells start
 155 decomposing nitrogenase to retrieve Fe in the buffer, preparing for photosynthesis (9). Also, during this
 156 period, the respiration rate gradually decreases as the N₂ fixation rate decreases despite the available C
 157 storage (8). In this model, we imposed linearly decreasing F_N^{ideal} and $R_{O2}^{potential}$ with time, reaching
 158 $F_N^{ideal} = 0$ and half of the $R_{O2}^{potential}$ at the initiation of the next light period.

159

160 **Computing cellular C and N.** We consider total cellular C (Q_C) and N (Q_N) quotas as a sum of two
161 pools: inflexible biomass (C_B and N_B) and flexible storage:

$$Q_C = C_B + C_S \quad [\text{eq. S34}]$$

$$Q_N = N_B + N_S. \quad [\text{eq. S35}]$$

162 Based on these values, the N:C ratio of the total biomass is obtained:

$$Y_{bio-all}^{N:C} = \frac{Q_N}{Q_C}. \quad [\text{eq. S36}]$$

163

164 **Impact of cell size.**

165 All the metabolic rates (except for N_{2fix}^{max}), maximum storage capacity and baseline biomass
166 concentrations are assumed proportional to the cell size.

167

168 **Modeling T dependence.**

169 To evaluate the effect of temperature (T), we multiplied $f_T(T)$ [eq. 3] to P_I , $\lambda^{Chl-ideal}$, $P_{CS}^{Chl-ideal}$,
170 $R_{enzyme}^{potential}$, and/or $N_{2fix}^{potential}$, depending on the simulation. To make all the metabolisms
171 (photosynthesis, respiration and N₂ fixation) T dependent, we multiply $f_T(T)$ to all of these. To make
172 only one of these metabolisms T dependent, the factor is multiplied only to P_I , $\lambda^{Chl-ideal}$ and $P_{CS}^{Chl-ideal}$
173 for photosynthesis, to $R_{enzyme}^{potential}$ for N₂ fixation or to $N_{2fix}^{potential}$ for respiration, respectively.

174

175 **Parameterization**

176 **Tunable parameters.** In order to represent the previous experimental data (8, 9) as well as the new data
177 from our study, we have visually estimated 34 parameters (Table S1). In addition, we have set that the

178 late dark period starts 7 hours after the initiation of the dark period. To reduce the number of free
179 parameters, we have applied same values for $P2$ and $P3$, and K_{Cs}^{dec} and K_{Cs}^{N2fix} . For the cellular Fe quota,
180 we have used a constant value of 4.87×10^6 molecules per cell based on the maximum value estimated
181 in the proteomic study (9).

182

183 **Non-tunable parameters.** In order to constrain other parameters, we have referred to Inomura et al.,
184 (2017) (1) for the following parameters; $Y_{synth}^{O2:Bio}$, $Y_{non-synth}^{O2:CH}$, $Y_{N2fix}^{O2:N}$, and E (with energy transfer
185 efficiency of 0.41 (11) and biomass of $C_5H_7O_2N_a$), where $a = 5Y_{bio-all}^{N:C}$ (C:H:O as suggested (1)). We
186 assume that 100% electron from hydrogen produced from N_2 fixation, which leads to $R_{H2} = 4/3$ (8).
187 Also, based on the electron balance between N_2 fixation and glucose decomposition, $Y_{N2fix}^{C:N} = 1$ (2, 3). We
188 have set the reference temperature $T_{ref} = 28$ °C (301.15 K) following the reference experiments (8, 9) as
189 well as the laboratory experiments in this study.

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