Notes on the 1967

Summer Study Program

in

GEOPHYSICAL FLUID DYNAMICS

at

The WOODS HOLE OCEANOGRAPHIC INSTITUTION

Reference No. 67-54

Contents of the Volumes

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Editors' Preface

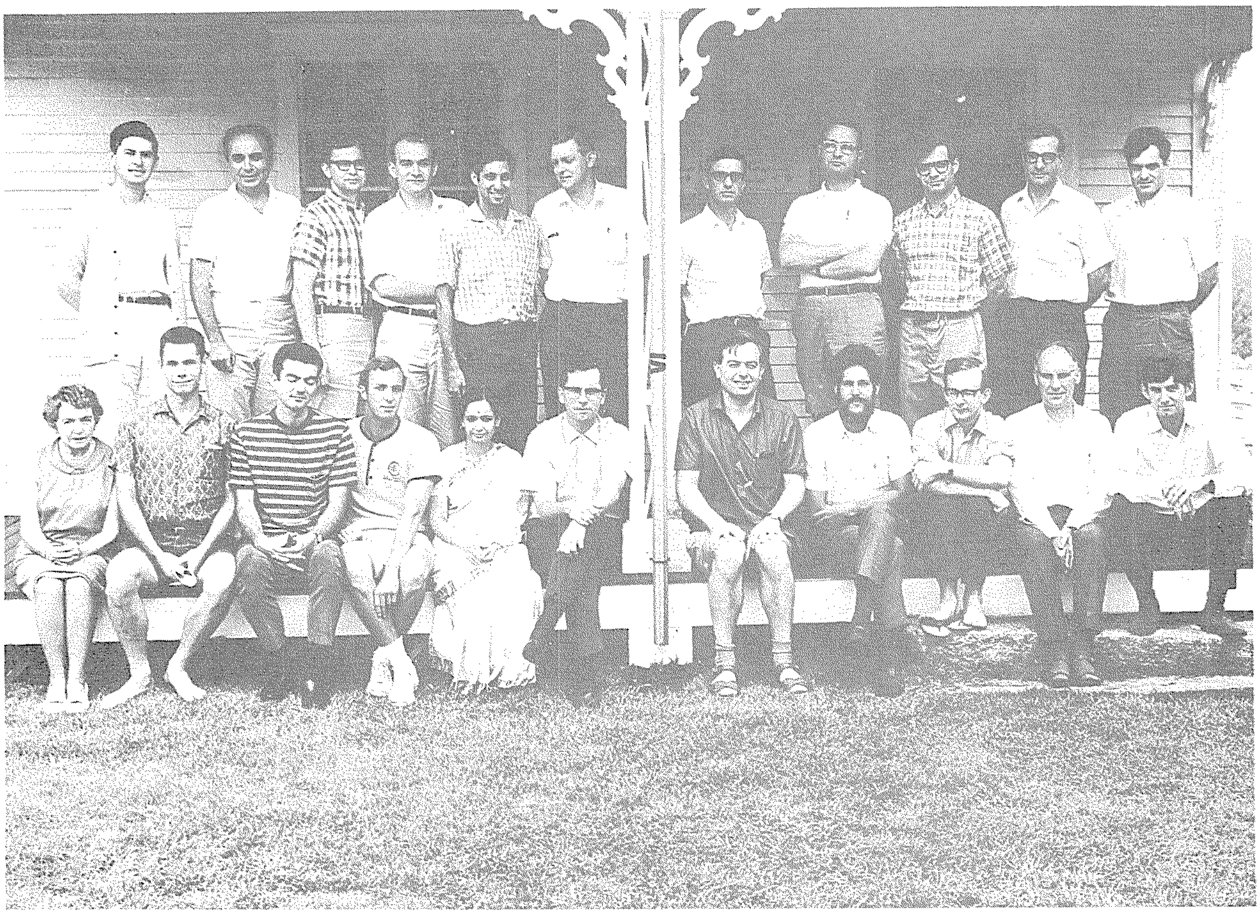
This volume contains the final reports by the student participants of their research activities of the summer as well as abstracts of seminars and discussions by the staff and by invited lecturers.

As in previous years the efforts reported by the students reflect varying degrees of originality and completeness. A few of the students were guided carefully; with others there was little contact between student and staff. Some of the students either posed or were given precisely formulated problems which could be essentially completed during the summer. Others accepted problems which were not well-posed and which involved a period of groping and searching for a tractable and reasonably finite problem with a significant goal. It is noteworthy that all six of the pre-doctoral participants started on research projects which they intended to develop into Ph.D. theses after they returned to their respective universities.

Time limitations did not allow the participants to rework the manuscripts and the present records must be interpreted as only interim reports.

We who took part in the program this summer are deeply indebted to the National Science Foundation for its continued support of the program and to the Woods Hole Oceanographic Institution for its support and encouragement and for the use of its facilities.

Mary C. Thayer
George Veronis



BACK ROW, LEFT TO RIGHT: Chorin, Onat, Buzyna, Denis, Blumsack, Howard, Malkus, Gilbert, Kraichnan, Veronis and Herring. FRONT ROW: Thayer, Thompson, Johnson, Thatcher, Gadgil, Busse, Roberts, Somerville, McIntyre, Hide and Childress.

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Formation of Layers in a Stably Stratified Fluid

Steven L. Blumsack

Introduction

The purpose of this report is to present a possible explanation for a phenomenon that has been observed for many years now. When a container is filled with a fluid, stably stratified (with respect to the gravitational field) with some slowly diffusing substance, such as salt, and is subsequently warmed uniformly at its surfaces, the formation of layers ensues. Each layer is nearly uniform in salinity, with rather thin interfaces between the layers. A possible mechanism for forming these layers will be discussed and an appropriate length scale will be found.

The method to be used will be as follows: the effects of certain parameters will be discussed and the experimental evidence will justify their omission from the mathematical model. This simplified model will be discussed both physically and mathematically, as an initial value problem, to determine the gross features of the basic flow. Since the mathematical flow will have no vertical structure, the experiment would then have to be explained by some instability; only an infinitesimal instability theory will be attempted, consequently, only the scale of the layers will be found, not their fine structure since this would require a more exact analysis.

Description of Experiment

An exact analysis of this problem would not only entail considerable labor, but the physical mechanism that is responsible for the formation of these layers might be obscured in its complexity. Therefore, an idealized model of the experiment will be formulated, hopefully containing the essential features of the relevant mechanism.

A standard experiment will now be defined. A 1000 ml graduated cylinder, six centimeters in diameter, was filled with (dyed) salty water at room temperature; the salinity increased nearly linearly downwards. Then the container was immersed in a hot-water bath, approximately 25°C warmer than room temperature. After about a minute, one could see the aforementioned layers, about one centimeter wide, in the final stages of formation.

These layers appeared as alternating bands of dark fluid and light fluid with the general trend being lighter near the top of the container. Although it was impossible to detect the flow during the formation of the layers, one could see small protrusions of dark fluid commencing at the wall and tilted down into the interior. As the layers became visible, these protrusions had become level.

The effects of the size of the container were checked first. Its height did not seem to be important, as the layers appeared to form almost simultaneously in the middle portion of the cylinder. It was noted that, in the time required to form the layers, the heat would have penetrated less than one centimeter into the fluid, implying that the diameter of the container should not be an important parameter, at least in the early stages of the formation of the layers.

It was noted that when the temperature difference across the glass was increased, the layers appeared to become wider and more irregular.

The discussion in the preceding paragraph suggests that an appropriate mathematical model would be a semi-infinite fluid, linearly stratified with salt, bounded on one side by an infinite plane which is heated uniformly beginning at a particular instant in time. Notice that no vertical structure is present in the model*, so that the vertical dependence will not be in the basic flow, but in those perturbations on the basic flow that grow with time. Since the perturbations need some source of energy in order to grow, the basic velocity, temperature, and salinity fields must be discussed before any stability analysis may be undertaken.

Basic Fields - Qualitative Discussion

As has been stated, at any instant in time the basic fields will be independent of the vertical coordinate, implying that there can be no horizontal flux of mass, consequently no velocity normal to the wall and no advection of either temperature or momentum.

When the wall is heated, the fluid nearest to the wall becomes warmer, thereby less dense, and would tend to rise, the motion being retarded by viscosity. As time progresses, the effects of the temperature

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*Although the salinity does have a vertical dependence, the dynamics only involves its gradient, which is assumed constant.

move slowly inward and the temperature of the fluid closest to the wall increases, but increases continually more slowly.

If the wall was not continually heated, but instead maintained at a constant temperature, the fluid would rise until it became neutrally buoyant and then tend to remain at that level. This is not quite the story for the case at hand, but since the temperature at the side wall is increasing so slowly, it does not seem too unreasonable to suspect that most of the fluid will maintain a neutrally buoyant state. In addition, away from the wall, the fluid is being warmed very slowly, thereby introducing very slow motions.

Although the basic velocity, temperature, and salinity fields are time dependent, and spatially rather complicated, the following crude approximations of these fields will be made now and the first two justified later.

(i) Lines of constant density are horizontal.

(ii) The temperature and salinity fields are independent of time.

(iii) The fluid is infinite in extent (no wall).

In order to analyze the stability problem, the lines of constant salinity (called isohalines) will be assumed to be straight; this will force the temperature gradient to be uniform in order to satisfy (i). Hopefully, this assumption is not critical to the physics, being needed only to simplify the mathematics.

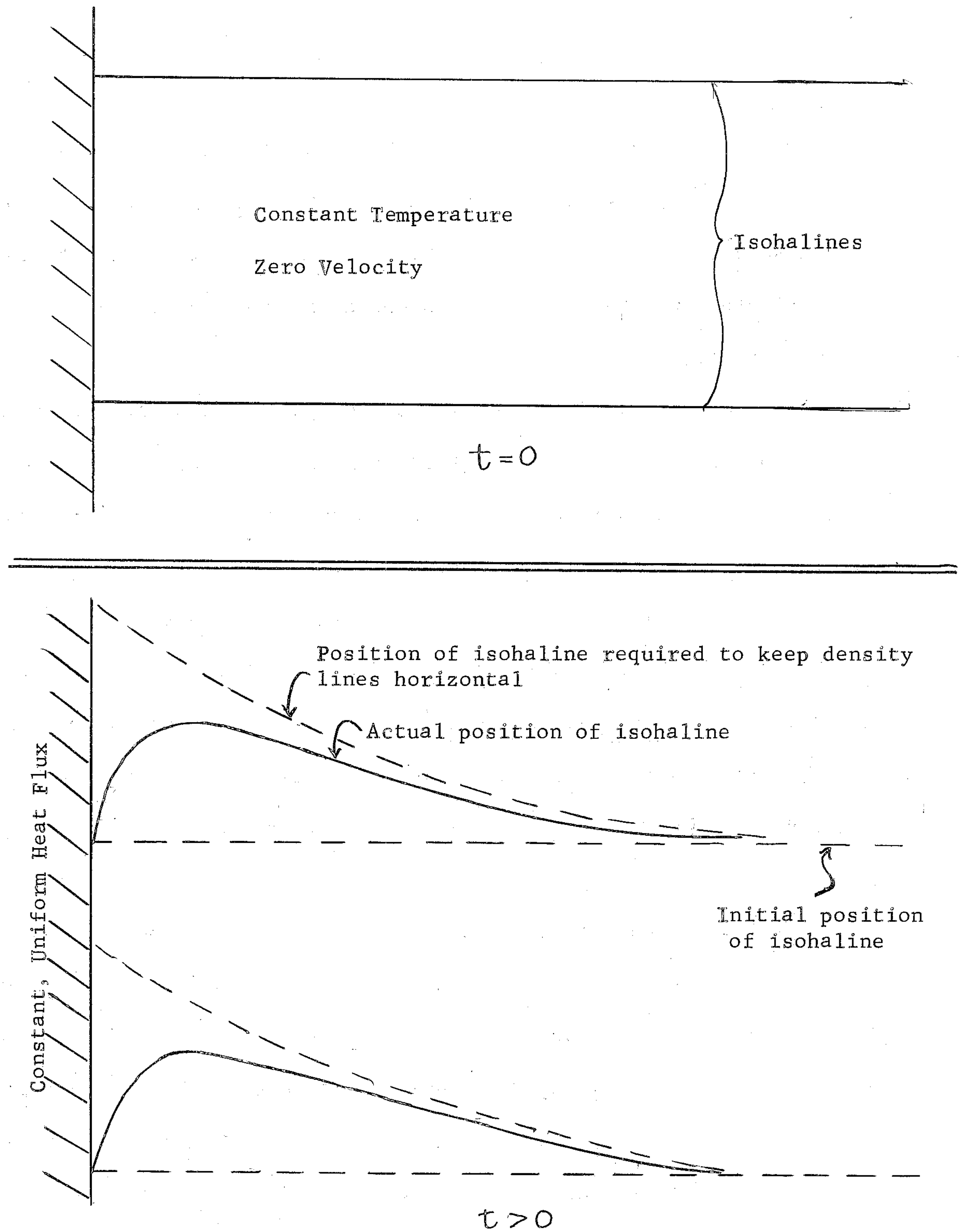

Figure 1. Time Dependence of Basic Flow

Release of Energy

An instability is dependent upon a source of energy and a mechanism for releasing this energy. For the model just described, there is no obvious source of energy since the kinetic energy of the basic field has been neglected and the lines of constant density are horizontal and stably oriented.

The density field is composed of two parts, temperature, which may be conducted away, and salinity, which tends to remain with the fluid particles; therefore, if the contribution of the density gradient due to temperature differences is negated, the remaining portion of the density surfaces will be tilted, providing some available potential energy.

More precisely, assume that the fluid is a perfect conductor of heat and is inviscid; also make the usual Boussinesq assumption on the conservation of mass equation so that the velocity field is non-divergent. The former two assumptions will be relaxed but are convenient to keep the physics in its most simple form at first.

The infinitesimal perturbation hypothesis allows one to look at the dynamics of a single plane wave since interactions between different plane waves occur only for finite amplitude disturbances. Let the wave vector of the plane wave be $\vec{k} = (\ell', m')$; the incompressibility condition says that only transverse waves exist, so that the velocity is perpendicular to the wave-vector, i.e., parallel to lines of constant phase. The dashed lines in Figures 2a and 2b are the nodes (where the perturbation velocity vanishes), and the tilted solid lines are isohalines.

In Figure 2a, both sets of lines are tilted in the same sense, but