ESTIMATES OF CRUSTAL TRANSMISSION LOSSES USING MLM ARRAY PROCESSING

by

EDWARD KURT SCHEER

S.B., Massachusetts Institute of Technology
(1972)

S.B., Massachusetts Institute of Technology
(1972)

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

OCEAN ENGINEER

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

and the

WOODS HOLE OCEANOGRAPHIC INSTITUTION

MAY 1982

Signature of Author...... ....
Department of Ocean Engineering,
May 1982

Certified by........... ....
Thesis Supervisor

Accepted by............. ....
Chairman, Joint Committee on Ocean Engineering,
WHOI/MIT Joint Program

ACKNOWLEDGMENTS

Of all the persons who have helped me during the past few years, I first thank Cathy for putting up with me in the most trying times, and for her love and support. Then I must thank Heidi for giving me the final push to get everything done. Art Baggeroer did some pushing too, and I am indebted to him for his valuable advice and guidance.

I am grateful to the MIT/WHOI Joint Program office for providing support for graphics and reproduction costs.

Thanks to Ken Prada and Dave Gever for answering many "stupid questions", and for letting someone like me get near their computer.

To my parents: Vielen Dank!!

I am especially grateful to Betsey Pratt for her figures, and to Gretchen McManamin for her Wang.

For helping maintain my sanity, I must thank Norbert's brother, Bill Weaner, Fifi Larue, Bill Cox, D.I.G., the staff and management of the Copley Plaza Hotel, Lulu, Taffy, Scummy, and the boys in the band.

This work was supported under ONR contract N-00014-75-C-0852.

CHAPTER I

INTRODUCTION

Background

Seismic refraction experiments have been used extensively in the past thirty five years in investigations of the structure of the oceanic crust. The longer range of the refraction or wide angle reflection technique, on the order of tens of kilometers, permits a deeper and wider area of examination, although with less resolution, than the spatially limited seismic reflection experiment. Observations of arrivals from the Mohorovicic discontinuity, at an average depth of seven kilometers below the sea floor, are routinely made.

The major focus in interpreting refraction data has been the analysis of travel time/range data and the "inversion" of this data for the purpose of determining a velocity versus depth profile of the crust. The most frequent application of this procedure is the geophysicist's use of velocities for postulating geologic structures and rock types below the sediment (Christensen & Salisbury, 1975). Another area using refraction data, less widely seen, falls into the ocean acoustician's domain. In studying the behaviour of sound in the ocean, the sea floor is often modelled as a boundary with a half space below, and with some form of reflection characteristic and/or loss mechanism. If acoustic energy, upon encountering the bottom, was either reflected or transmitted directly, this would be appropriate, and the determination of reflection and transmission coefficients for the sea-sediment interface would probably be sufficient. However, sound energy does penetrate beneath the sea floor and is both reflected and refracted back to the water. In an active acoustical experiment, especially at longer ranges, a significant amount of the received energy may come from waves that have interacted with the earth's crust and have been reinjected into the water. Since these arrivals can be detected in the ocean, their study is of concern for the acoustician.

The role of bottom interaction, especially at low frequencies, is now an area of intense research activity in modelling acoustic propagation. In particular, in the language of the sonar engineer, the TL, or transmission loss, of this energy is of major importance for i) predicting the character of the sound field at a receiver in future experiments, ii) for comparing crustal loss with the better known TL of paths remaining primarily in the water layer, and iii) expanding the role of arrival amplitudes in inversion theory. Just as there may be a number of possible paths in the sea between a source and receiver, each with a different loss characteristic, trajectories in the crust are variegated and exhibit different TL behaviors. It is important to be able to differentiate the energy partitioned among the different paths, and to determine which paths are most important.

Resolving the locus of a particular acoustic path is intimately tied to the problem of determining the velocity structure of a medium. To the limits of the geometrical optics approximation of acoustic behaviour, sometimes sorely pressed at low frequencies, a completely detailed knowledge of sound speed variations, both laterally and with depth, plus known source characteristics and attenuation losses in the medium, enables one in principle to predict signals observed at a receiver. For an ocean acoustician, the requirement of environmental knowledge of the sound speed profiles, both in water and crust, needed to predict the amplitude and timing of data, is clearly very burdensome. In the past twenty five years, however, models of the oceanic crust have been formulated which are statistically consistent over much of the oceans. These models divide the crust into three or more horizontal layers with certain average thicknesses and velocities (Raitt, 1963). At least within the confines of these models, if a typical transmission loss were known for each of these layers, an acoustician can make predictions of the expected strength and timing of crustal arrivals at other stations. Most of this environmental information has been obtained from refraction and/or wide angle reflection data, usually via travel time analysis. Little has been done in developing models accounting for amplitude dependence.

Arrays for Refraction Experiments

The standard technique in ocean refraction experiments has basically involved one ship and one or more receivers (sonobuoy or OBS), with increasing range between shots. With a dense shot spacing and large enough total range, the use of event arrival times, especially the first ones, for the most prominent features in the data has been sufficient for obtaining a reasonably good understanding of the velocity structure of the crust. The crustal model referred to above was developed from averaging experiments of this type from many diverse areas. Lately, a multichannel hydrophone array has replaced the single receiver in some experiments, with the array sometimes being towed by a second ship (Stoffa, Buhl, 1979). In the latter technique, termed an expanding spread profile (ESP), the two ships start at a common point and steam in opposite directions. In this way, a common depth point is shared by all shots. With the use of a Raydist apparatus, accurate range information, which is a sensitive parameter in the inversion methods, is also available. As with all arrays, the SNR for the detection of coherent energy can be improved with appropriate processing. Moreover, estimates of the received energy for different horizontal phase velocities can be made which, under the condition of horizontal crustal layering, provides us with crustal velocity estimates using just one shot. However, for a single offset, complete information concerning crustal structure is not be obtained since the SNR for certain events is range dependent.

Since receiver arrays have the ability of generating phase velocity information on a shot by shot basis, the process of traveltime analysis used in inversion studies can be somewhat automated. The original procedures of generating a travel time versus range plot for a sequence of densely spaced shots and visually picking arrivals can be improved by using an array velocity analysis technique that can assign velocities to arrivals in each shot trace. An expanded use of data received from one shot would minimize interpretation errors caused by uncertainties in range and source level variations. Clearly, once a composite of a number of shot traces is developed with estimated phase velocities along the trace for each shot, the problem of selecting different arrival times for a particular velocity is eased, and the intercept times can be found for use in traveltime inversion techniques, eg. the tau-p method (Stoffa, Diebold, & Buhl, 1981). Array processing techniques are also important in discriminating distinct phenomena that occur in the multipath reverberation one encounters after the first refracted arrival, and effects of local inhomogeneities such as bathymetric variations in exploration and/or oceanographic experiments.

Velocity Analysis

A conventional way of doing array velocity analysis employs a statistic that estimates the amount of trace to trace coherence across the array, for a given assumed phase velocity. All realistic velocities are scanned, and the normalized statistic, a "semblance coefficient", indicates the relative amount of energy in the data, at each velocity (Sereda and Hajnal, 1976). Another method, used throughout in what follows here, employs a data adaptive spectral estimator. Several data adaptive techniques were originally developed in various areas, particularly large aperture teleseismic arrays and sonars. The Maximum Liklihood Method (Capon, 1969; Edelblute, 1967; Lacoss, 1971) was used at Woods Hole originally in the processing of reflection data (Leverette,1977), and eventually extended to seismic refraction work (Baggeroer and Falconer,1981). The technique conceptually designs a beamformer based on the input data (hence data adaptive). This beamformer minimizes output power with the constraint that energy from a specific direction is passed undistorted. We shall see that the structure of this beamformer can be used to define an algorithm that estimates what is known as the frequency-wavenumber function of the acoustic field for a certain spatial frequency associated with a specific direction. Insofar as the directions of the arrivals at the array are related to the crustal sound speed of the paths the energy has traversed, estimated directions lead to estimated velocities. In the horizontal layering situation, this relationship is quite simple and the velocities estimated are very accurate, especially at high SNR.

In stochastic process theory, the power spectral density function is a measure of the partitioning of energy in a process with respect to frequency. The corresponding function for a wide sense stationary random process in space and time is the frequency-wavenumber function. It is a measure of the mean square power per unit bandwidth in temporal frequency arriving from a unit steradian in spatial frequency or wavenumber, which is uniquely related to horizontal phase velocity. The estimated function indicates the amount of energy that has arrived at the array via a particular path.

The acoustic field generated by an explosion, however, cannot be modelled as a stationary process. With the transient nature of the field, only a small part of the data is used. This "windowed" data must then be treated as if it were part of an ongoing, time invariant process. The power estimated in the hypothetical process is an indication of the actual energy, needed for true amplitude measurement, in the windowed data segments that were employed. The concept of windowing data to track nonstationary phenomena is extensively used in signal processing, particularly speech analysis. This technique is often referred to as "short time, spectral estimation".

The MLM estimate is known to be biased (Capon, 1969). An analytic expression for this bias has yet to be developed for all possible situations, however. We an empirical technique that can be used to evaluate the bias for the particular data set and array configuration discussed below. Given a accurate estimate of the frequency/wavenumber function and the energy spectrum of a source, the transmission loss for a certain ray path can be determined.

The Rose Experiment

The MLM algorithm and our transmission loss calculation procedure will be applied to a data set obtained from a large scale acoustic/seismic program (ROSE) conducted off the western coast of Mexico in January 1979, near the East Pacific Rise. Together with seventy ocean bottom seismometers, a vertical (MABS) and a horizontal (ESP) array were used to receive acoustic energy generated by a series of explosions. The horizontal array was towed so that data was received in the ESP format described above. The vertical array was stationary. The use of these two types of array deployments, and of the bottom receivers, resulted in one experiment employing most of the techniques currently used in seismic refraction work.

Insofar as the experiment occurred near an active plate boundary, the structural makeup of the crust was not "typical", and difficulties were experienced in relating the velocity estimates obtained from single shots to the simple layered models discussed above. As we shall see, the complex seafloor topography also limited the accuracy of our calculated velocities. However, interesting and useful results were obtained and estimates of crustal energy partitioning shall be presented.

Overview

In Chapter II, a summary of the standard theories of seismic refraction is given. The emphasis is on current ideas concerning the strength of refracted waves. Next we discuss the data set and describe the different experiments conducted in the ROSE project. Chapter IV deals with the velocity spectral algorithm and the method used to determine bias corrections. Chapter V presents some results of the computations done on the data with respect to velocity estimation. Next, we describe the compensations that were necessary to make the measurements obtained from the algorithm correspond to transmission loss estimates in physical units. Source levels, biases, surface effects, group beampatterns, sensitivities, and analog to digital conversion factors must all be included to arrive at estimates of path losses. Finally, a summary of transmission loss estimates from this data set is presented.

CHAPTER II

SEISMIC REFRACTION

In this chapter, the fundamental concepts underlying the refraction experiment are presented. In particular, we concentrate on factors influencing the travel time and amplitude of arrivals. The material discussed is mainly a review for the geophysicist, but may not be as familiar for the ocean acoustician.

We begin with the free space solution of the wave equation in a homogeneous, isotropic elastic solid. We then discuss acoustic propagation in a simple layered medium, with one interface separating two isovelocity half spaces. Using a high frequency, ray theory analysis, the concept of a critically refracted interface wave is presented. We show that this analysis, based on the "geometrical optics" model of sound propagation, does not explain empirical observations of remotely sensed acoustic events, and turn to a "wave theory" analysis in which the concept of "head waves" is introduced. Travel times in layered media are accurately predicted by head wave theory. A second interface, representing the sea surface, is added to the model and we define specific events observed in the ROSE data which can be represented in terms of head waves and surface reflections from this model. Since the ocean crust is not an isovelocity layer, the model is then extended to include multiple interfaces below the seabed. Events received at different horizontal offsets, based on the multiple layer model and head wave theory, are presented in the form of a theoretical travel time/offset (T-X) diagram. Because of the absence of events that correspond to expected interlayer reflections with this model in most refraction data, the model of the crust is finally generalized as a region with a continuous velocity gradient. The current perspective of oceanic crust is based on this last model, which provides better agreement with observed arrival amplitude behavior. We show, however, that some layers or interfaces of the classical layered model of the crust mentioned in Chapter I have counterparts as regions with very small or very large velocity gradients respectively in the continuous model. Finally, since we are concerned with energy partitioning in the crust, current theories of head wave (in layered media) and ray (in continuous models) amplitude behavior with respect to range are presented.

Free Space Propagation

Let Φ and Ψ be defined as the Fourier transforms of the dilational and rotational displacement potentials in an elastic solid. Under the conditions of homogeneity and isotropy, the Helmholtz equations in free space for these quantities are (Grant & West, 1965):

$$\nabla^2 \Phi + k_\alpha^2 \Phi = 0 \qquad (2\text{-}1a)$$

$$\nabla^2 \Psi + k_\beta^2 \Psi = 0 \qquad (2\text{-}1b)$$

where:

$$k_\alpha = \omega \Big/ \sqrt{\frac{\lambda + 2\mu}{\rho}} \triangleq \omega/\alpha \qquad (2\text{-}2a)$$

$$k_\beta = \omega \Big/ \sqrt{\lambda/\rho} \triangleq \omega/\beta \qquad (2\text{-}2b)$$

λ and μ are Lame's constants, ω is the radian frequency, and ρ is the density of the solid. In a homogeneous, free space, two dimensional geometry, a solution found by separating variables, is given by:

$$\Phi(x, z : \omega) = A(\omega)\, e^{j 2\pi \nu_\alpha (lx + nz)} \qquad (2\text{-}3)$$

where $\ell^2 + n^2 = 1$, $\nu_\alpha = k_\alpha / 2\pi$, and $A(\omega)$, corresponding to temporal behaviour, is an arbitrary function. This solution represents a compressional (P) plane wave traveling with velocity α in a direction with cosines $(\ell, 0, n)$. The "wavenumbers" k_α and ν_α represent spatial frequency in radians and cycles per unit distance, respectively. The solution for Ψ is the same, except that the phase speed is β , and the displacements are orthogonal to the direction of propagation, representing "shear" (S) waves.

Medium with one interface

We now turn from the free space model, and consider a medium with one horizontal plane boundary separating elastic half-spaces with velocities α_1 , β_1 and α_2, β_2 as in Fig. 2-1 (Telford et al, 1976). An incident compressional plane wave with amplitude A_0 imposes the boundary condition that apparent wave numbers in a direction parallel to the interface are constant. This leads to Snell's law:

$$\frac{\sin\theta_1}{\alpha_1} = \frac{\sin\theta_2}{\alpha_2} = \frac{\sin\lambda_1}{\beta_1} = \frac{\sin\lambda_2}{\beta_2} \triangleq p \qquad (2\text{-}4)$$

where θ_1 is the angle both of incidence and of P-wave reflection, (θ_2, λ_2) are the angles for P and S plane wavefronts that are "refracted" into the second layer. λ_1 is the angle of reflection for an S wave in the upper layer and the constant p is termed the ray parameter. If the sound velocity in the second layer is greater than α_2 , we see that there is a critical incidence angle, θ_c , when sin θ_2 = 1. At incident angles θ_c , a compressional plane wave solution exists that travels parallel to the boundary as an interface

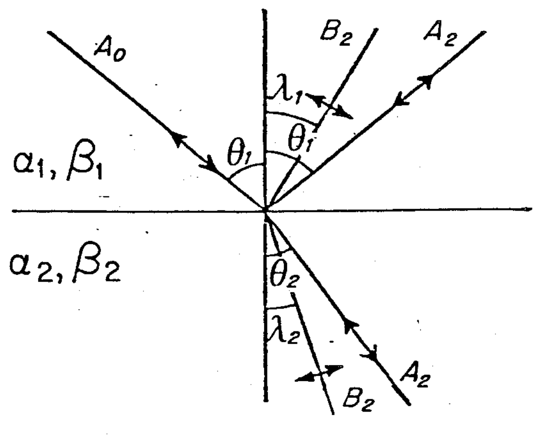


Fig. 2-1
Geometry of reflected and refracted waves at one interface.

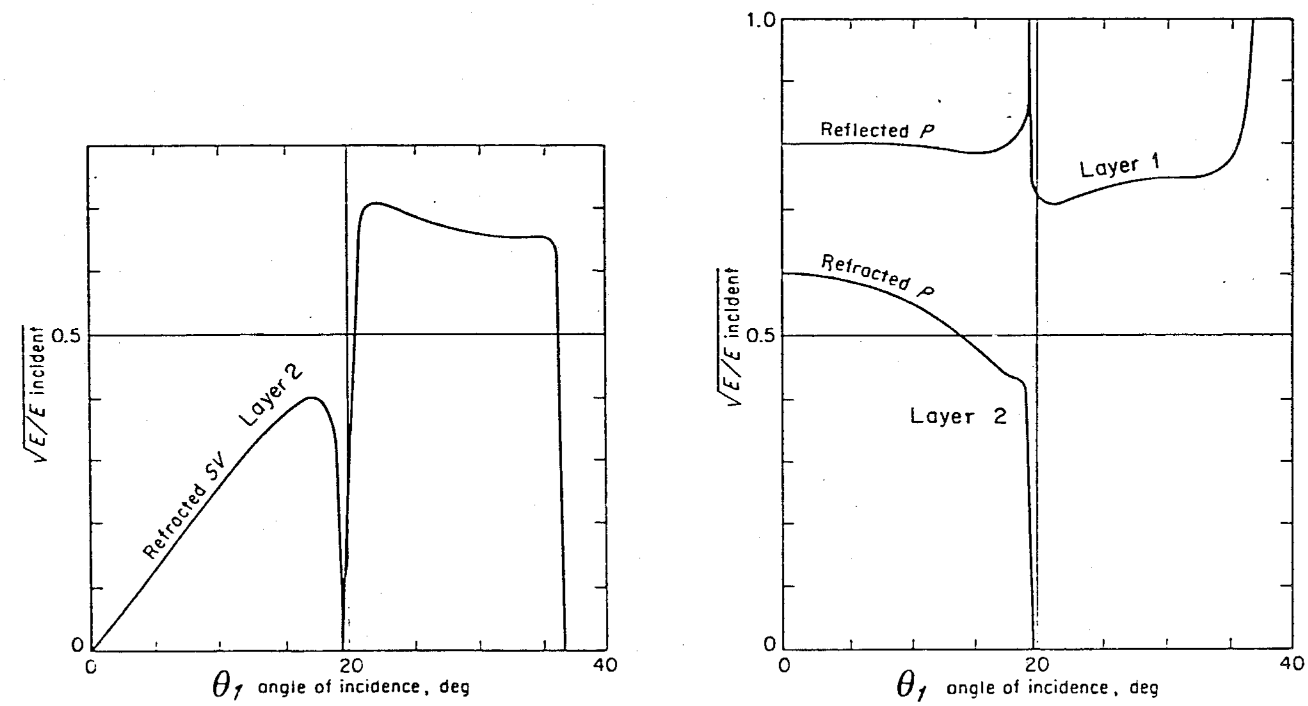


Fig.2-2
Amplitudes of reflected and refracted compressional (P) and shear (SV) waves versus angle of incidence at one interface for / = .33 and / =.6.
(from Grant & West, 1965)

wave. This solution is the basis of all simple refraction theories and formulae. The "critically refracted" wave travels with the higher speed, α_2, so that at large horizontal ranges, it should be the earliest arrival.

From this ray theory or geometrical optics viewpoint, however, the interface wave will not appear in the upper layer, and its predicted amplitude is zero. The latter fact is seen by applying six boundary conditions of continuity of stress and displacement at the interface to eqs. 2-1, whereby the Knott equations (Telford, 1976) in terms of the potential function amplitudes, or the Zoeppritz formulae for the displacement amplitudes (Grant & West, 1965) are derived. An example, calculated from these equations, is shown in fig. 2-2 (from Grant & West) in which ratios of incident amplitude to the refracted P and S amplitudes in the lower layer and to the reflected P amplitude in the upper layer are shown versus angle of incidence for a fluid-solid boundary. In these, $\alpha_1/\alpha_2 = 1/3$ and $\beta_1/\beta_2 = .6$. The critical angle for the compressional (P) and shear (SV) waves are thus $\sin^{-1}(1/3) = 19.5°$, and $\sin^{-1}(.6) = 37°$, respectively. In the ROSE experiment, typical critical angles for P waves were in the $10°$ to $15°$ range. Note that, in these figures, all energy in the upper layer is either incident or is reflected from the interface at angles other than critical, while amplitudes associated with interface waves at the critical angles is zero. It is observed, however, that significant energy with travel times much as one would calculate for an interface wave with speed α_2 refracting energy into the water at the critical angle, does appear in refraction experiments.

Head waves

The most widespread theory to explain this is based on Huygen's principle using curved instead of ideal planar wavefronts. It predicts the existence of "head waves" as shown in Fig. 2-3 (from Cerveny and Ravindra, 1971). In 2-3-a, a spherical wavefront originating at M_o impinges upon the interface for time $t > h/\alpha_1$. At the boundary it sets up a disturbance along OP and creates Huygen wavelets (Fig 2-3-b) which produce the reflected and refracted wavefronts where constructive interference occurs. The speed along the interface of P, the contact point with the incident wavefront, is $\alpha_1 / \sin\theta(P)$. $\theta(P)$ is the angle from PM_o to the horizontal axis. Beyond a critical horizontal distance, $X_c = h / ((\alpha_2/\alpha_1)^2 - 1)^{1/2}$, the speed of this point becomes less than α_2. At this range, the angle $\theta(P)$ has increased to the critical angle $\theta^* = \theta_c$. We now get the situation in fig. 2-3-c. The refracted wave in layer 2 is now ahead of the incident or reflected fronts. Again using a Huygen construction, M^*Q is seen to be a locus of constructive interference, and for constant α_1 and α_2, is a straight line (in 2 dimensions). In time Δt, the disturbance at point O^* will move both to Q along the boundary at speed α_2, and to point M^* in layer 1 at speed α_1. The angle of this "head wave" is seen to be: $\sin^{-1}\left(\frac{\alpha_1 \Delta t}{\alpha_2 \Delta t}\right) = \theta_c$. Wave theory thus predicts that the Snell's law interface wave constantly reflects energy, in the form of a head wave, back into the uppermost layer at the critical angle. The apparent horizontal phase velocity of the head wave in layer 1 will be:

$$\frac{\alpha_1}{\sin\theta_c} = \alpha_2 = 1/p \qquad (2\text{-}5)$$

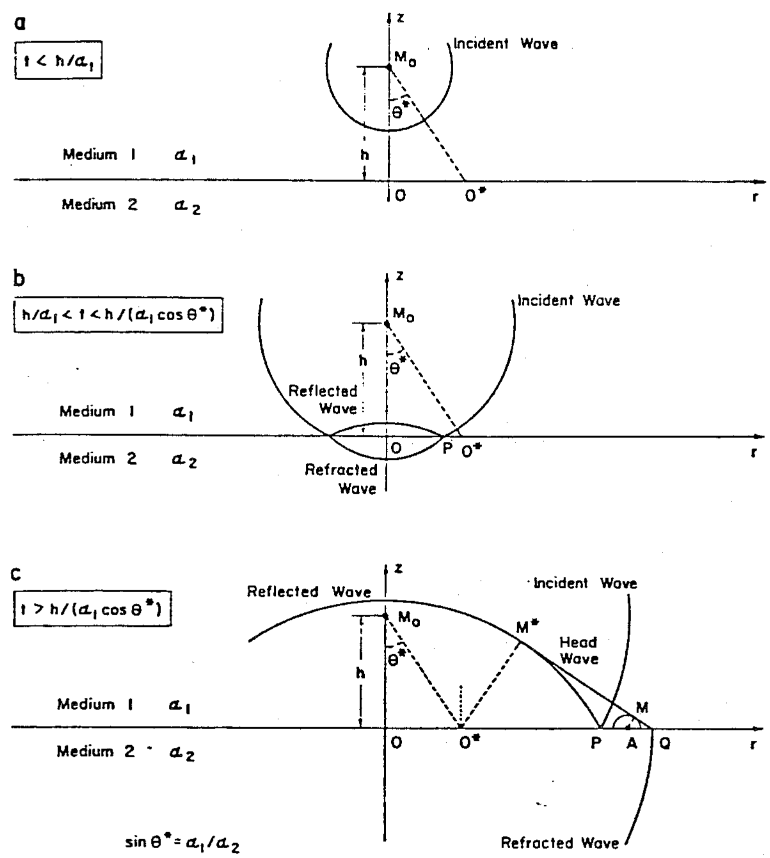


Fig.2-3

Construction of head wave using Huygen wavelets.
(from Cerveny & Ravindra, 1971)

Thus, if the direction of the refracted energy or the horizontal phase velocity of an emerging plane wavefront can be determined within the upper medium, the sound speed in layer 2 can be found remotely for a horizontally layered medium. This is the basis of classical inversion theory for two simple layers as discussed in Ewing, 1963.

Two interface model

We now modify the proceeding model by introducing a perfectly reflecting interface in the upper half space, representing the sea surface. Due to surface reflections, many arrivals other than those from emerging interface waves, will occur at a receiver with this model. Referring to Fig. 2-4, together with the critically refracted compressional wave labelled 1P, a converted shear wave (1S), a direct wave, and a series of water layer reflections (1W, 2W, etc), will be recorded at the array. Since 1P refracts energy continuously back into the water, a surface receiver may encounter energy which travels as an interface wave and refracts into the water. Upon reflection from the surface, this energy reenters the seabed, again as an interface wave, before finally refracting into the water and being detected. This "multiple refraction" is termed 2P in Fig. 2-4. More multiples of this type (3P, 4P, etc), for which an arrival has had a number of encounters with the surface, can be observed with velocity analysis in the array data presented subsequently. Often it is found that the amplitudes of 2P arrivals, and even those of higher multiples, are stronger than 1P. Since a multiple refraction arrival can be the sum of a large number of rays each travelling

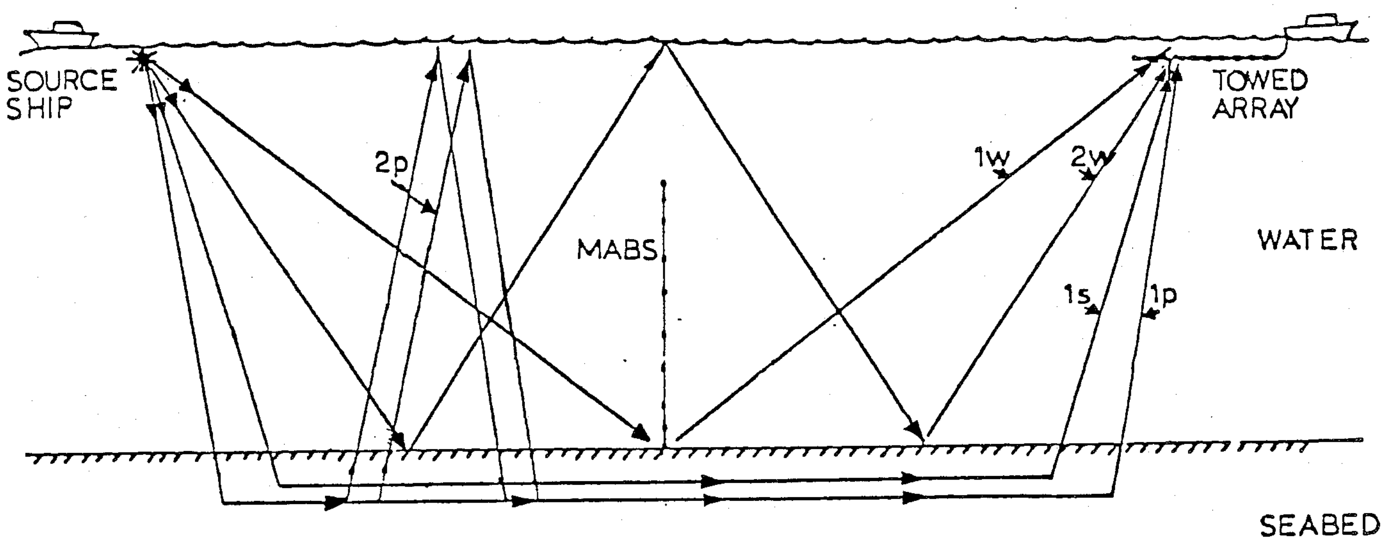


Fig.2-4
Possible ray paths with a two-interface model.

along a different path, this is possibly due to the constructive interference. While the exact acoustics solution still remains unsolved for these multiples, they are important from the ocean acoustician's perspective because of their relatively high energy levels.

Multiple layers

The upper layer in the two interface model discussed above represents the water. For the region below the seabed, we first introduce a multiple layer model and use head wave theory to predict events received at a horizontal offset X in the form of a travel time/offset (T-X) plot. As discussed in Chapter I, the original, "classical" model of the oceanic crust has 3 isovelocity layers above the mantle interface.

In a multiple layered case, the number of events one can expect is large, especially in sedimentary locales. Figure 2-5 depicts a situation with N interfaces. In constructing a time versus range (T-X) plot for this example, and concentrating only on critical refractions of first arrivals, we see that up to range X_{c1} , the first event is the "1P" from the layer with velocity V_0. When the range exceeds X_{c1}, the 1P event from the second boundary arrives earlier. With densely spaced sample points in range, the locus of the first arrival traces a straight line in the T-X plane with slope $1/V_1$. As range is increased beyond X_{c1}, the interface wave from the V_2 layer will eventually be the earliest arrival. This pattern continues until, at the largest distance, the slope of the first arrival line will be $1/V_N$. In this way, for a horizontal layered situation, in which layer velocities increase with depth,

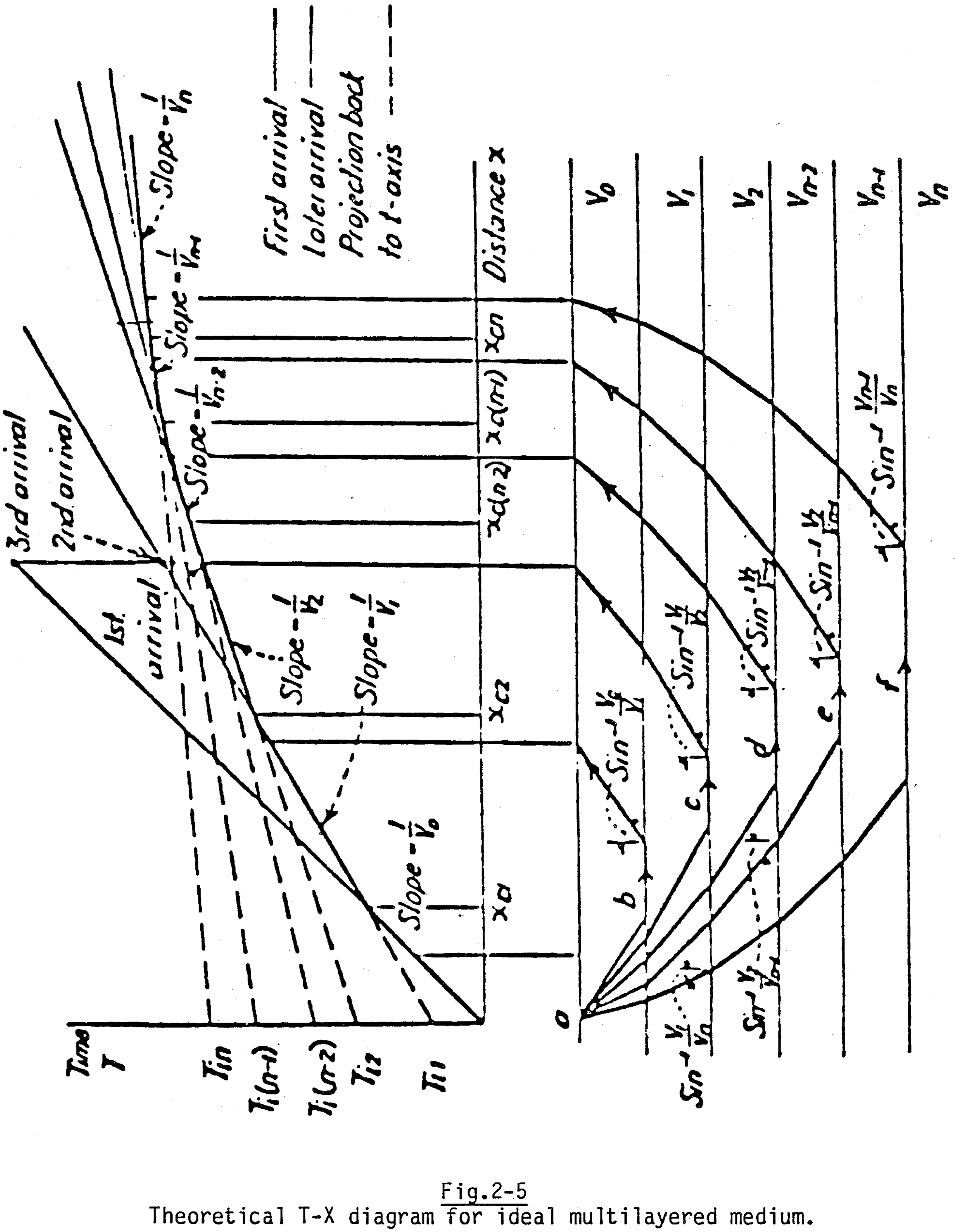
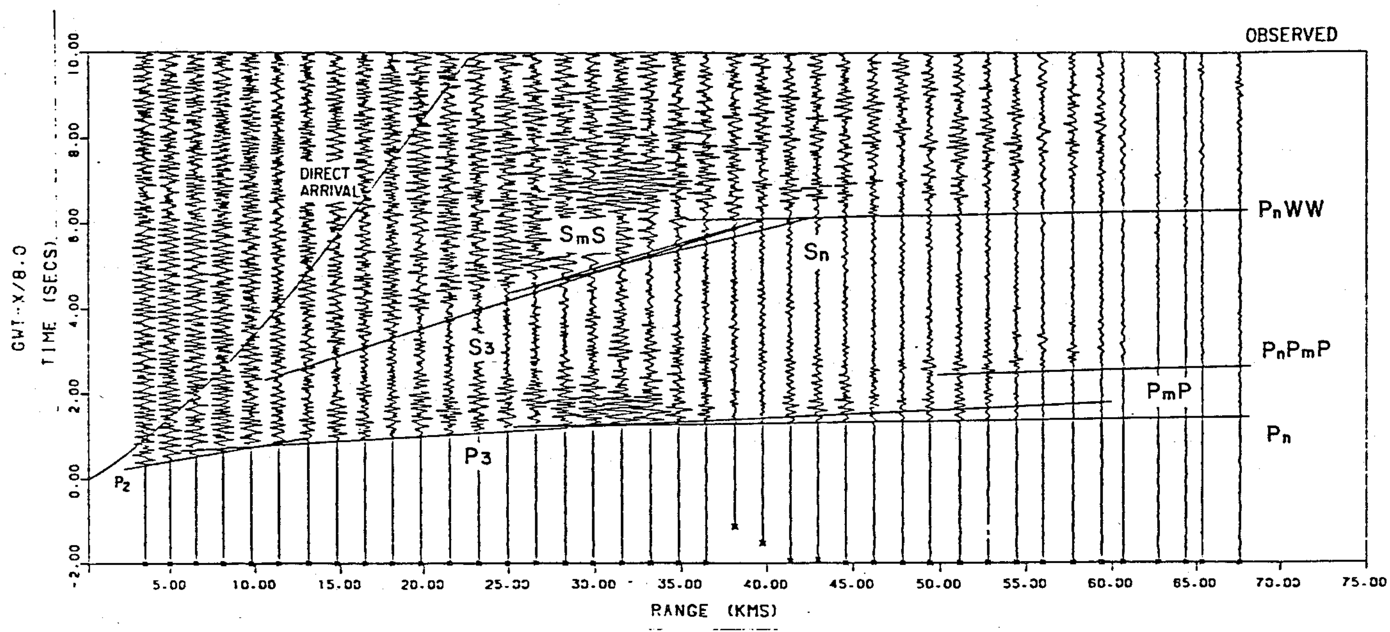
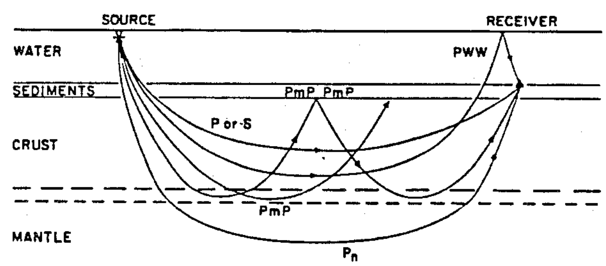


Fig.2-5
Theoretical T-X diagram for ideal multilayered medium.

the calculation of slopes from earliest arrivals on a T-X plot is sufficient for obtaining the layer velocities of interest.

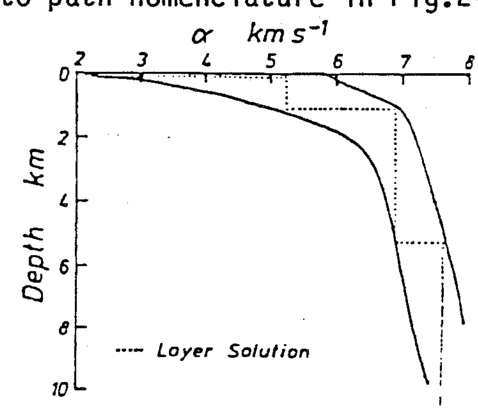
For the multiple layer case, we have discussed arrivals due to head waves only. A T-X diagram for the multi-layer model is more complex than this because multiple reflections and multiple refractions from each of the interfaces are present. These appear after the first arrival. Except for those involving the water surface, seafloor, sedimentary layered sequences, and the mantle interface, interlayer reflections are rarely seen (Ewing & Houtz, 1969) in refraction data, however. An example of an actual T-X plot is shown in figure 2-6a (Detrick & Purdy, 1980) for an experiment conducted near the Kane fracture zone. The locus of events that can be attributed to layer reflections are limited to those designated by PmP, PmPPmP, and SmS, for the mantle interface, and PnWW for the ocean surface boundary (see key to path nomenclature in fig. 2-6b). In the ROSE data presented subsequently, which is for a young area with little sediment coverage, the only clearly identifiable reflections we find involve the water layer, directly (1W, 2W), or indirectly (2P,etc.). Since strong reflections occur at areas of considerable contrast in elastic properties, e.g. at the interfaces in the model, the lack of reflected energy argues against clearly defined layering in the sub-basement. Because of this experimental evidence, a model based on a continuous velocity/ depth relationship in the crust is more appropriate, although more complex, especially in sedimentary regions.





<u>Fig.2-6a</u>
Actual T-X diagram from OBH data obtained near Kane fracture zone.
(from Detrick & Purdy, 1980)

<u>Fig.2-6b</u>
Key to path nomenclature in Fig.2-6a.



<u>Fig.2-7</u>
Representative bounds on velocity/depth profile
obtained from tau-p method of travel time inversion.
(from Kennett, 1977)

Continuous velocity/ depth profiles

In the analysis of acoustic propagation in regions with continuous velocity gradients, ray theory is the primary tool for analysis. Head wave theory is not applicable. Whereas is a layered model, the horizontal phase velocity of an emerging plane wavefront is equal to the layer velocity, in ray theory, it is equal to the velocity at which the ray turns upward. This velocity is also the reciprocal of the ray parameter, i.e. 1/p. Knowledge of arrival direction remains important as a tool for remotely obtaining information about velocity structure.

By conceptually allowing layer thicknesses in the multiple layer model to approach zero, we can see that any arbitrary velocity/depth relationship, provided that velocities increase with depth, may be determined from the slopes produced by the first arrival events on a T-X diagram. This is the basis of the Herglotz-Wiechert integrals (Aki, 1980), used for the inversion of travel time versus range data. The density in range with which data is available is of critical importance since the slope of the first arrival changes continuously with offset.

The tau-p method of travel time inversion (Stoffa, Diebold, & Buhl, 1981) assumes a continuous velocity gradient in the crust. Using first arrival times, it produces bounds on possible velocity-depth profiles which are compatible with the data. Many of these resulting profiles are similar to the example in figure 2-7 (Kennett, 1977). Although a continuous gradient is assumed, a consistent feature of these velocity/depth profiles is an area of small velocity change immediately above a sharper gradient at the mantle interface. This region corresponds to the "classical" layer 3 in Raitt (1963). Averaging from many experiments, this region has a mean compressional speed of 6.8 km/sec, a thickness of about 5 km, and begins at a mean depth of 2 km below basement. We shall see that velocities in this band were the most prevalent at the arrays in the Rose experiment. Rays that travel within this "layer" emerge as first arrivals at offsets of approximately 10 to 30 km in areas where ocean depth is on the order of 3 km. Beyond about 30 km, mantle reflections and mantle interface waves appear as the earliest events.

The model of the oceanic crust we have been employing has evolved from a simple one-interface case to a continuous velocity gradient representation. Although the latter is the most general, we point out that at least the mantle interface and the sea surface can be effectively considered from the simpler, layered model. Reflections from these interfaces are routinely observed in refraction work and the concept of head waves predicts travel times along the mantle interface accurately. In many instances, "layer 3" can also be treated as a homogeneous isovelocity layer.

Amplitude considerations

Since the main focus of this paper centers on energy partitioning in the crust, we now look at some theories concerning head wave amplitude behavior with range (for layered models) and the amplitude behavior of rays, (for the continuous velocity gradient case).

The behavior of head wave amplitude with range is a controversial issue. Cerveny and Ravindra (1971) use a first order ray series solution in solving the equations of motion for a single interface problem in contrast to the above "zeroeth" order plane wave or geometric optics solution. They obtain an amplitude distance curve for a "pure" head wave that behaves as $L^{-3/2} X^{-1/2}$ where X is the horizontal offset, and $L = (X-X_c)$ is the propagation distance along the interface. According to this equation, at large ranges, amplitude decreases as $1/X^2$ ("spherical spreading").

Alternatively, if we assume a velocity distribution that varies arbitrarily with depth, ray theory predicts that the pressure amplitudes will behave with distance as:

$$P^2 = \frac{P_o^2 R_o^2 \Delta\theta}{h X} \frac{c}{c_o} \left| \frac{\sin\theta_o}{\sin\theta} \right| \qquad (2\text{-}6)$$

(from Clay & Medwin (1977))

R_o is the reference range where sound speed is c_o. θ_o is the initial angle of a ray bundle of width $\Delta\theta$ and amplitude P_o. θ is the average angle of the bundle, with vertical height h, at horizontal range X where average sound speed is c (see fig. 2-8). This equation is valid at ranges where focusing of the ray bundle does not occur, so that θ closely approximates the angle of all rays in the bundle at X. For a source at the surface of an isovelocity layer lying above a half-space with a linear sound speed gradient with slope $\frac{dc(z)}{dz}$ =b, the equation for the mean square pressure at the surface at offset X reduces to:

$$P^2 \approx \frac{b R_o^2 P_o^2}{4 c_o X} \qquad (2\text{-}7)$$

Rays will behave with an X^{-1} amplitude dependence for a linear gradient. This type of geometrical behavior is termed "cylindrical spreading" and is also