PAPERS IN PHYSICAL OCEANOGRAPHY AND METEOROLOGY

PUBLISHED BY

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

AND

WOODS HOLE OCEANOGRAPHIC INSTITUTION

VOL. VII, NO. 2

FRONTS AND FRONTOGENESIS IN RELATION TO VORTICITY

BY

SVERRE PETTERSSEN AND JAMES M. AUSTIN

CAMBRIDGE AND WOODS HOLE, MASSACHUSETTS

January, 1942

CONTENTS

INTRODUCTION

Soon after the discovery of the polar front, it was realized that fronts were subject to processes which either increased or diminished their intensity. Thus, fronts may form in fields where the distribution of the meteorological elements is continuous; and, in other cases, fronts may dissolve and develop into a field of continuous distribution of the various elements.

The processes which lead to the formation of a front or the increase in intensity of an existing front, are called *frontogenetical processes;* and the processes which lead to the dissolution of fronts are called *frontolytical processes.*

In theoretical treatments of fronts it has been customary to simplify the problem by assuming that a frontal surface is a mathematical discontinuity, and doubts have been raised against the validity of this simplification. Petterssen [1] has shown that both the dynamic and the kinematic boundary conditions that hold for perfect discontinuities hold also for layers of transition of finite thickness within which the meteorological elements vary continuously. We are, therefore, justified in treating frontal surfaces and fronts as either strict discontinuities or as finite layers of transition. Frontogenesis may therefore be defined as the process that tends to create a surface of discontinuity in the atmosphere. Whether or not this process results in a strict discontinuity is immaterial.

The dynamic significance of a frontal surface resides in the inclination of the density surfaces relative to the pressure surfaces. If the density and the pressure surfaces coincide, the layer of transition contains no solenoids, and is therefore dynamically unimportant. A frontal surface is therefore commonly defined as an inclined layer of air within which the significant meteorological elements have a maximum of space variations. Thus when the x-axis is normal to the front (pointing towards the cold air), the z-axis is vertical, and ρ is the density, a front is present when simultaneously

$$\frac{\partial \rho}{\partial x} \text{ and } -\frac{\partial \rho}{\partial z}$$

have numerical maxima, provided that the dimensions of the layer of transition are small compared with those of the adjacent air masses.

The isobaric surfaces in the atmosphere are quasi-horizontal. Therefore, when the layer of transition is quasi-horizontal, the number of solenoids approaches zero. In this case

$$\frac{\partial \rho}{\partial x} \approx 0 \quad \text{and} \quad -\frac{\partial \rho}{\partial z} \text{ is a maximum.}$$

In this case the layer of transition between the two air masses is called an *inversion.* There is, of course, no distinct difference between a frontal surface and an inversion. Bergeron [2] has shown that, on the average, the number of solenoids contained in a frontal surface is about one hundred times larger than that contained within an inversion.

The first theory of frontogenesis was rendered by Bergeron [2], who considered the behavior of a linear field of potential temperature in a linear field of motion, and found that a front would form along the axis of dilatation in a stationary field of deformation. Although the case discussed by Bergeron is a rather special one, the fundamental idea has proved sound, and all subsequent investigations of frontogenesis are based on Bergeron's principle.

The simplifications introduced by Bergeron may be justified within the air that constitutes the layer of transition. But in order to explain frontogenesis more fully, it is necessary to extend the fields of motion and property to a wider area, and then it is not permissible to assume linear distributions. A detailed discussion of this problem has been rendered by Petterssen [3], who developed a formula for the intensity of frontogenesis which holds for any distribution of wind and property, the only restriction being that the property in question is assumed to be conservative. The general theory was then applied to the following cases:

(a) Linear field of motion and non-linear field of property;

(b) Linear field of property and non-linear field of motion;

(c) Both fields non-linear and of the types usually found in the region between two adjacent air mass sources.

As a result of his investigations, Petterssen found that case (b) was trivial, whereas frontogenesis may occur under the conditions (a) and (c). In agreement with Bergeron, Petterssen found that the field of deformation was the principal cause of frontogenesis, although convergence may contribute to the process. Furthermore, it was found that frontogenesis may occur in non-stationary fields of motion and that, in general, the line of frontogenesis does not coincide with the axis of dilatation.

The theories of frontogenesis referred to above are based mainly on kinematical considerations, and discuss the behavior of isotherms, or potential isotherms, in various types of motion. However, the dynamical theory of fronts requires not only a temperature discontinuity but also a number of other conditions which must be fulfilled at a front. A detailed discussion of the general and special frontal characteristics has been rendered by Petterssen [4] and others. One of the general characteristics is the presence of a wind discontinuity resulting in a maximum of cyclonic wind shear (or cyclonic vorticity) at the front. It is therefore of interest to investigate the factors creating and maintaining cyclonic vorticity at the fronts. The aim of this paper is to contribute to the solution of this problem.

CHAPTER I. FRONTAL CHARACTERISTICS

Before we proceed to discuss frontogenesis from a general point of view, it will be necessary to define the term *front intensity*. A front is recognized by a number of general frontal characteristics which apply to all fronts, and special characteristics which apply to the various types of fronts. We shall here only be concerned with the general characteristics.* Obviously, some of these characteristics are derived from others. We shall therefore endeavor to investigate whether the various general frontal characteristics can be derived from one (or more) fundamental characteristics.

1. Front Intensity

The dynamic significance of a frontal surface resides in its inclination relative to the horizontal, or, more precisely, in the field of isosteric-isobaric solenoids contained in the layer of transition between the adjacent air masses. Treating the frontal surface as an inclined layer of transition, and letting N denote the number of isosteric-isobaric solenoids per unit area, we obtain:

$$N = |\nabla\alpha \times (-\nabla p)| \tag{1}$$

where vertical bars indicate the magnitude of the vector, α is specific volume, and p is atmospheric pressure.

It is convenient to choose a right-hand system of coordinates with the z-axis normal to the earth's surface, and the x-axis perpendicular to the front and pointing toward the colder air. Then

$$\nabla\alpha \times (-\nabla p) = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \dfrac{\partial\alpha}{\partial x} & \dfrac{\partial\alpha}{\partial y} & \dfrac{\partial\alpha}{\partial z} \\ -\dfrac{\partial p}{\partial x} & -\dfrac{\partial p}{\partial y} & -\dfrac{\partial p}{\partial z} \end{vmatrix} \tag{2}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are the unit vectors along the coordinate axes. With this choice of axes, we may neglect $\partial\alpha/\partial y$ as against $\partial\alpha/\partial x$ and $\partial\alpha/\partial z$. This gives

$$\nabla\alpha \times (-\nabla p) = \frac{\partial\alpha}{\partial z}\frac{\partial p}{\partial y}\boldsymbol{i} + \left(\frac{\partial\alpha}{\partial x}\frac{\partial p}{\partial z} - \frac{\partial\alpha}{\partial z}\frac{\partial p}{\partial x}\right)\boldsymbol{j} - \frac{\partial\alpha}{\partial x}\frac{\partial p}{\partial y}\boldsymbol{k}.$$

To obtain maximum values for the various terms, we assume $p = 100$ cb, $T = 280°$A, the width of the frontal zone in the horizontal $= 50$ km, the temperature difference (ΔT) across the front 10°C, and $|\partial p/\partial x| = |\partial p/\partial y| = 5$ mb per 200 km within each of the air masses close to the front. Thus, in the MTS-system of units

$$\left|\frac{\partial\alpha}{\partial z}\frac{\partial p}{\partial y}\right| = \left|\frac{\partial\alpha}{\partial z}\frac{\partial p}{\partial x}\right| \approx 10^{-7}$$

* For a summary of general and special frontal characteristics, see S. Petterssen [4].

$$\left| \frac{\partial \alpha}{\partial x} \quad \frac{\partial p}{\partial z} \right| \approx 10^{-5}$$

$$\left| \frac{\partial \alpha}{\partial x} \quad \frac{\partial p}{\partial y} \right| \approx 10^{-9}.$$

It follows then that only the term containing $\partial p/\partial z$ need be considered. This is due to the fact that the vertical component of the pressure gradient is about 10^4 times the horizontal pressure gradient. We may therefore write:

$$\nabla \alpha \times (-\nabla p) = \frac{\partial \alpha}{\partial x} \frac{\partial p}{\partial z} j \tag{3}$$

or, when substituting from the hydrostatic equation

$$N = -\frac{\alpha - \alpha'}{\Delta x} \frac{1}{\alpha_m} g = \frac{\rho - \rho'}{\Delta x} \frac{1}{\rho_m} g. \tag{4}$$

Here subscript m denotes the mean values within the frontal zone, the letters with indices refer to the warm air, and g is the acceleration of gravity.

N in the above equation may be defined as the *specific front intensity*. Summing N over a cross section of unit height across the frontal zone, we obtain for the intensity I of the front

$$I = \frac{\rho - \rho'}{\rho_m} g. \tag{5}$$

Since the mean density within a unit sheet across the frontal zone does not vary much with time although $\rho - \rho'$ varies, we may, for all practical purposes, take $\Delta\rho = \rho - \rho'$ as a measure of the front intensity. The condition

$$\Delta\rho = \rho - \rho' > 0 \tag{6}$$

is therefore a *fundamental front characteristic.*

From the dynamical boundary condition and from the equation of state, we obtain with the same degree of approximation:

$$I = g \frac{\rho - \rho'}{\rho_m} = g \frac{\alpha' - \alpha}{\alpha_m} = g \cdot \frac{T' - T}{T_m} > 0$$

as equivalent expressions for the fundamental front characteristic and equivalent measures of the intensity of a front.

If the frontal surface be considered as a strict discontinuity, the discontinuity is of zero order with regard to density, specific volume and temperature.

2. Fronts in Relation to Pressure

We choose the same system of coordinates as in the previous paragraph. Let $p = p(x,y,z)$ and $p' = p'(x,y,z)$ represent the distribution of atmospheric pressure in the cold and in the warm air, respectively, at a given instant. Differentiating p and p' along the frontal surface, we obtain:

(1)
$$dp = \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial y} dy + \frac{\partial p}{\partial z} dz$$

(2)
$$dp' = \frac{\partial p'}{\partial x} dx + \frac{\partial p'}{\partial y} dy + \frac{\partial p'}{\partial z} dz.$$

Since the pressure is continuous at the front

$$p = p' \quad \text{and} \quad \frac{\partial p}{\partial y} = \frac{\partial p'}{\partial y}\cdot$$

Therefore, when $\tan \theta_d$ denotes the inclination of the front

(3)
$$\tan \theta_d = \frac{dz}{dx} = -\frac{\dfrac{\partial p}{\partial x} - \dfrac{\partial p'}{\partial x}}{\dfrac{\partial p}{\partial z} - \dfrac{\partial p'}{\partial z}}\cdot$$

Substituting from the equation of static equilibrium, we obtain:

(4)
$$\tan \theta_d = \frac{\dfrac{\partial p}{\partial x} - \dfrac{\partial p'}{\partial x}}{g(\rho - \rho')}\cdot$$

Since the x-axis points from the warm toward the cold air, we may distinguish between two cases: (a) the warmer air overruns the colder air, and (b) the colder air overruns the warmer air. In the case (a) $\tan \theta_d$ is positive, and in the case (b) $\tan \theta_d$ is negative.

Case a. Since $\rho > \rho'$ and since $\tan \theta_d > 0$, it follows that

$$\frac{\partial p}{\partial x} - \frac{\partial p'}{\partial x} > 0$$

which means that the pressure gradient is discontinuous at the front in such a way that there is an infinite cyclonic curvature of the isobars at the front. This result is independent of the choice of positive direction of the x-axis. Thus, if we choose the x-axis from the cold to the warm air, the inclination of the frontal surface is

$$\tan (\pi - \theta_d).$$

Deducting Eq. (1) from Eq. (2) we obtain

$$\tan (\pi - \theta_d) = \frac{\dfrac{\partial p'}{\partial x} - \dfrac{\partial p}{\partial x}}{g(\rho' - \rho)}\cdot$$

Since $\rho' < \rho$ and $\tan (\pi - \theta_d) < 0$, it follows that

$$\frac{\partial p'}{\partial x} - \frac{\partial p}{\partial x} > 0.$$

Thus, in all normal cases when the warmer air overruns the colder air, the front is characterized by a cyclonic kink in the isobars that cross the front. If the front is parallel to the isobars, it must either be situated in the trough, or the pressure gradient must be larger in the air mass where the pressure is higher.

Case b. In this case, $\tan \theta_d < 0$, while $\rho > \rho'$. It follows then that

$$\frac{\partial p}{\partial x} - \frac{\partial p'}{\partial x} < 0$$

which means that the pressure gradient is discontinuous at the front with an infinite anticyclonic curvature of the isobars at the front. Fronts of this type occur as squall lines and are usually associated with thunderstorms. The cold air overruns the warm air only along the lower portion of the frontal surface, as shown in Fig. 1. In the free atmosphere, the front has a normal slope, and as a result, the front will be associated with a large-scale pressure trough in the bottom of which is superimposed a small ridge of high pressure of the type recorded in the barograms during the passage of squall lines.

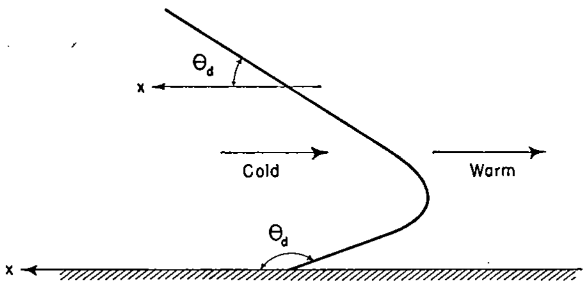

FIG. 1. Cross section through an overrunning cold front.

Along overrunning cold fronts, with cross sections as shown in Fig. 1, the vertical accelerations are usually considerable. Taking the vertical acceleration into account, we obtain:

$$\tan \theta_d = \frac{\dfrac{\partial p}{\partial x} - \dfrac{\partial p'}{\partial x}}{g(\rho - \rho') + \rho \dfrac{dv_z}{dt} - \rho' \dfrac{dv_z'}{dt}}. \qquad (5)$$

If the density difference is small, the terms depending on the vertical acceleration might become important, and the isobars in the immediate vicinity of such fronts may take various forms.

If the Coriolis terms are considered, the denominator in Eq. (5) would contain the terms:

$$2\Omega_x(\rho v_y - \rho' v_y') - 2\Omega_y(\rho v_x - \rho' v_x')$$

where Ω_x and Ω_y are the components of the angular velocity of the earth's rotation. Even when the temperature discontinuity is as small as 1°C, the wind velocity would have to be 100 to 200 m/sec for the Coriolis terms to equal the gravity term. Thus, the Coriolis terms are completely negligible.

It follows from Eq. (5) that the frontal surface will be vertical when

$$\rho \frac{dv_z}{dt} - \rho' \frac{dv_z'}{dt} = -g(\rho - \rho')$$

and a density discontinuity corresponding to a temperature discontinuity of 1°C would be balanced by vertical accelerations of 1 to 2 cm/sec². Vertical accelerations larger than these are likely to occur along overrunning cold fronts. In the following paragraphs we shall discuss only fronts with warm air overrunning cold air.

3. Inclination of Frontal Surfaces

Let $\boldsymbol{\Omega}$ denote the angular velocity of the earth's rotation, and $K=\mu/\rho$, where μ is the coefficient of eddy viscosity. The equations of motion along the x-axis (i.e., normal to the front) in the cold and in the warm air may then be written:

$$\text{(1)} \qquad \frac{dv_x}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - 2\Omega_y v_z + 2\Omega_z v_y + K\frac{\partial^2 v_x}{\partial z^2}$$

$$\text{(2)} \qquad \frac{dv_x'}{dt} = -\frac{1}{\rho'}\frac{\partial p'}{\partial x} - 2\Omega_y v_z' + 2\Omega_z v_y' + K'\frac{\partial^2 v_x'}{\partial z^2}.$$

Substituting from the above equations into Eq. 2(4), we obtain for the slope of the frontal surface:

$$\text{(3)} \qquad \tan\theta_d = \frac{2\Omega_z}{g}\frac{\rho v_y - \rho' v_y'}{\rho - \rho'} - \frac{2\Omega_y(\rho v_z - \rho' v_z')}{g(\rho-\rho')} + \frac{\rho K\dfrac{\partial^2 v_x}{\partial z^2} - \rho' K'\dfrac{\partial^2 v_x'}{\partial z^2}}{g(\rho-\rho')} - \frac{\rho\dfrac{dv_x}{dt} - \rho'\dfrac{dv_x'}{dt}}{g(\rho-\rho')}.$$

The first term on the right-hand side represents the Margules formula for the slope of the front. Putting:

$$\tan M = \frac{2\Omega_z}{g}\frac{\rho v_y - \rho' v_y'}{\rho - \rho'}$$

we obtain from Eq. (3):

$$\text{(4)} \qquad \tan\theta_d = \tan M\left[1 - \frac{2\Omega_y(\rho v_z - \rho' v_z')}{2\Omega_z(\rho v_y - \rho' v_y')} + \frac{\rho K\dfrac{\partial^2 v_x}{\partial z^2} - \rho' K'\dfrac{\partial^2 v_x'}{\partial z^2}}{2\Omega_z(\rho v_y - \rho' v_y')} - \frac{\rho\dfrac{dv_x}{dt} - \rho'\dfrac{dv_x'}{dt}}{2\Omega_z(\rho v_y - \rho' v_y')}\right]$$

or

$$\tan\theta_d = \tan M(1 + A + B + C)$$

where A, B and C represent the last three terms within the parentheses.

To obtain an idea of the order of magnitude of the various terms in Eq. (4), we shall assume the following values:

$T = 280°\text{A}$, $T' = 290°\text{A}$ $\qquad$ $p = p' = 100$ cb.

$2\Omega_z = 10^{-4}\ \text{sec}^{-1}$ (Lat. 45°N) $\qquad$ $2\Omega_y = 10^{-4}$ to $0\ \text{sec}^{-1}$ depending on the orientation of the front.

$v_z = \pm v_z' = \pm 10^{-1}$ m/sec $\qquad$ $v_y = \pm v_y' = \pm 10$ m/sec

Suppose that v_x increases from 10 to 15 m/sec in three hours; then

$$\frac{dv_x}{dt} = \pm \frac{dv_x'}{dt} = \pm 5 \times 10^{-4} \text{ m/sec}^2.$$

The order of magnitude of the frictional term may be obtained for steady motion by choosing, say, the x-axis along the isobars. Then

$$K \frac{\partial^2 v_x}{\partial z^2} = -2\Omega_z v_n$$

where v_n is the wind component normal to the isobars. Putting $v_n = 5$ m/sec, as a high value, we obtain

$$K \frac{\partial^2 v_x}{\partial z^2} = -5 \times 10^{-4} \text{ m/sec}^2.$$

Assuming the combination of signs which gives maximum values to the various terms in Eq. (4), we obtain the following numerical values for A, B and C:

$$A \approx 0.6 \qquad B \approx 30 \qquad C \approx 30.$$

These figures might seem to indicate that the Margules formula has no significance except when the motion is strictly geostrophic. Since none of the numerical values given above are excessive, it is of interest to investigate what combination of signs of velocities, accelerations and frictional terms can occur at fronts.

(1) *The frictional term.* The sign of this term depends on the curvature of the velocity-height curve, and is negative near the earth's surfaces and tends to zero at the top of the friction layer. Therefore, within the friction layer, the frictional terms in the cold and the warm air have the same sign and the magnitude decreases rapidly with elevation.

As will be shown later, there must always be cyclonic shear along all fronts when the warm air overruns the cold air. In discussing the term B in Eq. (4), we need therefore only consider the case when $v_y > v_y'$.

TABLE I

Value of term $B = \dfrac{\rho K \dfrac{\partial^2 v_x}{\partial z^2} - \rho' K' \dfrac{\partial^2 v_x'}{\partial z^2}}{2\Omega_z(\rho v_y - \rho' v_y')}$

v_y' m/sec \ v_y m/sec	12	10	8	6	4	2	0
−12	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01
−10	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.02
− 8	−0.01	−0.01	−0.01	−0.01	−0.01	−0.02	−0.02
− 6	−0.01	−0.01	−0.01	−0.01	−0.02	−0.02	−0.03
− 4	−0.01	−0.01	−0.01	−0.02	−0.02	−0.03	−0.05
− 2	−0.01	−0.01	−0.02	−0.02	−0.03	−0.04	−0.09
0	−0.01	−0.02	−0.02	−0.03	−0.04	−0.09	
+ 2	−0.02	−0.02	−0.03	−0.04	−0.08		
+ 4	−0.02	−0.03	−0.04	−0.08			
+ 6	−0.03	−0.04	−0.08				
+ 8	−0.04	−0.08					
+10	−0.07						

Table I shows the magnitude of the term B in Eq. (4), taking into account the signs of the frictional terms in both air masses. It will be seen that the frictional influence on the slope of the front may amount to almost 10 per cent of the Margules inclination when the wind shear along the front is very small, but is negligible when the shear is moderate or large. It should be noted that a temperature discontinuity of 10°C and a cross-isobar wind component of 5 m/sec are rather excessive values. If the temperature discontinuity is 5°C and the cross-isobar component is 2.5 m/sec, the frictional influence on the slope of the front would be only 25 per cent of the values given in Table I. We may therefore assume that the direct frictional influence on the slope of the front is negligible.

(2) *The accelerational term.* Since the pressure is continuous at the front, it follows that the geostrophic wind normal to the front in the warm air must be the same as in the cold air, except for a minute difference due to the variation in density. If dv_x/dt and dv_x'/dt could have opposite signs for any appreciable length of time, the two masses would deviate increasingly from the geostrophic balance and in opposite directions. Although dv_x/dt may differ from dv_x'/dt the accelerations normal to the front must have the same signs if the accelerations are of appreciable magnitude.

The vertical velocity at a front is, in normal cases,* about 5 cm/sec. With normal inclination of the front ($\tan \theta_d = 1/100$) $v'_x - v_x$ would then be about 5 m/sec. To obtain the magnitude of term C in Eq. (4) we assume that $v'_x - v_x = 0$ initially, and that after 12 hours $v'_x - v_x = 5$ m/sec. Thus

$$\frac{dv_x'}{dt} = \frac{dv_x}{dt} + \frac{5}{43{,}200} \text{ m/sec}^2.$$

Assuming, as before, that $T = 280°$A and $T' = 290°$A, and neglecting terms of second order, we obtain the values given in Table II, which apply to positive accelerations. If the accelerations were negative, the sign would be reversed. The values given in Table II

TABLE II

Value of term $C = \dfrac{\rho \dfrac{dv_x}{dt} - \rho' \dfrac{dv_x'}{dt}}{2\Omega_z(\rho v_y - \rho' v_y')}$

v_y' m/sec \ v_y m/sec	12	10	8	6	4	2	0
−12	0.05	0.05	0.06	0.07	0.07	0.08	0.10
−10	0.05	0.06	0.07	0.07	0.08	0.10	0.12
−8	0.06	0.06	0.07	0.08	0.10	0.12	0.15
−6	0.06	0.07	0.08	0.10	0.12	0.15	0.20
−4	0.07	0.08	0.10	0.12	0.15	0.20	0.30
−2	0.08	0.10	0.12	0.14	0.19	0.29	0.60
0	0.10	0.11	0.14	0.19	0.29	0.57	
+2	0.11	0.14	0.19	0.28	0.56		
+4	0.14	0.19	0.28	0.54			
+6	0.19	0.27	0.52				
+8	0.27	0.51					
+10	0.49						

* See Petterssen [4].

refer to rather extreme conditions. This is particularly true in all cases of slight shear along the front where the accelerational influence may amount to 50 per cent of the Margules inclination. In all cases with moderate or strong shear, the accelerational influence is less than 20 per cent, and the Margules formula gives a sufficiently accurate approximation to the true slope of the frontal surface.

(3) *The vertical velocity term.* This term vanishes when the front is orientated east-

TABLE III

Value of term $A = -\frac{\Omega_y}{\Omega_z}\frac{\rho v_z - \rho' v_z'}{\rho v_y - \rho' v_y'}$

v_y'm/sec \ v_ym/sec	12	10	8	6	4	2	0
−12	0.01	0.01	0.01	0.01	0.01	0.01	0.02
−10	0.01	0.01	0.01	0.01	0.01	0.02	0.02
− 8	0.01	0.01	0.01	0.01	0.02	0.02	0.03
− 6	0.01	0.01	0.01	0.02	0.02	0.03	0.03
− 4	0.01	0.01	0.02	0.02	0.02	0.03	0.05
− 2	0.01	0.02	0.02	0.02	0.03	0.05	0.10
0	0.02	0.02	0.02	0.03	0.05	0.10	
+ 2	0.02	0.02	0.03	0.05	0.09		
+ 4	0.02	0.03	0.05	0.09			
+ 6	0.03	0.05	0.09				
+ 8	0.05	0.09					
+10	0.08						

west because Ω_y is then zero regardless of the vertical velocity. To obtain an estimate of the maximum value of this term, we shall assume that

$$v_z' = -v_z = 10 \text{ cm/sec}$$

and that the other quantities have the same values as in the previous tables. Table III shows the maximum magnitudes of the term A. It will be seen that the influence of

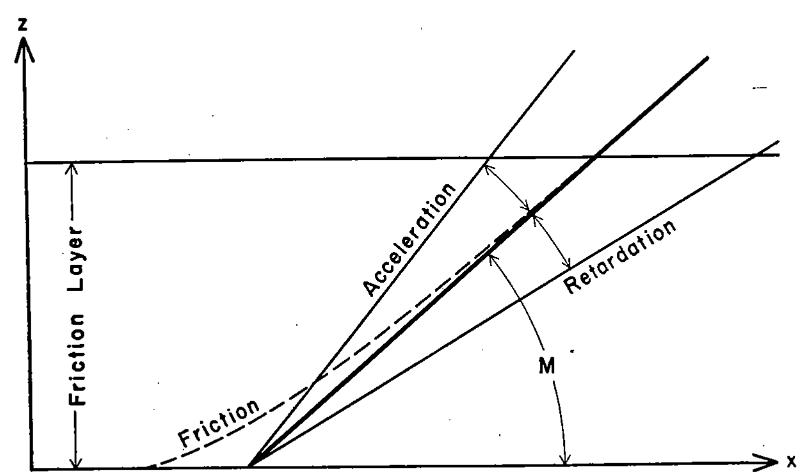

FIG. 2. Showing diagrammatically the influence of acceleration, retardation and friction on the inclination of a frontal surface. M indicates the inclination corresponding to the Margules formula.

vertical velocity on the inclination of frontal surfaces is altogether negligible in comparison to the term depending upon the acceleration.

Fig. 2 shows diagrammatically the influence of friction, acceleration and retardation on the inclination of a frontal surface in middle or high latitudes.

(4) *Variation with latitude.* Since Ω_z appears in the denominator of the terms A, B and C in Eq. (4), it follows that, other conditions being equal, these terms have their numerical minimum values at the poles and increase with decreasing latitude. Table IV shows the ratio of the values of the terms A, B and C in various latitudes to the values in latitude 45°. It will be seen that the ratio increases very rapidly below 30°. Since the

TABLE IV

Latitude	10	20	30	45	60	90
Ratio	4.0	2.0	1.4	1.0	0.8	0.7

accelerational term in Eq. (4) is much larger than the other terms, it is evident that in low latitudes this term will be as large as, or larger than, the Margules term. When the shear at the front is small, the slope will, in low latitudes, be controlled mainly by the accelerational term.

4. Fronts in Relation to Wind

It follows from the discussion in the foregoing paragraph that, in middle and high latitudes, the inclination of a frontal surface is mainly controlled by the geostrophic balance as expressed by the Margules formula. In all normal cases we may therefore put

$$\tan\theta_d = \frac{\dfrac{\partial p}{\partial x} - \dfrac{\partial p'}{\partial x}}{g(\rho - \rho')} = \frac{2\Omega_z(\rho v_y - \rho' v_y')}{g(\rho - \rho')} \tag{1}$$

or

$$\frac{\partial p}{\partial x} - \frac{\partial p'}{\partial x} = 2\Omega_z(\rho v_y - \rho' v_y'). \tag{2}$$

Although the accelerational term may influence the slope, it does not affect the sign of the inclination in middle and high latitudes. Furthermore, it was shown in Par. 2 that

$$\frac{\partial p}{\partial x} - \frac{\partial p'}{\partial x} > 0.$$

It follows then from Eq. (1) that

$$\rho v_y - \rho' v_y' > 0 \text{ (in the northern hemisphere)}$$

which means that there must be cyclonic shear of the specific momentum at a front. From this, it can be shown that the wind shear must be cyclonic at the front. Suppose that there is no wind shear at the front; then

$$v_y = v_y'$$

and Eq. (1) reduces to

$$\tan \theta_d = 2\Omega_z \frac{v_y}{g}$$

independent of the density discontinuity. In latitude 45°N

$$\frac{2\Omega_z}{g} \approx 10^{-5}$$

and to obtain a slope as small as 1/1000, v_y would have to be 100 m/sec. Hence, for fronts with slopes larger than 1/1000, v_y must be greater than v_y'; that is, the wind shear must be cyclonic at the front.

5. Summary of Frontal Characteristics

It has been shown in the foregoing paragraphs that all fronts are characterized by the following conditions:

(a) a zero order discontinuity in density, specific volume, temperature and potential temperature;

(b) a first order discontinuity in pressure as expressed by the inequality

$$\frac{\partial p}{\partial x} - \frac{\partial p'}{\partial x} > 0;$$

(c) a zero order discontinuity in specific momentum, i.e.

$$\rho v_y - \rho' v_y' > 0;$$

(d) a zero order discontinuity in wind, or a cyclonic wind shear, as expressed by

$$v_y - v_y' > 0.$$

From the derivation of the above criteria, it is evident that the density discontinuity is a fundamental front characteristic inasmuch as it expresses the field of solenoids within the frontal zone. The criteria (b)–(d) are partly dependent on the density discontinuity and partly upon the inclination of the frontal surface. The cyclonic vorticity at a front is therefore not uniquely determined by the density discontinuity, and should be regarded as a fundamental front characteristic. The process of frontogenesis is therefore one that tends to create a density discontinuity as well as cyclonic wind shear. Whereas the kinematical theory of frontogenesis applies to the creation of a density discontinuity, the aim of the following paragraphs will be to investigate the processes which lead to the formation of cyclonic vorticity at a front.

CHAPTER II. FRONTS AND VORTICITY

Since cyclonic wind shear is an important front characteristic, and since it is the only unstabilizing factor in the creation of cyclone waves, it is of particular interest to investigate the processes that tend to create wind shear, or vorticity, along fronts. If vorticity were a conservative property, the process would be a kinematical one, and the kinematical theory, which applies to the field of density (or temperature) would hold also for the vorticity field. The wind shear at the front would then be created through horizontal transport of the available vorticity lines towards the front. It is, therefore, necessary to discuss the conservatism of vorticity in relative motion.

6. Vorticity Expressions

Let $\boldsymbol{v}$ denote the wind velocity, and $\boldsymbol{q}$ the vorticity. Then

(1)
$$\boldsymbol{q} = \text{curl } \boldsymbol{v} = \nabla \times \boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

the vertical component of which is

(2)
$$q_z = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \cdot$$

In a polar system of coordinates

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$$

where $\boldsymbol{r}$ is the radius of curvature of the stream lines, and $\boldsymbol{\omega}$ is the angular velocity. The vorticity is then expressed by

(3)
$$\boldsymbol{q} = \nabla \times (\boldsymbol{\omega} \times \boldsymbol{r}) = 2\boldsymbol{\omega} - (\nabla \cdot \boldsymbol{\omega})\boldsymbol{r} + \boldsymbol{r} \cdot \nabla \boldsymbol{\omega}$$

and the vertical component is

(4)
$$q_z = \frac{v}{r} + \frac{\partial v}{\partial r} \cdot$$

We consider now the front as a zone of transition and choose the x-axis normal to the front pointing toward the colder air. The term $\partial v_x/\partial y$ in Eq. (2) is then small and vanishes if the conditions are uniform along the frontal zone.

The term $\partial v_y/\partial x$ represents the wind variation in the direction normal to the frontal zone, or the wind shear. Therefore, along a front, the vertical component of the vorticity expresses the horizontal wind shear.

7. Conservatism of Vorticity

The equations of three-dimensional motion may be written as

$$\frac{d\boldsymbol{v}}{dt} = -\alpha \nabla p - 2\boldsymbol{\Omega} \times \boldsymbol{v} - \nabla \phi$$

where ϕ denotes geopotential. The curl of the acceleration is thus:

$$\nabla \times \frac{d\boldsymbol{v}}{dt} = \nabla\alpha \times (-\ \nabla p) - 2\nabla \times (\boldsymbol{\Omega} \times \boldsymbol{v})$$

$$= \nabla\alpha \times (-\ \nabla p) + 2\boldsymbol{\Omega}\cdot \nabla \boldsymbol{v} - 2\boldsymbol{\Omega}(\nabla \cdot \boldsymbol{v}\ .$$

Since

$$\nabla \times \frac{d\boldsymbol{v}}{dt} = \frac{\partial}{\partial t}(\nabla \times \boldsymbol{v}) + \nabla \times (\boldsymbol{v}\cdot \nabla \boldsymbol{v})$$

$$\boldsymbol{q} = \nabla \times \boldsymbol{v}$$

$$\nabla \cdot \boldsymbol{q} = 0$$

we obtain after rearranging the terms

$$\text{(1)} \qquad \frac{d\boldsymbol{q}}{dt} = \nabla\alpha \times (-\ \nabla p) + (\boldsymbol{q} + 2\boldsymbol{\Omega})\cdot \nabla \boldsymbol{v} - (\boldsymbol{q} + 2\boldsymbol{\Omega})\nabla\cdot \boldsymbol{v}$$

which expresses the increase per unit time of the vorticity of an individual unit of air. This equation is the equivalent of Helmholtz's vorticity equation, with the exception that the latter applies to absolute motion while Eq. (1) refers to relative motion.

The first term on the right-hand side represents the number of isobaric-isosteric solenoids contained in a unit area normal to the isobaric and isosteric surfaces. The two last terms represent an inertia effect. Since we are here concerned with the increase in the vertical component of vorticity, we multiply Eq. (1) scalarly by the vertical unit vector $\boldsymbol{k}$, and obtain after some vector transformations:

$$\text{(2)} \qquad \frac{dq_z}{dt} = N_h + (q_x + 2\Omega_x)\frac{\partial v_z}{\partial x} + (q_y + 2\Omega_y)\frac{\partial v_z}{\partial y} - (q_z + 2\Omega_z)\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right).$$

Here, N_h denotes the number of solenoids contained in a unit horizontal area. This term may create or destroy vorticity, depending on whether the circulation around the solenoids is positive or negative.

The term

$$(q_x + 2\Omega_x)\frac{\partial v_z}{\partial x} + (q_y + 2\Omega_y)\frac{\partial v_z}{\partial y}$$

may be written as

$$(\boldsymbol{q}_h + 2\boldsymbol{\Omega}_h)\cdot \nabla_h v_z$$

where subscripts h indicate the horizontal components. This term expresses the effect of the distribution of the vertical velocity in the horizontal plane. It is positive when the vector $\boldsymbol{q}_h + 2\boldsymbol{\Omega}_h$ has a component in the direction of increasing vertical velocity, and negative when it has a component in the opposite direction. The term vanishes at the earth's surface except in singular lines or points in the horizontal motion. It will be shown later that the term may become quite important within the frontal zones.

Finally, the term

$$- (q_z + 2\Omega_z)\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right)$$

represents the effect of horizontal divergence and convergence. Since $q_z+2\Omega_z$ is always positive, it follows that convergence tends to increase and divergence tends to decrease vorticity.

If vorticity were a conservative property

$$\frac{dq_x}{dt} = \frac{dq_y}{dt} = \frac{dq_z}{dt} = 0.$$

It is obvious from Eq. (2) that, in general, the vertical component of vorticity can be conservative only when (a) there are no solenoids in the horizontal plane; (b) the horizontal distribution of vertical velocity is uniform; and (c) there is no horizontal convergence or divergence. Particularly within frontal zones, none of these conditions are fulfilled, and vorticity must be regarded as a non-conservative property.

Before we proceed to discuss the order of magnitude of the terms in Eq. (2), it is of interest to remark that Rossby [5] has introduced a vorticity expression, called potential vorticity, which, under certain circumstances, may be conservative. The expression derived by Rossby is:

$$q_{z_0} = \frac{2\Omega_z + q_z}{D/D_0} - 2\Omega_{z_0}.$$

Where D is the thickness of the air column and letters with subscripts zero refer to the initial conditions and letters without subscripts zero refer to later conditions. Potential vorticity, however, is conservative only in the case of autobarotropy, and although it may be a useful quantity in the discussion of homogeneous air masses, its application to frontal phenomena is largely restricted on account of the underlying assumptions.

8. Factors Influencing the Horizontal Shear at Frontal Surfaces

We shall now return to Eq. (2) in the foregoing paragraph and discuss in some detail the relative importance of the terms that affect the increase or decrease of shear in frontal zones. We choose the x-axis normal to the front pointing from the warm toward the cold air, and we assume that the conditions within the frontal zone are such that the variation along the frontal zone is negligible against the much larger variations normal to the frontal zone. Then

$$\frac{\partial v_y}{\partial y} = \frac{\partial v_z}{\partial y} = 0$$

and Eq. 7(2) reduces to

$$\frac{dq_z}{dt} = N_h + (q_x + 2\Omega_x)\frac{\partial v_z}{\partial x} - (q_z + 2\Omega_z)\frac{\partial v_x}{\partial x} \tag{1}$$

or

$$\frac{dq_z}{dt} = N_h + S$$

when S denotes the sum of the inertia terms.

(1) *The solenoid term:* To obtain an idea of the sign and the magnitude of this term, we consider Fig. 3, which represents diagrammatically the conditions at a warm and a

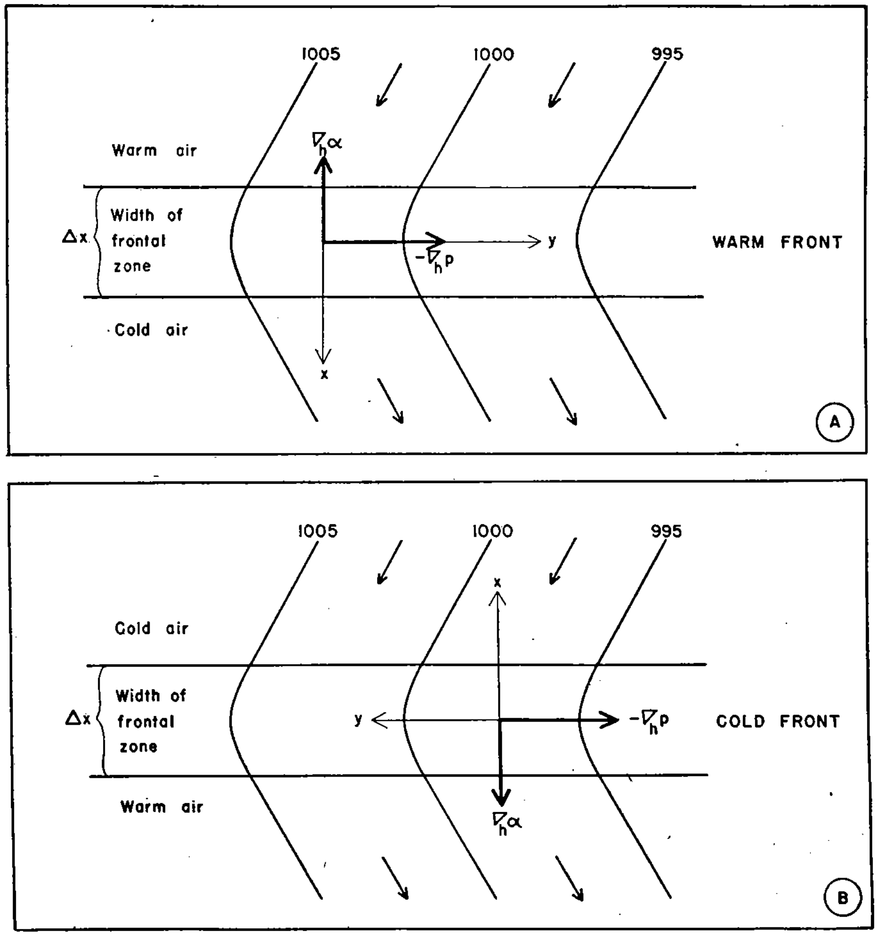

FIG. 3

cold front. In both cases, we assume that the temperature variation along the front is negligible against the temperature variation normal to the front. In the case of a warm front (Fig. 3A) the horizontal ascendant of specific volume (i.e., $\nabla_h\alpha$) points along the negative x-axis, while the horizontal pressure gradient points along the positive y-axis. The circulation around the solenoids is then negative and the density gradient within the frontal zone tends to decrease the vorticity at a warm front.

In the case of a cold front (Fig. 3B) $-\nabla_h p$ points along the negative y-axis. The circulation around the solenoids is now positive and the density gradient within the frontal zone tends to increase the cyclonic shear at the front. Therefore, other conditions being equal, cold fronts tend to develop stronger wind shear than do warm fronts, a fact that is corroborated by observations.

To obtain an estimate of the magnitude of the solenoid term, we consider the number of solenoids within a horizontal area $\Delta x \Delta y$, where Δx is the width of the front, and $\Delta y = 1$. We assume that the temperature in the warm air is 290°A. Table V then gives the number of solenoids contained within a horizontal strip of unit width through the frontal zone for various temperature differences (ΔT) and pressure gradients normal to the front and normal sea level pressure.

TABLE V

RATE OF INCREASE IN SHEAR, TIMES 10^5, PER SECOND^2 DUE TO THE SOLENOID TERM: COLD FRONTS (+) AND WARM FRONTS (−).

$\frac{\partial p}{\partial y}$	Temperature Difference (ΔT)				
	2°C	4°C	6°C	8°C	10°C
5 mb/100 km	±2.87	±5.74	±8.61	±11.48	±14.35
5 mb/200 km	±1.44	±2.87	±4.31	± 5.74	± 7.18
5 mb/400 km	±0.72	±1.44	±2.15	± 2.87	± 3.59
5 mb/600 km	±0.48	±0.96	±1.44	± 1.91	± 2.39
5 mb/800 km	±0.36	±0.72	±1.08	± 1.44	± 1.79
5 mb/1000 km	±0.29	±0.57	±0.86	± 1.15	± 1.44

It will be seen from Table V that, if the temperature difference were 6°C and the pressure gradient along the front 5 mb/400 km, the shear at a warm front would decrease by 2 m/sec in 24 hours, and the shear at a cold front would increase by the same amount. Although it is observed that cold fronts develop stronger shear than do warm fronts, it is evident that the other terms in Eq. (1) must be more important than the solenoid term, because, otherwise, cyclonic shear could not be maintained at warm fronts.

(2) *The inertia terms:* The last two terms in Eq. (1) may be re-arranged as follows. Since Ω_x vanishes when the x-axis is along a latitude circle, and since the x-axis is chosen normal to the front, the magnitude of Ω_x will depend on the orientation of the front. Let β denote the angle from the north to the positive y-axis (which is tangent to the front). We then obtain, at latitude ϕ,

(2) $$\Omega_x = -\Omega \cos \phi \sin \beta.$$

Furthermore

(3) $$\Omega_z = \Omega \sin \phi.$$

From the kinematical boundary condition we obtain

$$v_z - v_z' = (v_x - v_x') \tan \theta_d.$$

Within the frontal zone the mean values of $\partial v_z/\partial x$ and $\partial v_x/\partial x$ may be expressed as follows

(4) $$\frac{\partial v_z}{\partial x} = \frac{v_z - v_z'}{\Delta x} \quad \text{and} \quad \frac{\partial v_x}{\partial x} = \frac{v_x - v_x'}{\Delta x}$$

where Δx is the width of the frontal zone. This gives

$$\frac{\partial v_z}{\partial x} = \frac{v_x - v_x'}{\Delta x} \tan \theta_d. \quad (5)$$

Substituting these expressions into Eq. (1), we obtain for the sum of the inertia terms

$$S = [(q_x \tan \theta_d - q_z) - 2\Omega(\cos \phi \sin \beta \tan \theta_d + \sin \phi)] \frac{v_x - v_x'}{\Delta x}. \quad (6)$$

It is readily seen that the term $(q_x \tan \theta_d - q_z)$ is negligible. We have

$$q_x = \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}.$$

Since the conditions are uniform along the y-axis, we obtain

$$q_x = -\frac{\partial v_y}{\partial z} = -\frac{v_y'' - v_y}{\Delta z}$$

where v_y'' refers to the point C in Fig. 4. Similarly

$$q_z = \frac{\partial v_y}{\partial x} = \frac{v_y - v_y'}{\Delta x}.$$

Unless v_y varies rapidly in the warm air from A to C, we may put

$$v_y' \approx v_y''.$$

Therefore

$$q_x \tan \theta_d = q_z$$

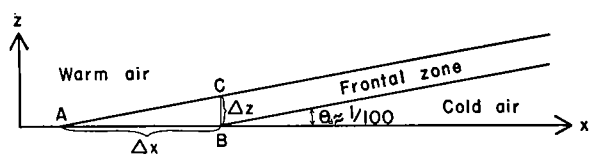

Fig. 4

and the first term on the right-hand side of Eq. (6) vanishes. This simplification holds particularly above the layer directly influenced by friction where the wind variation with height is slight. Since the later discussion will be restricted to the conditions above the friction layer, we may neglect the term containing q_x and q_z. The inertia term then reduces to:

$$S = -2\Omega(\cos \phi \sin \beta \tan \theta_d + \sin \phi) \frac{v_x - v_x'}{\Delta x}. \quad (7)$$

Summing S over the width of the frontal zone, we obtain:

$$S = -2\Omega(\cos \phi \sin \beta \tan \theta_d + \sin \phi)(v_x - v_x'). \quad (8)$$

To obtain an estimate of the magnitude of this term, we consider a front in latitude 45°N. Here

$$2\Omega \cos \phi = 2\Omega \sin \phi \approx 10^{-4}.$$

Substituting into Eq. (8) we obtain:

$$S = -10^{-4}(\sin \beta \tan \theta_d + 1)(v_x - v_x'). \quad (9)$$

Normally tan θ_d is about 1/100 and the first term within the parentheses is negligible against unity. In such cases, Eq. (9) reduces to:

(10) $$S = -10^{-4}(v_x - v_x').$$

Table VI gives the value of S for fronts with normal slope.

TABLE VI

$v_x' - v_x$(m/sec)	± 2	± 4	± 6	± 8	± 10	± 12	± 14
$S \times 10^6$	±20	±40	±60	±80	±100	±120	±140

Because the sign of S is determined by the sign of $v_x' - v_x$, it follows that in the case of cold fronts there will be an increase in shear, provided that the x-component of the velocity in the cold air is numerically greater than the x-component of the velocity in the warm air. Likewise, increasing shear at warm fronts requires that

$$v_x' > v_x.$$

By comparing Table VI with Table V we see that the inertia term, in general, is much larger than the solenoid term. Consequently, with a front of normal slope, the rate of increase of shear is chiefly determined by the differences in the x-components of the velocity, or the convergence within the frontal zone.

From the above and the kinematic boundary condition, it follows that there will be increasing shear at the front only when

$$v_z' > v_z,$$

i.e., when the warm air ascends relative to the cold.

With fronts of steep slope, e.g., steep cold fronts, the term sin β tan θ_d in Eq. (9) may become important. This term has its maximum effect when the front is orientated east-west with the cold air to the north. Then

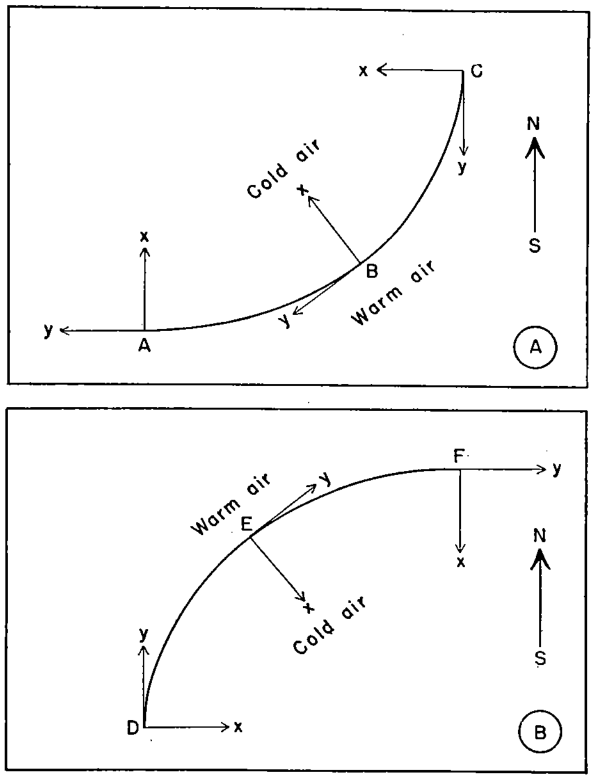

FIG. 5

$$\sin\beta \tan\theta_d = -\tan\theta_d.$$

It follows from Eq. (9) that the rate of increase of shear will be less than in Table VI, due to the steepness of the frontal surface. Should the warm air be north of the front, then sin β tan θ_d = tan θ_d and the rate of increase of shear will exceed that given in Table VI. When the front is orientated north and south, this term is zero. Only when the front is very steep can $\sin\beta \tan\theta_d$ change the sign of the inertia term. This will occur when

$$\tan\theta_d > -\frac{1}{\sin\beta}.$$

Fig. 5A shows the normal case of fronts with cold air to the north and warm air to the south. In rare cases, the cold air may be to the south of the warm air, as in Fig. 5B. Table VII gives the limiting values of θ_d when the inertia term vanishes. The letters in Table VII refer to the points in Fig. 5.

TABLE VII

Point	*A*	*B*	*C*	*D*	*E*	*F*
θ_d	45°	55°	90°	90°	125°	135°

It will be seen from Table VII that, when the slope of a cold front is very large, the inertia term becomes unimportant, and when the slope is excessively large, the term is negative. It is of interest to note that cold fronts with strong shear and with slopes of the magnitudes indicated in Table VII actually occur, particularly in connection with line squalls. It should be noted, however, that these fronts are steep only within the friction layer, where friction tends to reduce the amount of shear.

(3) *Conclusions:* From the above discussion, it follows that:

(a) Vorticity is not a conservative property, and its variation is mainly due to divergence, and only to a lesser degree due to solenoids.

(b) The inertia term is positive at fronts with normal slope, provided that the warm air ascends relative to the cold. The inertia term is therefore normally positive along the lower portion of warm and cold fronts, but is usually negative along the upper portions of frontal surfaces.* The upper portion of frontal surfaces is therefore accompanied by slight wind shear.

(c) The inertia term is generally larger than the solenoid term at fronts with normal slope.

(d) The solenoid term is negative at warm fronts and positive at cold fronts, with the result that cold fronts tend to develop stronger wind shear than do warm fronts.

(e) The inertia term depends on the slope and the orientation of the frontal surface and may become negative at very steep fronts, even when the warm air ascends relative to the cold air.

(f) The effect of friction is to decrease the shear at all fronts.

* For a summary of types of fronts, see Petterssen [4].

CHAPTER III. SYNOPTIC ASPECTS

The aim of this chapter is to show how the vorticity field can be evaluated from ordinary weather maps, and to present some examples of synoptic situations in order to test the foregoing conclusions. Besides the regular weather maps, vorticity maps have been constructed to show the creation of vorticity associated with increasing front intensity. Only a small number of stations have been plotted on the reproduced maps, although the actual computations of vorticity were based on a very dense network of stations.

9. Evaluation of the Vorticity Field

The vertical component of the vorticity can be obtained either by using the actually observed winds or by computing the gradient wind from the pressure distribution. The latter method was adopted, since the wind observations often are non-representative, and since it is synoptically more significant to map the conditions at the top of the friction layer than at the earth's surface.

TABLE VIII

VALUES OF THE CYCLOSTROPHIC TERM FOR CYCLONIC CURVATURE

(All values are negative)

Radius of Curvature in Kilometers		10^4	5×10^3	2×10^3	$1{\cdot}5\times10^3$	10^3	8×10^2	6×10^2	4×10^2	2×10^2	10^2	50
Geostrophic wind m/sec	Latitude degrees											
5	25	—	0.1	0.2	0.2	0.3	0.4	0.6	0.7	1.1	1.7	2.3
	35	—	—	0.1	0.2	0.3	0.4	0.5	0.6	1.0	1.5	2.1
	45	—	—	0.1	0.2	0.2	0.3	0.4	0.5	0.8	1.3	1.9
	55	—	—	0.1	0.1	0.2	0.3	0.3	0.4	0.7	1.2	1.8
10	25	0.2	0.5	0.8	1.0	1.3	1.5	1.8	2.4	3.5	4.6	5.7
	35	0.1	0.4	0.5	0.7	1.0	1.2	1.5	2.0	3.0	4.1	5.2
	45	0.1	0.3	0.5	0.6	0.8	1.0	1.2	1.7	2.7	3.8	4.9
	55	—	0.2	0.4	0.6	0.8	0.9	1.0	1.5	2.4	3.5	4.7
15	25	0.4	0.8	1.5	1.8	2.5	3.0	3.6	4.5	6.1	8.0	9.6
	35	0.3	0.6	1.1	1.5	2.1	2.4	3.0	3.8	5.4	7.2	8.8
	45	0.3	0.5	0.9	1.3	1.7	2.1	2.6	3.4	4.9	6.7	8.4
	55	0.2	0.4	0.8	1.2	1.5	1.9	2.3	3.1	4.5	6.3	8.1
20	25	0.6	1.2	2.5	3.0	4.2	4.7	5.6	6.8	9.3	11.6	13.6
	35	0.5	1.0	1.9	2.4	3.4	3.9	4.8	5.9	8.3	10.4	12.7
	45	0.5	0.8	1.6	2.2	2.9	3.4	4.2	5.1	7.5	9.7	12.0
	55	0.5	0.6	1.4	2.0	2.6	3.1	3.8	4.7	6.9	9.4	11.7
25	25	1.0	1.9	3.7	4.5	5.9	6.6	7.9	9.6	12.5	15.3	17.6
	35	0.8	1.4	2.8	3.6	4.9	5.6	6.7	8.3	11.3	14.1	16.5
	45	0.7	1.1	2.5	3.2	4.2	4.9	6.0	7.4	10.4	13.3	15.8
	55	0.6	0.9	2.2	2.8	3.9	4.5	5.4	6.9	9.7	12.7	15.4
30	25	1.5	2.4	4.9	6.0	7.9	9.0	10.4	12.5	15.9	19.1	21.8
	35	1.0	1.8	4.0	4.9	6.6	7.6	8.9	10.9	14.4	17.7	20.7
	45	0.9	1.6	3.4	4.4	5.8	6.7	8.0	9.9	13.4	16.8	19.8
	55	0.7	1.4	3.0	3.8	5.4	6.0	7.3	9.1	12.6	16.2	19.3

The vertical component of the vorticity is expressed by Eq. 6(4), viz.,

$$q_z = \frac{v}{r} + \frac{\partial v}{\partial r} \tag{1}$$

where v now denotes the gradient wind. The method of computing q_z from the pressure field may be described briefly as follows:

(a) Isobars were drawn for each 2.5 mb. By means of an appropriate wind scale, the geostrophic wind was evaluated for about 200 points within the area under investigation.

(b) The radius of curvature of the isobars was measured at each point. Because the gradient wind coincides with the isobars, the radius of curvature of the isobars can be substituted for the radius of curvature of the streamlines at the gradient wind level.

(c) Since the cyclostrophic component of the wind depends on the radius of curvature of the trajectory (r_t), it was necessary to evaluate r_t. Instead of drawing trajectories, which is a tedious process, r_t was computed by the formula*

$$\frac{1}{r_t} = \frac{1}{r}\left(1 - \frac{c}{v}\cos\psi\right)$$

where c denotes the velocity of the pressure system, and ψ is the angle between the direc-

TABLE IX

VALUES OF CYCLOSTROPHIC TERM FOR ANTICYCLONIC CURVATURE

(All values are positive)

Radius of Curvature in Kilometers		10^4	5×10^3	2×10^3	1.5×10^3	10^3	8×10^2	6×10^2	4×10^2
Geostrophic wind m/sec	Latitude degrees								
5	25	—	0.1	0.2	0.3	0.6	0.6	1.0	2.0
	35	—	—	0.1	0.2	0.4	0.4	0.6	1.1
	45	—	—	0.1	0.2	0.3	0.4	0.5	0.8
	55	—	—	0.1	0.1	0.2	0.3	0.4	0.7
10	25	0.2	0.6	1.0	1.5	2.6	4.0	—	—
	35	0.1	0.3	0.7	1.0	1.6	2.2	3.8	—
	45	0.1	0.2	0.6	0.8	1.2	1.7	2.6	6.0
	55	0.1	0.2	0.5	0.7	1.0	1.3	2.0	4.2
15	25	0.4	0.9	2.5	3.8	11.0	—	—	
	35	0.3	0.7	1.7	2.4	4.6	7.8	—	
	45	0.3	0.6	1.3	1.9	3.3	4.7	10.7	
	55	0.2	0.5	1.1	1.5	2.6	3.6	6.3	
20	25	0.6	1.5	5.1	9.2	—	—		
	35	0.5	1.1	3.2	5.0	13.3	—		
	45	0.5	1.0	2.5	3.6	7.2	14.3		
	55	0.3	0.8	2.0	2.9	5.4	8.6		
25	25	1.1	2.6	9.9	—	—			
	35	0.9	1.8	5.6	9.5	—			
	45	0.8	1.4	4.2	6.4	17.9			
	55	0.6	1.0	3.3	5.0	10.5			
30	25	1.8	3.7	22.0	—				
	35	1.3	2.4	9.3	19.9				
	45	1.0	2.0	6.6	10.5				
	55	0.8	1.8	5.2	8.1				

* See Petterssen [4] p. 225.