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SHORT PERIOD VERTICAL OSCILLATIONS IN THE WESTERN BASIN OF THE NORTH ATLANTIC

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INTRODUCTION

Because of general interest in the subject of vertical oscillations in the sea and because such information is scanty for the ocean basins, an investigation of the question in the western North Atlantic was initiated by the establishment of "Atlantis" station 2639, July 9 to 13, 1936.

The significance of vertical oscillations in the sea has been known from the earlier work of Helland-Hansen and Nansen, and, in 1926, these authors summarized their conception of the problem as follows: "By earlier investigations we have found that there are probably considerable vertical oscillations of the water layers in various regions of the ocean. Hence the occasional vertical series of observations cannot be expected always to represent the average conditions at any particular station. It is therefore of great importance for the discussion of the general conditions in a sea-area on the basis of the observations made, to study how far these actual observations at the different stations and different depths may be regarded as representative." Also, in this same paper we find the statements: "It has already been mentioned that the oscillations described have obviously to a great extent some connection with the tides; but how the tidal wave can produce vertical movements of such dimensions in the different strata of the sea seems to us at present to be inexplicable. We have here a phenomena of fundamental importance to oceanography, which has to be made the subject of special methodical investigations."

The present discussion is based on the short period temperature, salinity, and oxygen variations which were observed to occur at various fixed levels at "Atlantis" station 2639 during a four and one quarter day period of observation, $9^h\ 30'$, July 9 to $15^h\ 45'$, July 13, 1936 (L.C.T.). In addition to a brief theoretical treatment, the results are here analyzed from a standpoint of practical oceanography, and, in so far as the observations warrant, the effect of short period oscillations on temperature, salinity, and oxygen distribution is discussed with certain of the more prominent results generally deduced from these factors.

It is necessary to bear in mind that the tentative conclusions from this investigation are based on the results of continuous observations that extended over a period of little more than four days, and frequently the details are confined to the more complete set of observations taken in the upper twelve hundred meters during the final 24 hour period, July 12 to 13, so that they are applicable to oceanographic investigations only in a very general way. It seems that while short period oscillations appear to be correlated with daily and half daily tidal periods, there are other factors which disturb the periodic recurrence of the observed phenomena in varying degrees of intensity. Hence, the results obtained during a twenty-four lunar hour period (the principle period considered for purposes of analysis) are not necessarily applicable to those of any other day. However, while the degree and nature of the variance with time is not known, we are not prevented from drawing certain qualitative conclusions for the region represented by station 2639, and, in some cases, the conclusions may be extended to the ocean as a whole. In this instance may be mentioned especially the order of magnitude of the effect of vertical displacements of the water column on dynamic calculations of total transport between two fixed verticals in the sea, as are generally calculated from temperature and salinity observations, and the discussion and tabulation of the probability of occurrence of various degrees of difference between random oxygen observations at the same fixed levels of the same region.

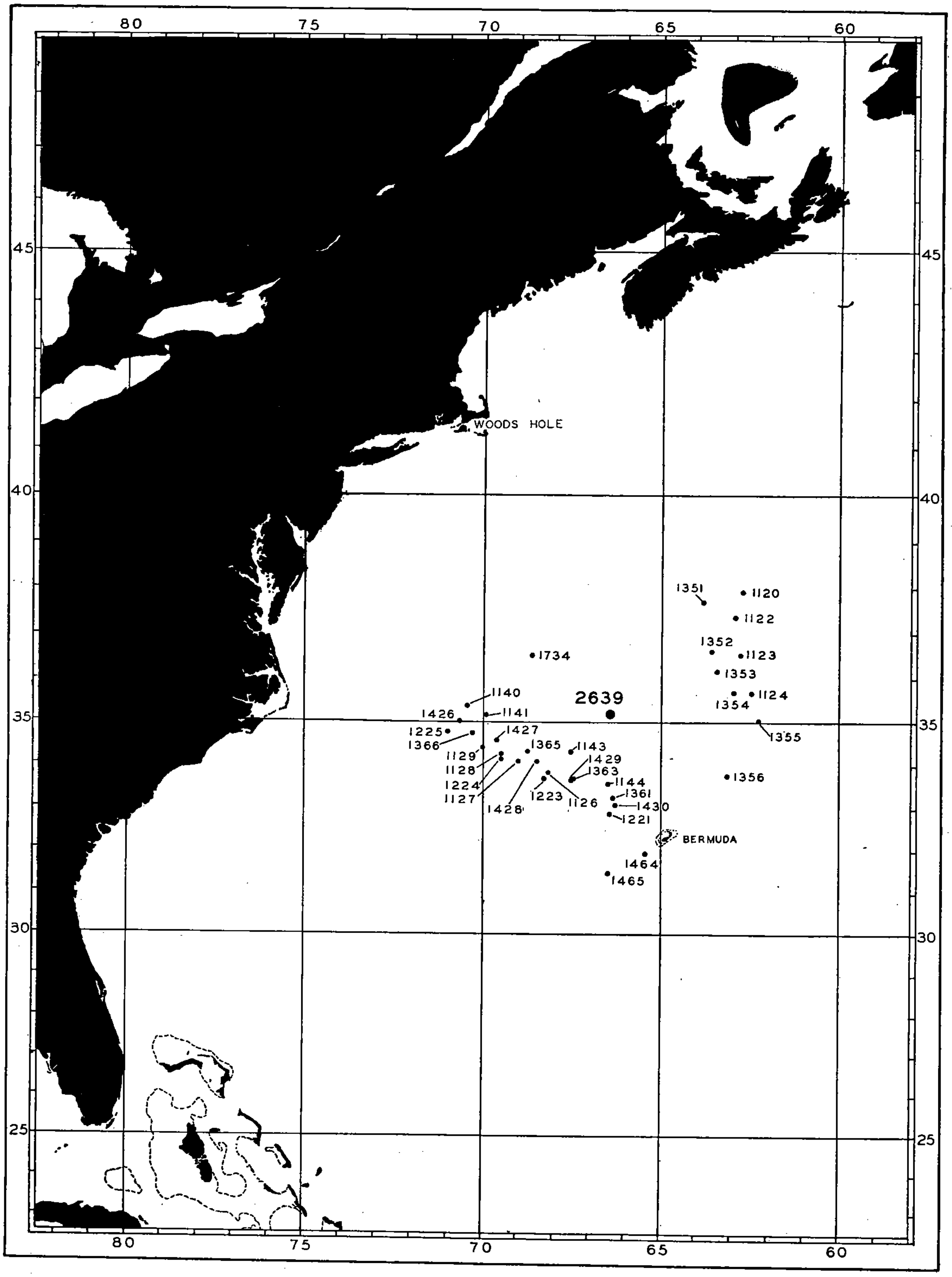

FIG. 1.—Location of "Atlantis" oceanographic stations referred to in this report.

THE STATION

For purposes of this investigation it was desirable to locate a station where horizontal gradients of hydrographic characteristics of the water mass should be as small as possible for considerable distance in all directions. Such a condition was attained by locating station 2639 at a position about 180 miles northwest of Bermuda (mean position: 66° 25′W, 35° 07′N); this being somewhat more than 150 miles from the eastern edge of the American coastwise convergence (Fig. 1). Horizontal gradients of the fundamental characteristics (temperature, salinity, oxygen, etc.) of the water are very small (page 16) and isolines representing these properties maintain relatively level positions in the oceanographic sections.[1]

The total period of actual operation at this station extended from 9^h 30′ on July 9 to 15^h 45′ on July 13, 1936. During this time the "Atlantis" was not anchored and occasional steaming back to position was necessary. Checks on the amount and direction of the ship's drift (aided by good weather conditions) were maintained by frequent astronomical observations by the ship's officers. Thirty-eight of the forty-four individual lowerings of water bottles and thermometers (90%) were made within a five mile radius of the mean position of the station. Of the remaining six lowerings, the three comprising series C and the first lowering of series D are located approximately 10 and 16 miles, respectively, northwest of the mean position, while series U and V (one lowering each) are located 7½ and 11 miles, respectively, to the north. The drift of "Atlantis" during the period of observation is summarized in table 1.

TABLE 1

Date July 1936	Time Interval		ΔT	ΔD	Direction	$\frac{\Delta D}{\Delta t}$	Lowerings
9	$08^h40'$	$12^h00'$	3.3	1.25	75°W	0.38	A_1, A_2, A_3
9	$18^h40'$	$20^h13'$	1.6	?	75°W	?	B_1, B_2, B_3
9–10	$20^h13'$	$04^h50'$	8.6	5.75	77°W	0.67	C_1, C_2, C_3
10	$04^h50'$	$06^h51'$	2.0	1.00	73°W	0.50	D_1
10	$09^h00'$	$12^h00'$	3.0	0.75	73°W	0.25	D_2
10	$12^h00'$	$16^h00'$	4.0	6.40	73°W	1.60	D_3
10	$17^h20'$	$20^h12'$	2.9	1.75	94°W	0.61	E_1, E_2
10–11	$20^h12'$	$04^h50'$	8.6	5.00	94°W	0.58	E_3, F_1, F_2, F_3
11	$04^h50'$	$06^h10'$	1.3	1.00	88°W	0.75	—
11	$07^h00'$	$12^h00'$	5.0	1.50	5°E	0.30	G_1, G_2, G_3
11	$13^h32'$	$20^h20'$	6.8	2.60	86°W	0.38	H_1, H_2, H_3
11–12	$20^h20'$	$04^h45'$	8.4	1.25	7°E	0.15	I_1, I_2, I_3, J_1, J_2
12	$04^h45'$	$08^h19'$	3.6	3.25	79°W	0.91	J_3
12	$08^h45'$	$12^h00'$	3.3	2.70	2°W	0.83	K_1, K_2
12	$12^h00'$	$15^h08'$	3.1	4.00	91°E	1.28	K_3
12	$15^h50'$	$20^h08'$	4.3	3.50	35°W	0.81	L, M
12–13	$20^h08'$	$04^h45'$	8.6	3.00	45°W	0.35	N, O, P, Q, R
13	$04^h45'$	$05^h43'$	1.0	0.60	13°W	0.60	—
13	$06^h10'$	$13^h14'$	7.1	11.1	7°E	1.57	S, T, U, V
13	$15^h28'$	$20^h20'$	4.9	4.80	N	0.98	W

Time is recorded as 60th meridian; to correct to L.C.T. subtract 26 minutes. ΔD is amount of drift in nautical miles; drift direction is measured east and west of north; drift speed is given in nautical miles per hour.

[1] Oceanographic sections and horizontal projections illustrating horizontal gradients of temperature, salinity, and oxygen in this locality are given by figures 14, 15, 17, 18, and 22 to 36 in Seiwell (1934).

METHODS

Water samples were obtained by means of Nansen type reversing water bottles to each of which were attached two Richter and Wieser reversing thermometers.

Serial Observations

The total time of actual operation at station 2639 was approximately four and one quarter days; during the first three and one quarter days the entire water column from surface to bottom (5090 meters) was sampled, but, during the final twenty-four hours, sampling was limited to the upper 1200 meters. The vertical distance between samples was approximately: 0 to 100 meters = 50 meters; 100 to 1400 meters = 100 meters; 1400 to 2000 meters = 200 meters; and 2000 to 5000 meters = 250 meters. To complete a sampling of the entire water column required three separate lowerings of the water bottles and thermometers for which about six hours were required; sampling of the upper 1200 meters was completed with one lowering of water bottles and thermometers and required only about two hours. Thus, within the first three and one quarter days, eleven complete samplings of the entire water column were made, and, for the final twenty-four hours, twelve samplings of the upper 1200 meters were obtained.

Two reversing thermometers were always attached to each water bottle and in each lowering three or four of the water bottles carried both protected and unprotected thermometers, spaced at appropriate intervals. The difference in readings of unprotected and protected thermometers for the same level is used for the calculation of the sampling depths.[2]

Probable Errors of the Observations

The probable error is so defined that the chances are even whether the deviation exceeds it in absolute magnitude or is less than it. Further, the probability that the error exceeds 2.4 times the probable error is 1/10, 3.8 times the probable error is 1/100, and 4.9 times the probable error is 1/1000.

The probable errors of the various determinations carried out on board "Atlantis" during the investigation were:

Oxygen = 0.03 cc/liter
Salinity = 0.02 ‰
Temperature = 0.01° to 0.02° (depending on the particular thermometer).

Determination of Exact Time of Sampling

For purposes of this investigation it was necessary to know the times at which the water bottles and thermometers actually sampled the water mass. This was determined from information obtained on board "Atlantis" which showed that for average wire angles ranging from 5° to 35°, between surface and 500 meters, the falling velocity of the messengers ranged from about 295 to 205 meters per minute; the decrease in falling velocity of the messenger being approximately 15 meters per minute for each 5 degrees the average angle of the hydrographic wire was increased.

The sampling time results recorded in table 13 are probably not in error by more than ±3 minutes.

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[2] For a discussion of this method see: Wüst, Böhnecke and Meyer (1932).

VARIATIONS OF TEMPERATURE AND OXYGEN AT STANDARD DEPTHS DURING THE 24 HOUR PERIOD[3] BEGINNING 15^h 34′, JULY 12, 1936

50 METERS

The 50 meter depth falls in the summer thermocline where a small vertical displacement of the water column will cause correspondingly large variations of temperature (Fig. 3) because of the large negative rate of change of temperature with increasing depth.

Temperature (Fig. 2). The first evident change in the course of the temperature time curve indicates that the minimum temperature (19.93°) occurred at about 18^h 00′ which was followed by a maximum (21.94°) at around 24^h, or approximately 6 hours later. Still later the temperature dropped to 20.21° at 4^h 00′, then fluctuated within 0.15° until 10^h 00′ when it rapidly dropped to a second minimum (19.54°) at 12^h, or about 12 hours after the preceding temperature maximum.

Oxygen. Analysis of oxygen variations with time at the 50 meter level are complicated by the occurrence of an oxygen maximum at a nearby depth (Fig. 4). During July 9 to 12, oxygen variations were inverse to temperature variations but the observation intervals were too wide to show details. In the more detailed twenty-four hour period, there was a general correspondence between the occurrence of oxygen maximum values and temperature minimum values and vice versa, although a certain amount of discrepancy is to be expected from the effects of biological activity (photosynthesis and respiration) at this level. It appears from the general correspondence of oxygen maxima and temperature minima that the normal position of the oxygen maximum was at a depth somewhat below 50 meters.

That increases in oxygen content, which corresponded in general with decreases in temperature, cannot be entirely the result of photosynthesis is shown by the fact that these increases occurred between the time of sunset and sunrise (indicated by the heavy lines, Fig. 2) as well as between the time of sunrise and sunset.

100 METERS

The 100 meter level represented the approximate lower limit of the summer thermocline, and the vertical variation of temperature in the overlying water was somewhat greater than that in the water below (Fig. 3).

Temperature (Fig. 2). At 100 meters only one distinct temperature minimum and one maximum occurred in 24 hours, both of which corresponded respectively to the time of the first minimum and maximum of the 50 meter layer. The temperature minimum (18.25°) occurred at 18^h 00′ and was followed by a temperature maximum (18.91°) about six hours later at 24^h 00′. Still later the temperature decreased to 18.50° at 4^h 00′ and then fluctuated by not more than 0.09° to the end of the observational period at 15^h 34′.

Oxygen. In general the temperature oxygen relationship at the 100 meter level was opposite to that recorded for 50 meters; a rising oxygen content corresponding to a rising temperature and vice versa. Thus, corresponding with the temperature minimum at 18^h 00′ and maximum at 24^h 00′ there were agreeing peaks and valleys in the oxvgen curve.

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[3] Local civil time of mean station position.

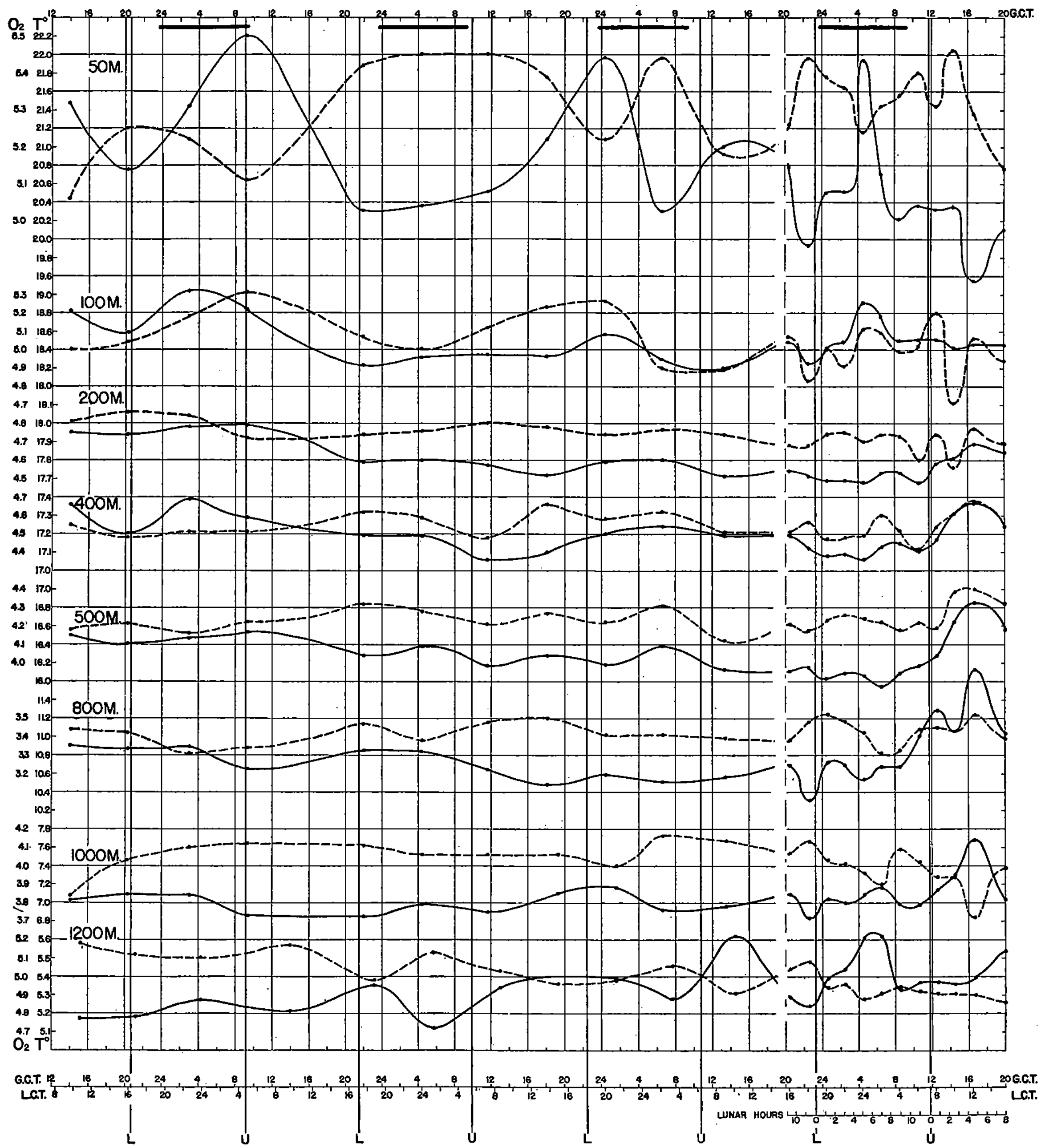

FIG. 2.—Time variation of temperature and oxygen for various depths at "Atlantis" station 2639 during 9^h 30′ July 9 to 15^h 45′ July 13, 1936 (local civil time), the final 24 hour observation period is more detailed (see text). Symbols *L* and *U* refer to times of lower and upper culminations of the moon. Temperature is indicated by full line, oxygen by broken line.

However, it is shown by the plotted data for the 100 meter level (Fig. 2) that oscillations of the oxygen curve may not agree necessarily with the temperature curve. On July 12, between 20^h and 22^h while the temperature increased from 18.41° to 18.48°, the oxygen decreased from 5.00 to 4.91 cc per liter; and between 4^h and 16^h (July 13) when the maximum temperature change was from 18.42° to 18.51°, the oxygen curve

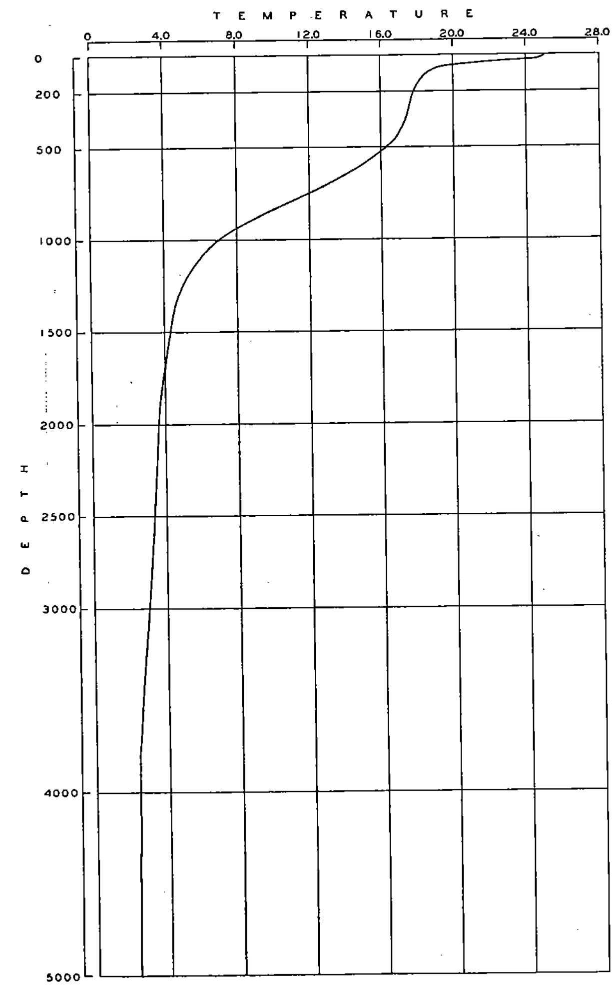

FIG. 3.—Mean vertical distribution of temperature at "Atlantis" station 2639.

passed through two complete well defined oscillations. If we may assume that vertical oscillations of the water column are indicated by temperature changes, as seems reasonable, then the oxygen nonconformities were likely the result of the biological influences at this depth.

200 METERS

The 200 meter level occurred below the summer thermocline and above the main thermocline; the water above and below was relatively homogeneous and vertical displacements produced less variation than at either the 50 or 100 meter depth.

Temperature. From a minimum value of 17.69° at 00ʰ 06′ on July 13 the temperature increased to a not well defined maxima of 17.74° in about three hours (3ʰ 18′). Still later at 6ʰ 06′ a second minimum of 17.68° was reached to be followed by a maximum of 17.89° at 12ʰ 18′. The time between the first minima and final maxima was a little more than twelve hours.

Oxygen. In general the rise and fall of the oxygen curve was similar to that of the temperature curve. Small irregularities occurred, but the courses of both curves were similar and the principal points of agreement were well defined.

400 Meters

The 400 meter depth marked the upper part of the main thermocline; the oxygen and temperature gradients being greatest in the underlying water.

Temperature. The first, but not well defined, temperature minimum (17.06°) occurred

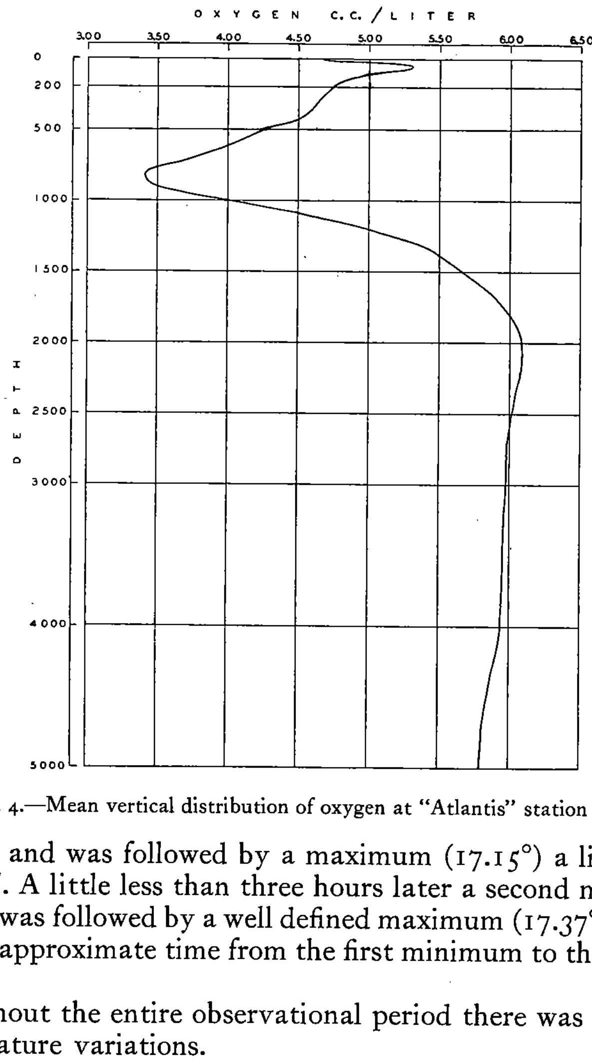

Fig. 4.—Mean vertical distribution of oxygen at "Atlantis" station 2639.

at 00ʰ 06′ (July 13) and was followed by a maximum (17.15°) a little more than three hours later at 3ʰ 18′. A little less than three hours later a second minimum (17.11°) occurred at 6ʰ 06′ and was followed by a well defined maximum (17.37°) at 12ʰ 06′, or about six hours later. The approximate time from the first minimum to the final maximum was twelve hours.

Oxygen. Throughout the entire observational period there was a close agreement of oxygen and temperature variations.

500 Meters

The 500 meter level laid within the principal thermocline.

Temperature. The first depression in the temperature time curve (16.03°) occurred at $19^h\ 30'$ (July 12) and was followed by a peak (16.10°) at $22^h\ 30'$ and then a second depression (15.95°) at $1^h\ 42'$ (July 13) and a second peak (16.86°) at $12^h\ 42'$. The intervals between peaks and valleys were approximately 3 and 11 hours, suggesting the presence of 6 and 24 hour periods.

Oxygen. The variations of oxygen and temperature were in agreement; small discrepancies occurred but their magnitude was not sufficient to alter the main trend.

800 Meters

The 800 meter level occurred in the principal thermocline.

Temperature. The first minimum (10.31°) was found at $18^h\ 00'$ (July 12) and was followed by a maximum (10.75°) about three hours later at $21^h\ 00'$. Three hours later a second minimum (10.53°) had occurred at $00^h\ 00'$ and twelve and one half hours later a second maximum (11.75°) at $12^h\ 30'$ (July 13).

Oxygen. Variations in the oxygen curve are complicated since the normal depth o the minimum oxygen content at this station was at about 850 meters (Fig. 4), and there was no particular correspondence between the trajectories of oxygen and temperature values.

1000 Meters

The 1000 meter level laid in the lower part of the principal thermocline.

Temperature. The course of the temperature curve indicates a minimum value (6.84°) at $18^h\ 30'$ (July 12) and about three and one half hours later a second minimum (7.00°) at $22^h\ 00'$ with a maximum about midway between. Still later the curve passed through a second maximum (7.17°) at about $2^h\ 00'$ (July 13), followed by the third minimum (6.96°) three and one half hours later at $5^h\ 30'$ and the third maximum (7.69°) at $12^h\ 00'$.

Oxygen. The direction of the oxygen gradient at 1000 meters was such that vertical movements of the water column will cause the oxygen curve to vary in the opposite direction to temperature (Fig. 4). Thus, a comparison of results above and below the oxygen minimum concentration suggests strongly that the observed variations of oceanographic factors are caused primarily by vertical displacements of the water column.

1200 Meters

The 1200 meter level was very near the lower limit of the principal thermocline.

Temperature. The temperature rose from its first minimum value (5.24°) at $18^h\ 10'$ (July 12) to a maximum (5.64°) at $1^h\ 00'$ (7 hour interval); it then decreased to a second minimum (5.33°) at $4^h\ 24'$ and finally was approaching a second maximum value ($>5.54°$) when observations ended at $15^h\ 44'$.

Oxygen. Throughout the observational period oxygen increases corresponded to temperature decreases and vice versa as in the case of the 1000 meter level.

A summary of the times of occurrence of maxima and minima values of temperature at various fixed levels during the final 24 hour observation period is given in table 2. It is seen that the time intervals between minima and maxima, or vice versa, divide themselves into three distinct classes with values of 2.8 to 3.5, 6 to 6.8 and 11 to 12.5 solar hours, the latter two of which may correspond to periods of 12 and 24 lunar hours and be connected with the tides.

TABLE 2

Meters Depth	Minimum Value	Minimum Time	Minimum Cul.	Approx. Interval	$\frac{dT^\circ}{dt}$	Maximum Value	Maximum Time	Maximum Cul.	Approx. Interval	$\frac{dT^\circ}{dt}$	Minimum Value	Minimum Time	Minimum Cul.	Approx. Interval	$\frac{dT^\circ}{dt}$	Maximum Value	Maximum Time	Maximum Cul.	Approx. Interval
50	19.93°	$18^h\ 00'$	$L-0.9^h$	6^h	0.335	21.94°	$00^h\ 00'$	$L+5.1^h$	12^h	0.20	19.54°	12^h	$U+4.7^h$						
100	18.25°	$18^h\ 00'$	$L-0.9^h$	6^h	0.11	18.91°	$00^h\ 00'$	$L+5.1^h$											
200	17.69°	$00^h\ 06'$	$L+5.2^h$	3.2^h	0.016	17.74°	$3^h\ 18'$	$L+8.4^h$	2.8^h	0.021	17.68°	$6^h\ 06'$	$L+11.2^h$	6.2^h	0.034	17.89°	$12^h\ 18'$	$U+5^h$	
400	17.06°	$00^h\ 06'$	$L+5.2^h$	3.2^h	0.028	17.15°	$3^h\ 18'$	$L+8.4^h$	2.8^h	0.014	17.11°	$6^h\ 06'$	$L+11.2^h$	6^h	0.043	17.37°	$12^h\ 06'$	$U+4.8^h$	
500	16.03°	$19^h\ 30'$	$L+0.6^h$	3^h	0.023	16.10°	$22^h\ 30'$	$L+3.6^h$	3.2^h	0.047	15.95°	$1^h\ 42'$	$L+6.8^h$	11^h	0.083	16.86°	$12^h\ 42'$	$U+5.4^h$	
800	10.31°	$18^h\ 00'$	$L-0.9^h$	3^h	0.147	10.75°	$21^h\ 00'$	$L+2.1^h$	3^h	0.073	10.53°	$00^h\ 00'$	$L+5.1^h$	12.5^h	0.10	11.75°	$12^h\ 30'$	$U+5.2^h$	
1000 {	6.84°	$18^h\ 30'$	$L-0.4^h$	—	—	—	—	—	3.5^h	—	7.00°	$22^h\ 00'$	$L+3.1^h$	4^h	0.043	7.17°	$2^h\ 00'$	$L+7.1^h$	3.5^h
	6.96°	$5^h\ 30'$	$U-1.8^h$	6.5^h	0.112	7.69°	$12^h\ 00'$	$U+4.7^h$											
1200	5.24°	$18^h\ 10'$	$L-0.7^h$	6.8^h	0.058	5.64°	$1^h\ 00'$	$L+6.1^h$	3.4^h	0.091	5.33°	$4^h\ 24'$	$L+9.5^h$	11.3^h+		5.54°+	$15^h\ 44'+$		

Scaled values for maxima and minima of temperature curves for various depths during 24 hour period: July 12, $15^h\ 34'$ to July 13, $15^h\ 34'$. The column headed Cul. gives the number of hours before or after the culminations of the moon (L and U). The occurrence of temperature maxima and minima with respect to the moon's culminations vary from depth to depth, but frequently there was a temperature minimum within one hour before the moon's lower culmination, on July 12, and a maximum 4 to 6 hours afterwards; similarly a temperature maximum frequently occurred about 5 hours after the moon's upper culmination. The rate of change of temperature with time is indicated approximately by dT^0/dt, degrees per hour. This value in part depends on the rate of change of temperature with depth and is of significant magnitude.

The times of the occurrence of maxima and minima temperature values with respect to the moon's culmination for the 24 hour period beginning $15^h\,34'$, July 12, are indicated in table 2, and, while these vary from depth to depth, it is seen for most of the depths examined that there was a temperature minimum within one hour before the lower culmination of the moon on July 12 and a maximum about 4 to 6 hours afterwards. Also, a temperature maximum frequently occurred about 5 hours after the upper culmination of the moon.

Temperature variations at standard depths with respect to the culminations of the moon during the entire observational period may be obtained from figure 2, but, with the exception of the final 24 hours, the relationship has little meaning because the observations were too far apart (approximately 6 hours). Thus, considering temperatures at the time of the culminations of the moon at $18^h\,52'$ on July 12 (lower) and at $7^h\,20'$ on July 13 (upper, L.C.T.) there were, for all depths examined, higher temperatures at the time of the moon's upper culmination.

The rate of change of temperature with time between 50 and 1200 meters is indicated approximately by dT°/dt in table 2. The rate of change of temperature with time will be determined largely by the rate of change of temperature with depth; even at 1200 meters it may be relatively large, e.g., $dT^\circ/dt=0.091$. The maximum dT°/dt value recorded was 0.335° per hour at 50 meters depth.

DEPTH VARIATIONS OF ISOTHERMS

Table 3 summarizes the variation in depth of selected isotherms between mean depths of 60 and 4067 meters as observed during the 102 hour period at station 2639; the variations may be used as indicators of vertical movements of the water column (page 17). Below 100 meters the vertical displacement of the isotherms ranged from 52 to 185 meters (the error of estimating the amount of displacement increases for great depths where the temperature gradient is very small), and during a single twenty-four hour period the isotherms between 60 and 1265 meters underwent vertical displacements of 34 to 74 meters.

TABLE 3

Isotherm	Mean Depth	Total Depth Range $9^h30'$, 7/9 to $15^h34'$, 7/13		Total Displacement	Total Depth Range $15^h34'$, 7/12 to $15^h34'$, 7/13		Total Displacement
20.0°	60	42	82	40	42	76	34
18.5°	101	83	135	52	83	118	35
17.0°	428	408	482	74	408	482	74
15.0°	591	560	633	73	571	633	62
10.0°	816	777	841	64	777	830	53
5.0°	1265	1240	1305	65	1242	1285	43
4.0°	1675	1647	1728	81			
3.0°	3000	2958	3036	78			
2.4°	4067	3973	4158	185			

Vertical range of selected isotherms between 60 and 4067 meters observed at Station 2639.

The approximate times at which the highest and lowest vertical positions of various isotherms (20.0°, 18.5°, 17.0°, 15.0°, 10.0°) were reached during the 24 hour observational period, beginning $15^h\,30'$ on July 12, are given in table 4, and possible connections with the tidal wave are suggested by the approximate 6 and 12 hour intervals (Fig. 5). The data also indicate the presence of other unknown disturbances which appear to play a significant part in these variations.

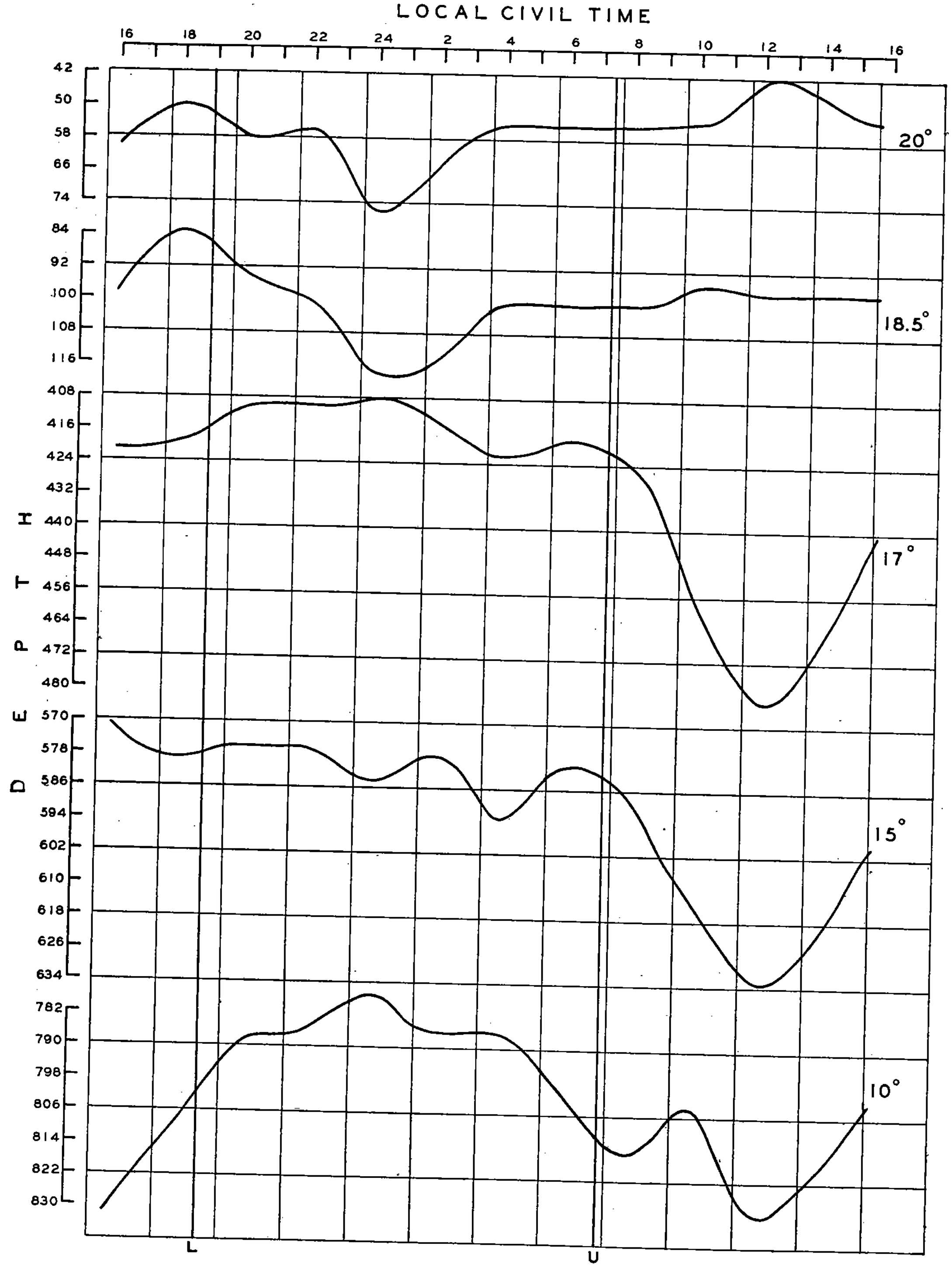

FIG. 5.—Time depth variation of 20°, 18.5°, 17°, 15°, and 10° isotherms at "Atlantis" station 2639 for the twenty-four hour period beginning 15h 30′ July 12, 1936 (see text). Symbols *L* and *U* refer to times of lower and upper culminations of moon.

TABLE 4

Isotherm	High Depth	High Time	Interval	Low Depth	Low Time	Interval	High Depth	High Time	Interval	Low Depth	Low Time	Interval
20.0°	50	18h00′	6h	76	24h00′	12h24′	42	12h24′	—	—	—	—
18.5°	83	18h00′	6h12′	118	00h12′	—	—	—	—	—	—	—
17.0°	408	00h12′	3h48′	422	4h00′	2h15′	418	6h15′	6h05′	482	12h20′	—
15.0° {	—	—	—	579	18h00′	—	—	—	6h10′	584	00h10′	1h50′
15.0°	578	02h00′	2h08′	593	4h08′	2h10′	580	6h18′	6h07′	633	12h25′	—
10.0°	777	00h10′	8h00′	815	8h10′	2h00′	803	10h10′	2h20′	830	12h30′	—

Depths (meters) and times (L.C.T.) at which high and low positions of various isotherms were reached during the twenty four hour period beginning July 12, 1936, 15h34′. Local civil time refers to the mean position of station 2639 (66°25′W, 37°07′N).

CAUSE OF VERTICAL DISPLACEMENTS OF ISOTHERMS

Before proceeding to a quantitative discussion of the vertical oscillations of the water column, evidence supporting the contention that vertical oscillations of the isotherms at station 2639 are primarily the result of vertical movements of the water column and not of horizontal shifts will be considered by studying the horizontal gradients of the 17° and 10° isotherms in the vicinity of station 2639.

Long Period Variations in Depths of 17° and 10° Isotherms in the Vicinity of Station 2639

Temperature data from 33 "Atlantis" stations, within a radius of about 200 miles from station 2639 (Fig. 1), were examined and the depths of the 17° and 10° isotherms scaled from temperature depth curves. The observations at these stations were taken between November 24, 1931 and July 28, 1933; the depth variations of the 10° and 17° isotherms are summarized in table 5.

TABLE 5

	1931 Nov. 24–26	1931 Dec. 6–7	1932 Feb. 14–17	1932 Apr. 18–20	1932 Aug. 18–31	1932 Dec. 2–4	1933 Feb. 12–13	Totals
10° range	810–916	888–1000	885–1025	840–930	825–907	818–915	863–935	810–1025
10° mean	878	944	974	902	872	872	899	903
$\Delta Z_{10°}$	106	112	140	90	82	97	72	215
17° range	476–535	510–600	510–605	475–600	475–535	452–545	478–605	452–605
17° mean	507	560	571	528	496	503	542	525
$\Delta Z_{17°}$	59	90	95	125	60	93	127	153
No. of stations	4	4	5	4	9	5	2	33

Depth range of 10° and 17° isotherms at "Atlantis" stations within a 200 mile radius of station 2639. ΔZ = vertical range of isotherm during indicated time interval.

The mean depth of the 10° isotherm at station 2639 during July 9 to 13, 1936 was calculated to be 816 meters and its vertical range during this period 64 meters (page 13). However, table 5 shows that from November 24, 1931 to February 13, 1933 the mean depth of this isotherm was 903 meters and its total vertical range 215 meters. Hence, during this 15 month interval the 10° isotherm was subjected to influences which greatly outweigh the effects of daily and half-daily tides. Table 5 illustrates certain trends, significant of which are, that between November 1931 and February 1933 the 10° isotherm reached its greatest mean depth near the end of winter, and its least mean depth in mid summer, or at the same season that station 2639 was made. This seasonal difference, in part, explains the high position of the 10° isotherm in the water column at station 2639.

The amount of vertical displacement of the 10° isotherm also appears to show a seasonal effect; table 5 illustrating that the maximum observed displacement occurred near the end of winter, coinciding with the maximum mean depth of this isotherm and a significantly smaller displacement occurred in summer when the mean depth of the isotherm was least.

The mean depth of the 17° isotherm at station 2639 during July 9 to 13, 1936 was calculated to be 428 meters, and its maximum vertical range 74 meters (page 13). The data in table 5 show that the greatest mean depth (571 meters) occurred near the end of winter and the least mean depth (496 meters) near mid summer; and that the maximum observed vertical displacement occurred in winter and the minimum in mid summer.

Thus, the 10° and 17° isotherms in the vicinity of station 2639 are similar in their seasonal changes; in winter both isotherms appear to occupy their greatest mean depth in the water column and go through their maximum vertical displacement; whereas in summer they both occupy higher mean positions in the water column and exhibit less vertical displacement. The fact that station 2639 was made in mid summer may account for the relatively high mean depths of the 10° and 17° isotherms. However, since table 5 shows a not too good agreement between the December 1931 and February 1932 values and the December 1932 and February 1933 values there is the suggestion of an annual variation superimposed on the seasonal variation of the mean depth of these isotherms which is no doubt the result of annual changes in the circulation. Thus, a seasonal variation is defined as:

$$f(x+\alpha) = f(x)$$

where α is the period. The fact that values of the mean depth do not recur after a fixed interval is due partly to the effect of diurnal and semidiurnal variations as well as probably the effect of an annual variation. The incompleteness of the data, however, prevent the specification of the magnitude of seasonal and annual variation.

Determination of Maximum Horizontal Gradients of 17° and 10° Isotherms in Vicinity of Station 2639

Since the possibility exists that the vertical oscillations of the isotherms at station 2639 to some extent may be caused by periodic horizontal displacements of the water mass (accompanying the tidal wave) it seemed advisable to examine the horizontal gradients of various isotherms in the vicinity of this station. The foregoing discussion of the short and long period variations of isothermal depths illustrates the difficulty of determining horizontal gradients of isotherms (or any other characteristic for a particular time) in the vicinity of station 2639. Consequently, for purposes of argument, the maximum horizontal gradient which might possibly exist in the vicinity of this station at any time (from the data at hand) has been calculated and the magnitude of the required horizontal displacement to produce the observed vertical displacements of the isotherms has been determined therefrom. This treatment should supply information regarding the reasonableness of suggesting that vertical displacements of the isotherms in the vicinity of station 2639 were caused by horizontal displacements of the water layers which may be associated with the tidal wave.

The maximum horizontal gradient of the 10° and 17° isotherms have been determined on the basis of the assumption that the depth variations of the 10° and 17° isotherms observed between November 24, 1931 and July 28, 1933 (table 5) could exist at any time in the vicinity of station 2639 (this, being no doubt an impossibility, serves for argumentative purposes). The following results were obtained:

1. Maximum gradient of 10° isotherm: 1.33 meters/kilometer
2. Maximum gradient of 17° isotherm: 1.00 meters/kilometer

The Horizontal Displacements Required to Produce the Observed Vertical Displacements of the 10° and 17° Isotherms

10° isotherm. From the time depth curve of this isotherm (Fig. 5) the following data for the 24 hour period of July 12–13 are obtained:

highest level: 777 meters, occurred about $00^h\ 00'$ (L.C.T.)
lowest level: 830 meters, occurred about $12^h\ 30'$ (L.C.T.)

The average rate of change of elevation during this period was 4.24 meters per hour. To produce this same effect by a pure horizontal displacement on the basis of a maximum horizontal gradient of 1.33 meters/kilometer, a mean horizontal velocity of the water particles of 3.2 kilometers per hour or 89 centimeters per second would have been required. Since the normal horizontal gradient is probably only one half to one third of the amount used in the above calculation, the improbable horizontal velocities of around 200 centimeters per second would have been required to produce the observed vertical displacements of this isotherm.

17° isotherm. From the time depth curve of this isotherm (Fig. 5) the following data for the 24 hour period of July 12–13 are obtained:

highest level: 408 meters, occurred about $00^h\ 00'$ (L.C.T.)
lowest level: 482 meters, occurred about $12^h\ 30'$ (L.C.T.)

The average rate of change of elevation during this period was 5.92 meters per hour. To produce this same effect by a pure horizontal displacement on the basis of a maximum horizontal gradient of 1.00 meters per kilometer, a mean horizontal velocity of the water particles of 5.9 kilometers per hour or 164 centimeters per second would have been required. Since the normal horizontal gradient is probably one half to one third of the amount used in this calculation, the absurd horizontal velocities of about 350 to 450 centimeters per second would have been required to produce the observed vertical displacements of this isotherm.

Thus, on the basis of the foregoing discussion it is reasonable to conclude that vertical oscillations of temperature (and other properties of sea water) in the locality represented by station 2639 resulted primarily from vertical oscillations of the water column rather than from horizontal displacements of the water layers.

ANALYSIS OF TIME VARIATIONS IN DEPTH OF ISOTHERMS DURING THE 24 HOUR PERIOD

Method of Harmonic Analysis

Since the preceding discussion suggests a connection between vertical oscillations of the isotherms and the tides the resulting amplitudes and phases of the oscillations may be determined by fitting a Fourier series with a limited number of terms to the observed data. This means that we shall first decompose the isotherm time depth curves (Fig. 5) into simpler waves having periods the same as the tidal wave (24 and 12 lunar hours). The depths of the various isotherms plotted as a function of time individually specify graphically the functions for which we wish to find Fourier series, each of which will represent, with a limited number of terms, as closely as possible the individual graphs of the observations. The general method used in this investigation for obtaining Fourier coefficients is outlined briefly in the following discussion.

From the given graph (Fig. 5) the values of the function, $u(x)$, corresponding to the values,

$$0, \quad \frac{2\pi}{n}, \cdots \frac{2(n-1)\pi}{n} \text{ of } x$$

are obtained. And, it is desired to find a sum:

$$\begin{aligned} f(x) &= a_0 + a_1 \cos x + a_2 \cos 2x + \cdots a_r \cos rx \\ &\quad + b_1 \sin x + b_2 \sin 2x + \cdots b_r \sin rx \end{aligned}$$

which furnishes the best possible representation of the function $u(x)$ (where $2r+1 \leqq n$). Denoting $u(0)$ by u_0, $u(2\pi/n)$ by u_1 etc. and substituting in the above equation the following equations of condition for the determination of the coefficients $a_0, a_1, \cdots a_r; b_1, b_2, \cdots b_r$ are obtained:

$$\begin{aligned} u_0 &= a_0 + a_1 + a_2 + \cdots a_r \\ u_1 &= a_0 + \sum_{k=1}^{k=r} a_k \cos \frac{2k\pi}{n} + \sum_{k=1}^{k=r} b_k \sin \frac{2k\pi}{n} \\ &\vdots \\ u_p &= a_0 + \sum_{k=1}^{k=r} a_k \cos \frac{2kp\pi}{n} + \sum_{k=1}^{k=r} b_k \sin \frac{2kp\pi}{n} \\ &\vdots \\ u_{n-1} &= a_0 + \sum_{k=1}^{k=r} a_k \cos \frac{2k(n-1)\pi}{n} + \sum_{k=1}^{k=r} b_k \sin \frac{2k(n-1)\pi}{n}. \end{aligned}$$

For the case where there are more equations than unknowns the method of least squares is now applied to find the best possible solution of these equations; and as a final result the normal equations for the coefficients $a_0, a_1, \cdots a_r$ and $b_1, b_2, \cdots b_r$ are obtained:

$$\begin{aligned} na_0 &= u_0 + u_1 + u_2 + \cdots u_{n-1} \\ \frac{n}{2} a_r &= u_0 + u_1 \cos \frac{2r\pi}{n} + u_2 \cos \frac{4r\pi}{n} + \cdots + u_{n-1} \cos \frac{2(n-1)\pi}{n} \\ \frac{n}{2} b_r &= u_1 \sin \frac{2r\pi}{n} + u_2 \sin \frac{4r\pi}{n} + \cdots + u_{n-1} \sin \frac{2(n-1)\pi}{n}. \end{aligned}$$

These equations give the coefficients $a_0, a_1, \cdots a_r; b_1, \cdots b_r$ which enable the arbitrary function, $f(x)$, to be expressed as nearly as possible as the sum of a limited number of sine and cosine terms.[4]

[4] For a more detailed discussion of this method the reader is referred to Whittaker and Robinson (1929).

As a final step the resulting equation is put in the form:

$$c_0 + c_1 \cos (x - \alpha_1) + c_2 \cos (2x - \alpha_2)$$

the amplitudes, c, and the phases, α, are connected with the a's and b's by the relations:

$$\alpha = \cos^{-1} \frac{a}{b}$$

$$c = \sqrt{a^2 + b^2}$$

This analysis is applied to the 20° (42 to 63 meters), 17° (408 to 481 meters) and 10° (777 to 830 meters) isotherms. The 20° isotherm lies within the summer or temporary thermocline, the 17° isotherm marks approximately the beginning of the main thermocline, and the 10° isotherm lies near the mid point of the main thermocline (Fig. 3).

The data for carrying out the analysis were obtained during the 24 hour period which began at about $15^h\ 30'$ (L.C.T.) on July 12, 1936; the observations were separated by approximate two hour intervals. At the mean position of the station (page 5) the lower culmination of the moon occurred at $18^h\ 52'$ on July 12 and the upper culmination at $7^h\ 20'$ on July 13. Beginning with the time of the moon's lower culmination, the entire period was divided into twenty-four lunar hours (12.46 solar hours = 12 lunar hours) and the depths of the isotherms concerned were scaled for each lunar hour (Fig. 5). The lunar time beginning from the moon's lower culmination is designated as t. However, since the observations did not extend for a full twenty-four lunar hours beyond the time of the moon's lower culmination on July 12, data for three lunar hours previous to this time were used. This reference time (which begins three hours before the moon's lower culmination on July 12) is designated as t'[5] and was used for the harmonic analysis. The final results for the phase determinations are represented as the number of lunar hours after the moon's lower culmination; the translation depending on the simple relation:

$$t = t' + 3.$$

RESULTS OF HARMONIC ANALYSIS

20° isotherm. Harmonic analysis gives for variations in depth of this isotherm:

$$H_{20^\circ} = 55.4 + 7.44 \cos \frac{2\pi}{24} (t - 5.2^h) + 1.81 \cos \frac{2\pi}{12} (t - 5.8^h) \qquad (1)$$

where, t, represents the number of lunar hours after the lower culmination of the moon.

The amplitude of the 24 hour wave is about four times that of the 12 hour wave. Since the phases of both waves are approximately the same, the maximum downward displacement of the water column at a depth of 55 meters should occur about five and one half lunar hours after the lower culmination of the moon.

The residue, R_{20°, is defined:

$$R_{20^\circ} = O_{20^\circ} - H_{20^\circ} \qquad (2)$$

where O_{20° is the observed depth of the 20° isotherm (Fig. 5) and H_{20° the depth as calculated from equation 1; it is graphically illustrated in figure 6, and the points represent

[5] Thus, when $t = 0^h$., $t' = -3^h$.