PAPERS IN PHYSICAL OCEANOGRAPHY AND METEOROLOGY
PUBLISHED BY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
AND
WOODS HOLE OCEANOGRAPHIC INSTITUTION

VOL. X, NO. 4

RESULTS OF RESEARCH ON SURFACE WAVES OF THE WESTERN NORTH ATLANTIC

I

INVESTIGATION OF BOTTOM PRESSURE FLUCTUATIONS AND SURFACE WAVES

II

RESULTS OF SEA SURFACE ROUGHNESS DETERMINATIONS IN THE VICINITY OF WOODS HOLE, MASSACHUSETTS AND BERMUDA

BY

H. R. SEIWELL

Contribution No. 436 from the Woods Hole Oceanographic Institution

---

CAMBRIDGE AND WOODS HOLE, MASSACHUSETTS
AUGUST, 1948

PART I

INVESTIGATION OF BOTTOM PRESSURE FLUCTUATIONS AND SURFACE WAVES

CONTENTS

PART II

RESULTS OF SEA SURFACE ROUGHNESS DETERMINATIONS IN THE VICINITY OF WOODS HOLE, MASSACHUSETTS AND BERMUDA

CONTENTS

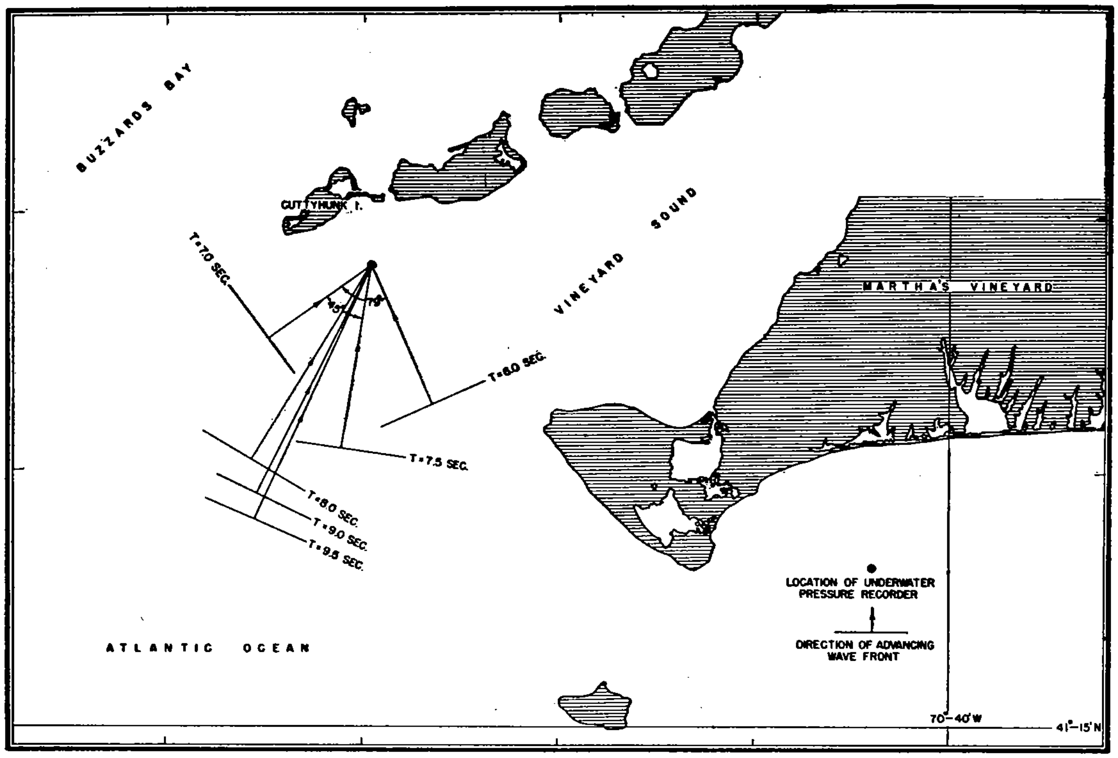

Fig. 1. Location of Underwater Pressure Recorder off Cuttyhunk Island (June 1946), and directions of advancing wave fronts, Experiment II (see text).

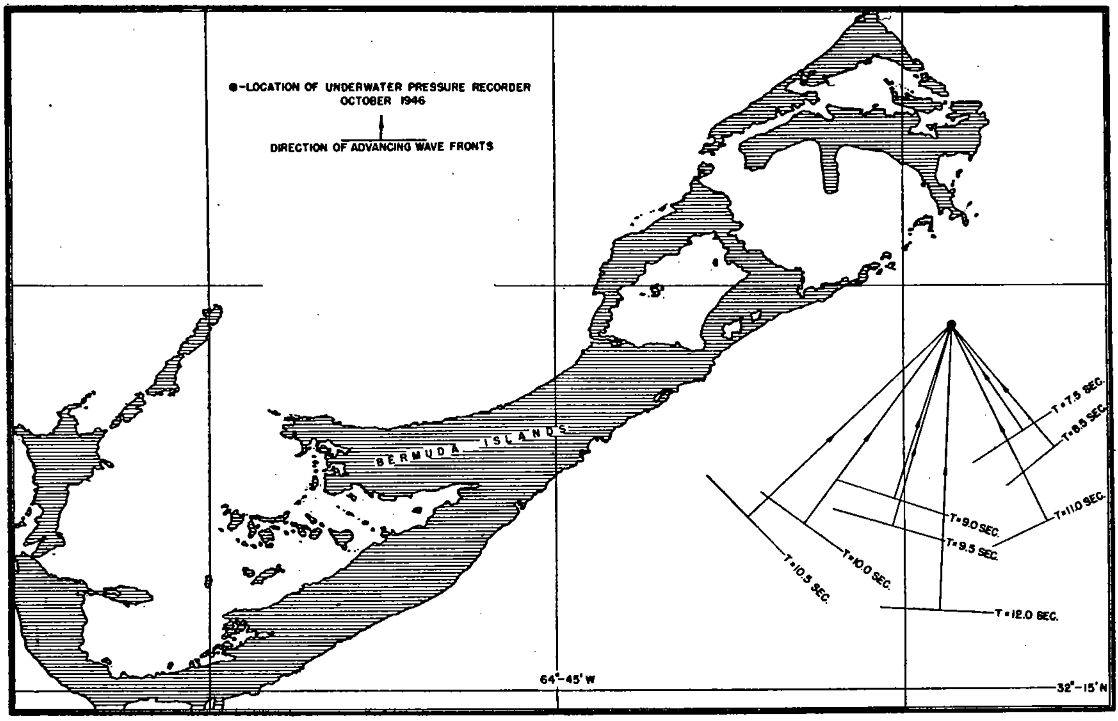

Fig. 2. Location of Underwater Pressure Recorder off Bermuda (October 1946), and directions of advancing wave fronts, Experiment V (see text).

PART I

INVESTIGATION OF SIMULTANEOUS PRESSURE FLUCTUATIONS AT THE BOTTOM AND SURFACE OF THE OCEAN

I. INTRODUCTION

It is to be expected that in the future, measurements of pressure fluctuations beneath the ocean surface will provide basic data for solution of many practical problems on the state of the sea. As a result of the wartime impetus, several types of underwater pressure recorders (of similar instrumentation principles) were developed both in this country and abroad. Essentially, the instrument consists of an underwater unit[1] which electrically transmits pressure impulses near the sea bottom to a clockwork recorder installed on the shore. The underwater pressure unit is adjusted for pressure fluctuations resulting from surface waves within the spectrum band of periods set up by winds acting on the sea surface. The resulting records may be scaled for height and period of the pressure fluctuations over known time intervals. An accessory wave analyzer has been constructed for rapid periodogram analysis of the pressure records.

This investigation is concerned with a comparative study of observed sea surface waves and recorded sea bottom pressure fluctuations. It was undertaken for the purpose of evaluating sea surface wave heights from sea bottom pressure recordings in the vicnity of Woods Hole and Bermuda[2].

II. THE PROBLEM

Amplitudes of pressure fluctuations within the sea are not identical with those of the overlying physical surface, and records of such need be considered in light of the hydrodynamic properties of the water column. The damping of surface pressure fluctuations with depth is related to wave length and depth. The damping phenomenon is selective to the extent that longer period waves, generally obscured at the sea surface, become recognized in the underwater pressure records. However, by the same token, shorter period waves are eliminated, and if the location of the instrument is too deep, the resulting records may be unsuitable for determination of operational interference patterns at the sea surface. Hence, at the outset, it is required to investigate quantitative relationships of simultaneous pressure fluctuations at the surface and bottom and to find a means for determining sea surface patterns solely from underwater pressure records.

In view of the increasing requirements for quantitative data on height, period, steepness, and other physical characteristics of surface waves, the problem of recording and analyzing the observations has become of first order importance in physical oceanography. The present investigation is fundamental, in that it provides a method for evaluating sea surface conditions from continuous underwater pressure records. Basic data, illustrating conditions of the experiments, are given in detail for use of future related work.

---

[1] The underwater pressure instrument was constructed by Mr. Arthur A. Klebba under U. S. Navy Contract NObs-2083 at the Woods Hole Oceanographic Institution. This instrument is based on an original design by Dr. Maurice Ewing of this Institution.

[2] Sea surface observations and simultaneous underwater pressure records were obtained with the combined facilities of the Woods Hole Oceanographic Institution and those provided by U. S. Navy Contract NObs-2083.

III. EXPERIMENTAL METHODS

The approach to the problem has been to compare theoretical sea surface wave heights, computed from amplitudes and periods of bottom pressure fluctuations (recorded by the underwater pressure meter), with observed heights and periods of waves at the overlying sea surface. The evaluation of differences between observed and theoretical values permits computation of empirical correction factors, and their probable errors, for later use in estimating state of the sea surface from underwater pressure recordings alone.

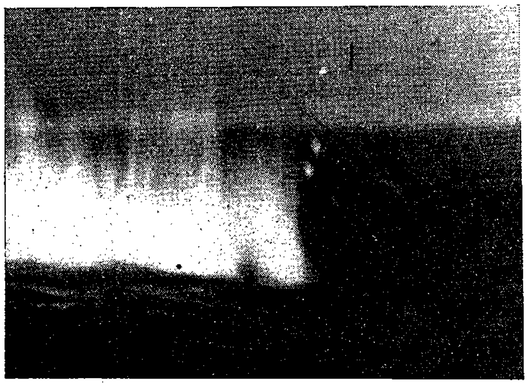

FIG. 3. Sea surface geyser from exploding 75 feet of primacord; off Cuttyhunk, June 1946.

The investigation comprised three series of experiments, two in the vicinity of Woods Hole (off Cuttyhunk Island) and one at Bermuda. The Woods Hole location was one and one-quarter miles south of Cuttyhunk Island (Figure 1), and the Bermuda location about three-fourths of a mile southeast of Castle Harbor entrance (Figure 2). At each location, an underwater pressure recording unit had been in operation for several months.

Measurements of sea surface waves over the underwater instruments were obtained by photographing changes in sea surface height against an anchored floating graduated pole with a 16 millimeter motion picture camera (Figures 3 and 4.) The pole was anchored about fifty feet distant from the underwater instrument[3]. It was positioned

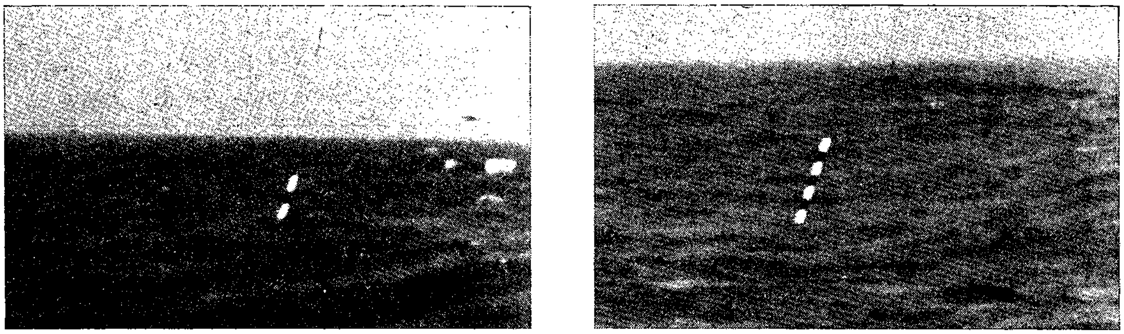

FIG. 4. Two successive relative positions of graduated pole resulting from sea surface fluctuations, off Cuttyhunk, June 27, 1946.

---

[3] This distance was computed from instrument location data, provided by Navy Contract NObs-2083, taking into account directions of the tidal currents.

with reference to a can buoy marking the underwater pressure unit. Synchronization of observations was obtained by exploding seventy-five feet of single strand primacord, shortly after the camera was set in operation. The explosion set up a geyser at the sea surface to be photographed against the background of the graduated pole (Figure 3), and a recognizable tick was simultaneously produced on the underwater recorder tape (Figure 5). The camera speed was accurately known; nearly 16 frames per second; each photographic run was for approximately 50 seconds. After processing, the individual frames were scaled and plotted against time[4]; the curves represent actual time changes in the sea surface patterns. The floating pole was graduated into six inch alternate black and white divisions. Scalings of water levels from enlarged films were made to one-tenth of a division, or one-twentieth of a foot. The probable error of the observations is one-twentieth (1/20) foot.

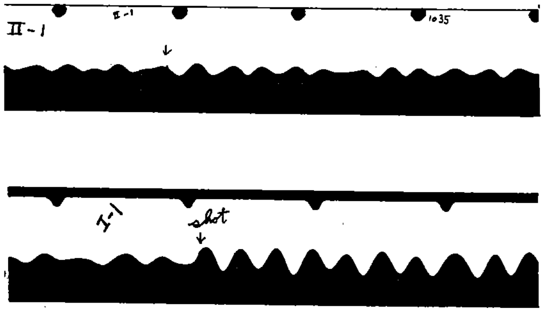

FIG. 5. Underwater recorder tapes for Experiments I-1 and II-1 showing explosion ticks.

TABLE I

EXPERIMENT	PHOTO RUN	STARTING TIME	FRAMES EXPOSED	TOTAL TIME	AVERAGE HEIGHT	AVERAGE PERIOD
I	1	10 58 21	761			
Cuttyhunk	2	11 10 32	759			
June 14	5	11 48 52	759			
	6	12 01 26	570	1h 4m	1.19 ft.	4.34 secs.
II	1	10 33 30	759			
Cuttyhunk	2	10 48 51	759			
June 27	3	10 52 07	754		(1.10 ft.)	(2.8)
	5	11 11 01	758		(2.00 ft.)	(3.7)
	6	11 21 39	760	0h 49m	1.42 ft.	3.16 secs.
V	1	14 53 00	700			
Bermuda	2	14 57 00	740			
October 25	5	15 12 00	740	0h 20m	1.5 ft.	3.0 secs.

Field experimental data and average crest height and period of surface waves during observational time. Sub averages bracketed in Experiment II for 10h33′30″ to 10h52′07″ and 11h11′01″ to 11h21′39″.

Data for the Cuttyhunk experiments (I and II) are shown in Figures 6 to 14, that for Bermuda (Experiment V) in Figure 15. The reproduced underwater records for Experiments I and II are also shown in Figures 6 to 14. Table 1 summarizes the field data of the experiments.

---

[4] Experiments in which the floating graduated pole leaned more than fifteen degrees from the vertical were rejected.

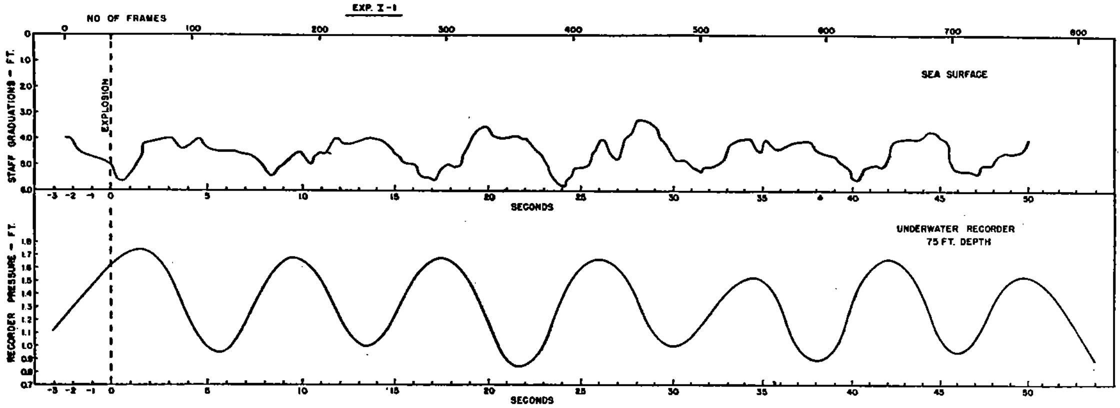

FIG. 6. Sea surface and underwater recorder records for Experiment I-1, off Cuttyhunk, 14 June 1946. Starting time: 10 58 21 EST.

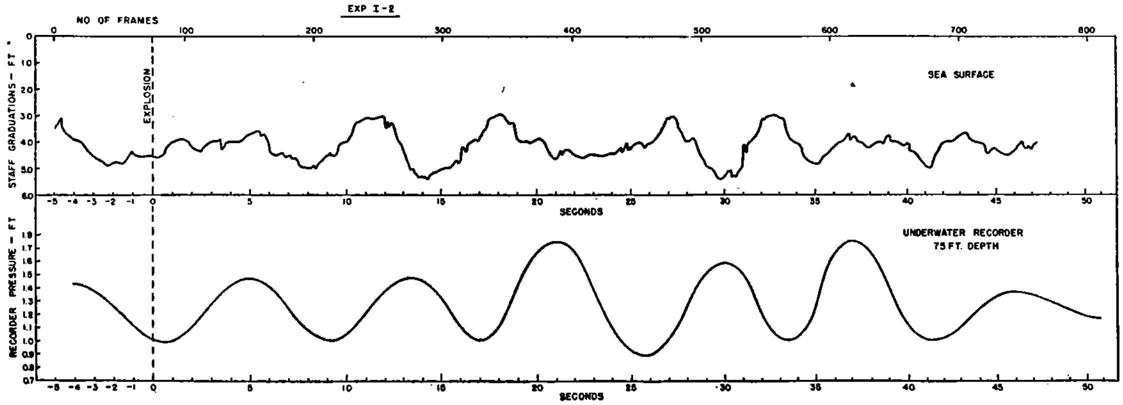

FIG. 7. Sea surface and underwater recorder records for Experiment I-2 off Cuttyhunk, 14 June 1946. Starting time: 11 10 32 EST.

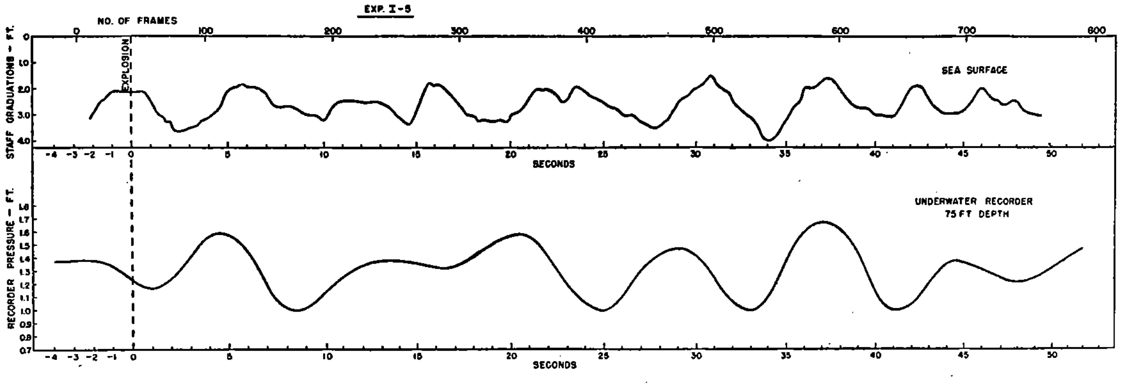

FIG. 8. Sea surface and underwater recorder records for Experiment I-5, off Cuttyhunk, 14 June 1946. Starting time: 11 48 52 EST.

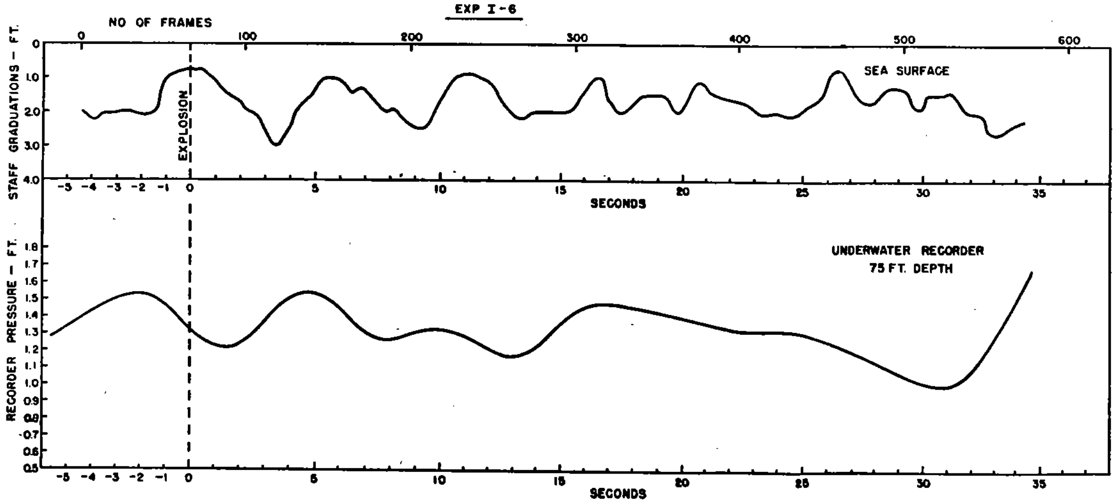

FIG. 9. Sea surface and underwater recorder record for Experiment I-6, off Cuttyhunk, 14 June 1946. Starting time: 12 01 26 EST.

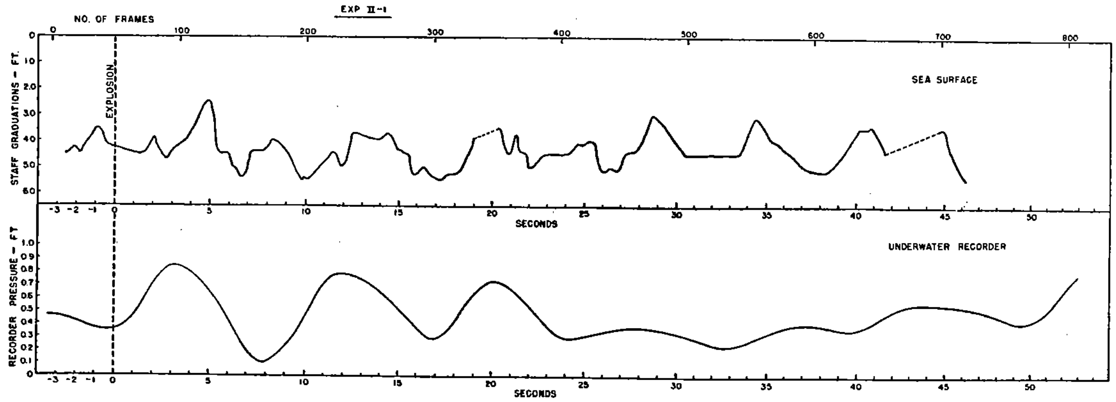

FIG. 10. Sea surface and underwater recorder records for Experiment II-1, off Cuttyhunk, 27 June 1946. Starting time: 10 33 30 EST.

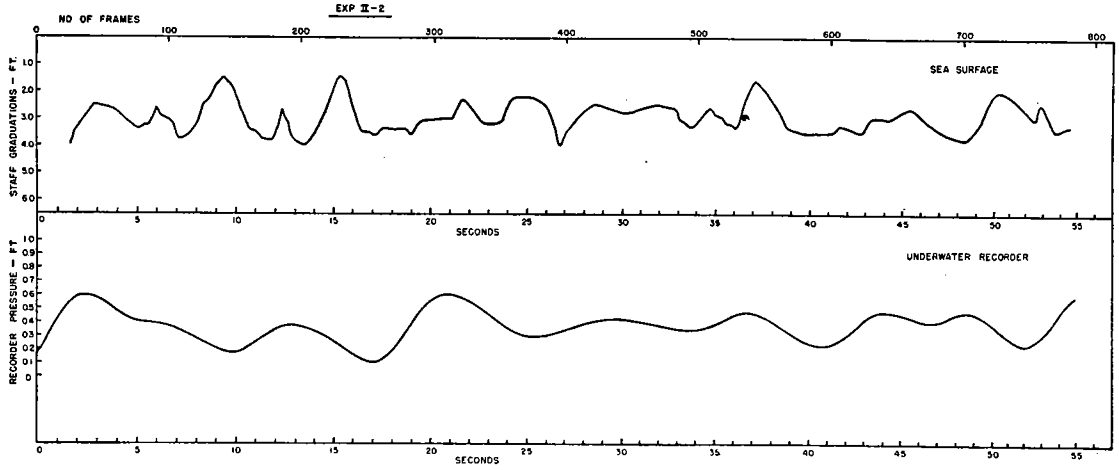

FIG. 11. Sea surface and underwater recorder records for Experiment II-2, off Cuttyhunk, 27 June 1946. Starting time: 10 48 51 EST.

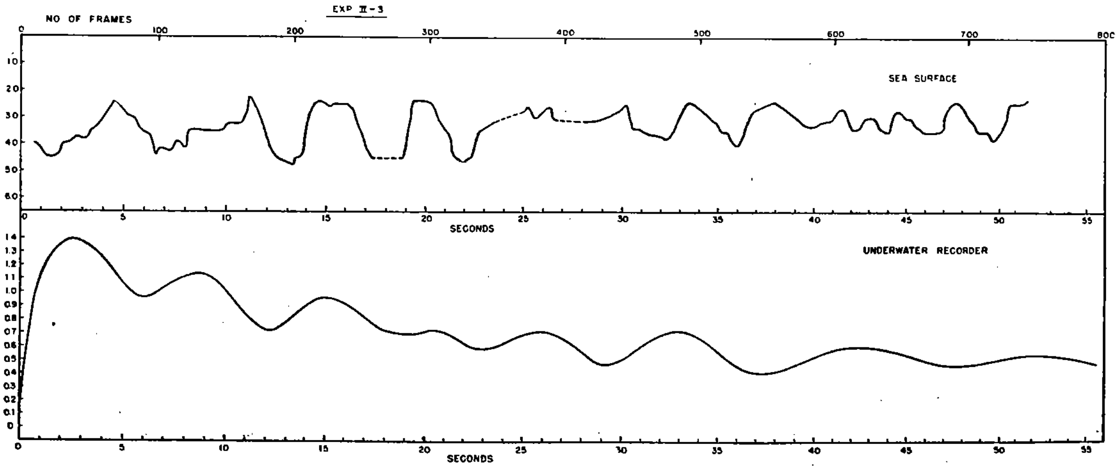

FIG. 12. Sea surface and underwater recorder record for Experiment II-3, off Cuttyhunk, 27 June 1946. Starting time: 10 52 07 EST.

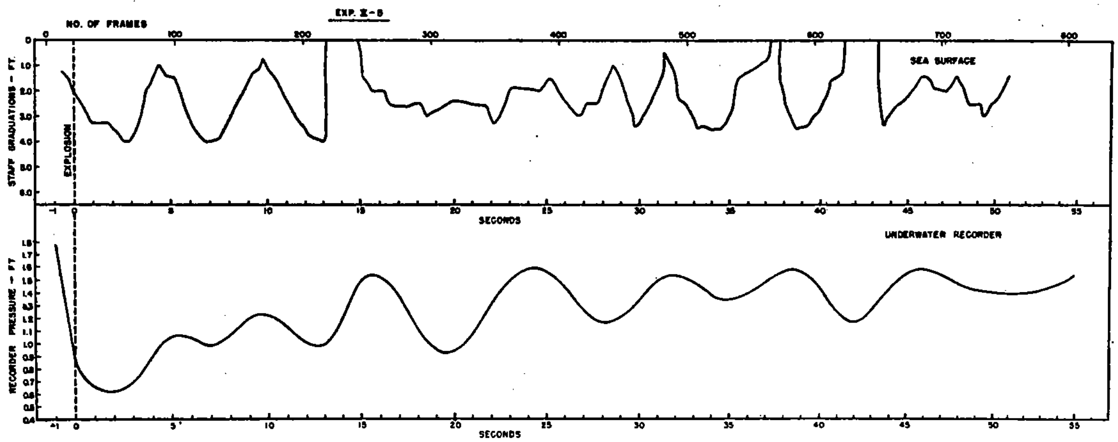

FIG. 13. Sea surface and underwater recorder record for Experiment II-5, off Cuttyhunk, 27 June 1946. Starting time: 11 11 01 EST.

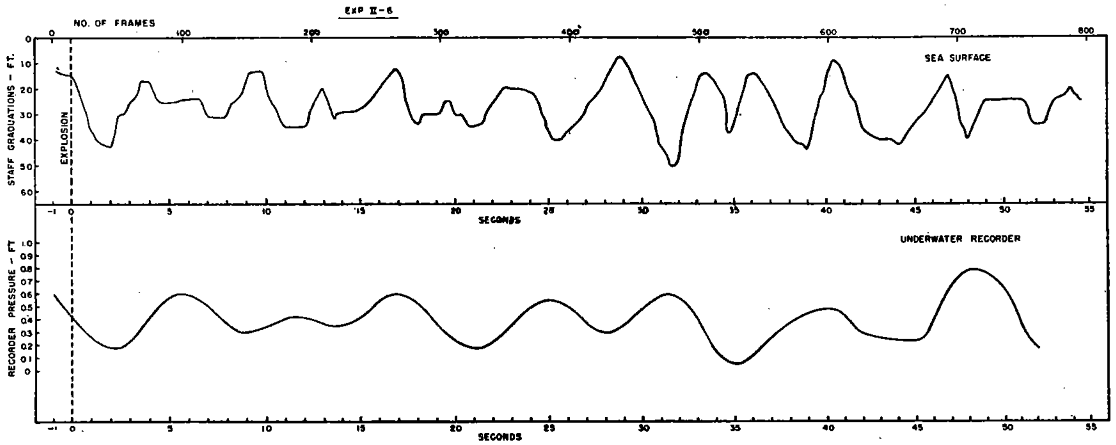

FIG. 14. Sea surface and underwater recorder records for Experiment II-6, off Cuttyhunk, 27 June 1946. Starting time: 11 21 39 EST.

IV. CONDITIONS AT THE SEA SURFACE

1. Interference Patterns

The state of the sea as it affects an observer may be indicated by interference patterns, composed of short time interval averages of wave height and wave period. Such patterns of approximately 50 second intervals, for the two Cuttyhunk and the one Bermuda experiment are given in Table 2. Average values of height and period of the sea surface, for the total duration of each experiment, computed from data in Table 2, are shown in Table 1. Each experiment illustrates characteristics for latter analyses.

It is apparent that sea surface conditions were continuously changing. Those observed off Cuttyhunk Island on June 14 represent the only time the sea surface remained relatively constant during an observational period. On June 27, at this location, both height and average period nearly doubled during the latter ten minutes of the observational period. Similarly, off Bermuda, on October 25, the average sea surface height diminished, and the average period increased during nineteen minutes of observation (Table 1). The rapid sea surface changes appear to result chiefly from variations of the shorter period components set up by local winds; limits of the variability being determined by variations in wind strength and dimensions of local disturbances. The more consistent effects of long period oscillations, induced from sea surface disturbances at greater distances are frequently obscured by local variations.

2. Periodogram and Harmonic Analyses

a. Periodogram Analysis

To obtain supplemental information on individual components making up the sea surface wave patterns, certain of the experiments were subjected to approximate periodogram analyses (5 photo runs of Experiment II) and detailed harmonic analyses (II-1, II-6, V-1, V-2, and V-5).

In carrying out the periodogram analysis[5], scaled values (for each half-second) of the sea surface curves (Figures 10 to 14) were successively arranged in series of horizontal rows, for groups of columns ranging from two to twenty-two. After twenty-one arrangements had been obtained, differences between the highest and lowest sums of each arrangement ($\Sigma_{max}-\Sigma_{min}$) were plotted to form the periodograms of Figure 16.

The dominating periodicities, indicated by variations of $\Sigma_{max}-\Sigma_{min}$ are tabulated in Table 3. The limits for periods exceeding 1.5 times the average $\Sigma_{max}-\Sigma_{min}$ are between four and eight seconds; the most frequent occurrence being 4.5 to 5.5 seconds.

---

[5] This method, established for rapid period analysis of economic time series, is described in handbooks of economic statistics.

TABLE 2

SUMMARY

INTERFERENCE PATTERNS FOR SURFACE OBSERVATIONS AND UNDERWATER PRESSURE RECORDS

Experiment No.	I-1	I-2	I-5	I-6	II-1	II-2	II-3	II-5	II-6	V-1	V-2	V-5
Experiment Date	14 June 1946					27 June 1946				25 Oct. 1946		
Time	1058	1111	1149	1201	1034	1049	1052	1112	1122	1453	1457	1512
SEA SURFACE												
Average Height, H_m	1.20	1.17	1.32	1.05	1.0	1.0	1.1	1.9	2.1	1.7	1.4	1.3
Height Range	0.2–2.1	0.4–2.4	0.2–2.4	0.3–2.1	0.3–2.2	0.2–2.5	0.3–2.4	0.1–4.0	0.1–3.6	0.5–4.0	0.5–3.0	0.5–2.0
Av Departure From H_m	0.5	0.6	0.6	0.6	0.6	0.6	0.6	1.15	0.8			
Average Period, T_m	4.33	4.24	4.98	3.81	2.8	3.0	2.7	3.4	3.9	2.5	2.9	3.6
Period Range	1.2–7.2	1.9–9.0	1.5–7.5	1.7–5.8	1.1–6.0	1.7–4.9	0.8–5.6	1.3–6.1	2.5–6.5	0.8–6.3	1.6–4.8	1.5–4.7
Av Departure From T_m	1.8	1.7	1.9	1.6	1.1	0.7	1.3	1.1	0.9			
BOTTOM												
Depth (ft)	75	75	75	75	75	75	75	75	75	120	120	120
Average Height, H_m	0.69	0.58	0.40	0.28	0.35	0.23	0.29	0.39	0.35	0.30	0.33	0.28
Height Range	0.53–0.83	0.37–0.75	0.26–0.67	0.01–0.68	0.08–0.68	0.08–0.50	0.03–1.22	0.15–0.68	0.12–0.55	0.13–0.48	0.11–.58	0.16–40.0
Av Departure From H_m	0.09	0.14	0.06	0.19	0.17	0.13	0.23	0.14	0.11			
Average Period, T_m	8.0	8.3	7.9	7.2	8.3	7.5	7.1	7.0	7.1	9.5	9.3	9.3
Period Range	7.5–8.5	7.0–9.0	7.0–9.0	5.5–10.2	6.8–9.7	5.3–10.5	5.5–10.5	4.3–9.5	5.1–9.0	8.0–13.0	8.0–11.0	9.0–10.0
Av Departure From T_m	0.33	0.58	0.58	1.28	0.80	1.50	1.47	1.12	1.22			
BOTTOM-SURFACE RELATION												
Bottom H_m/Surface H_m	0.575	0.496	0.303	0.267	0.35	0.23	0.26	0.21	0.17	0.18	0.24	0.22
$\Delta P_h/\Delta P_o$ (calc.)	0.444	0.492	0.435	0.329	0.492	0.376	0.314	0.299	0.314	0.377	0.354	0.354
$\Delta P_h/\Delta P_o - H_h/H_o$	−0.131	−0.004	0.102	0.062	0.142	0.146	0.054	0.089	0.144	0.20	0.11	0.13

TABLE 3

EXPERIMENT	PERIODS EXCEEDING AVERAGE Σmax-Σmin	PERIODS EXCEEDING 1.5 Average Σmax-Σmin
II-1	4.0, 5.0, 5.5, 7.0, 7.5, 8.0, 9.5, 10.0	5.0, 4.0, 8.0
II-2	3.0, 4.5, 5.0, 5.5. 6.0, 6.5, 7.0, 7.5, 9.5, 11.0	7.0, 6.0
II-3	3.0, 3.5, 4.5, 5.0, 5.5, 7.0, 9.5, 10.0, 11.0	5.0, 4.5
II-5	3.5, 4.0, 4.5, 5.5, 6.5, 7.0, 8.0, 9.5, 11.0	6.0, 6.5, 4.0
II-6	3.5, 4.0, 4.5, 5.0, 6.0, 6.5, 7.0, 8.0, 9.0, 10.0, 10.5	6.0, 6.5, 4.0

Results of sea surface periodogram analysis for Experiment II.

b. Harmonic Analysis

Harmonic analyses were performed on the scaled sea surface data arranged for the above periodogram analyses. As they were already grouped into one-half second intervals it was only necessary to divide the sums of each column by the number of rows to obtain average values. It being assumed that the best representation of variations for selected periods was by means of these average values. Amplitudes and phase values for waves selected from results of the periodogram analysis were computed by the method of least squares[6]. They are tabulated in Table 4.

In this manner, the physical patterns of the sea surface (Experiments II-1, II-6, V-1, V-2, and V-5) are represented by series of sine waves. The probable errors of the amplitudes for surface fluctuations[7] do not exceed 0.02 foot, and recombination of the coefficients (Table 4) reproduce the main features of the observed curves of Figures 10, 14, and 15. However, the individual waves do not necessarily correspond with reality — a doubt which exists in any harmonic analysis of physical data. The principal value of this type of analysis is its convenience in representing data. It is to be noted that the conclusions from this study are not based on the Fourier coefficients, determined as above. The coefficients are used to supplement conclusions arrived at from a direct comparison of surface and bottom pressure patterns.

Both Experiments II and V indicate large time variations of amplitudes for identical waves. Thus, the maximum (Table 4) amplitude change during the forty-nine minute interval of Experiment II was from 0.18 to 0.67 feet for the 6.0 second wave, from 0.47 to 0.96 feet for the 4.5 second wave for the twenty minute interval of Experiment V. The larger variations occurred in amplitudes of the shorter period waves. Table 5 shows that over fifty per cent of the total variations occurred between $T = 4.0$ and $T = 6.0$ seconds for Cuttyhunk, and between $T = 4.5$ and $T = 8.5$ for Bermuda. This apparently results from influence of local generating winds.

---

[6] For discussion of this procedure, see: Short Period Vertical Oscillations in the Western Basin of the North Atlantic, Papers in Physical Oceanography and Meteorology. Vol. V., No. 2. May 1937.

[7] Probable errors of the Fourier coefficients were computed from the equation $P. E. = e \sqrt{2/N}$, where e is the probable error of the N observations.

TABLE 4

RESULTS OF HARMONIC ANALYSIS

SURFACE (o) AND BOTTOM (h) OBSERVATIONS

	Wave Length, L (ft)	82		105		127		184		214		251		284		328	
	Velocity, c, ft/sec	20.4		23.0		25.5		30.7		33.3		35.8		38.4		41.0	
	Period, T (secs)	4.0		4.5		5.0		6.0		6.5		7.0		7.5		8.0	
	Depth	o	h	o	h	o	h	o	h	o	h	o	h	o	h	o	h
EXPERIMENT II-1	Amplitude, A (ft)	0.34		0.04		0.42		0.18				0.28		0.40		0.49	
	Phase, α (secs)	0.4		3.9		4.4		1.6				6.1		5.2		5.2	
II-6	Amplitude, A (ft)	0.36		0.34		0.26		0.67	0.05			0.21	0.09	0.10	0.09	0.16	0.07
	Phase, α (secs)	0.8		0.6		4.4		4.6	4.7			0.2	4.6	5.4	2.5	4.2	0.7
	$\alpha_h - \alpha_o$							0.1				4.4		−2.9		−3.5	
V-1	Amplitude, A (ft)			0.58	0.138					0.26	0.019			0.41	0.013		
	Phase, α (secs)			0.5	3.4					0.2	5.2			5.8	0.3		
	$\alpha_h - \alpha_o$			2.9						5.0				2.0			
V-2	Amplitude, A (ft)			0.47	0.014					0.25	0.009			0.11	0.032		
	Phase, α (secs)			0.9	2.9					4.9	3.0			1.2	2.5		
	$\alpha_h - \alpha_o$			2.0						4.6				1.3			
V-5	Amplitude, A (ft)			0.96	0.003					0.29	0.009			0.24	0.015		
	Phase, α (secs)			2.1	0.1					4.9	2.7			1.4	2.5		
	$\alpha_h - \alpha_o$			2.5						4.3				1.1			

	Wave Length, L (ft)	365		415		461		512		536		618		736	
	Velocity, c, ft/sec	43.5		46.0		48.6		51.2		53.7		56.4		61.5	
	Period, T, (secs)	8.5		9.0		9.5		10.0		10.5		11.0		12.0	
	Depth	o	h	o	h	o	h	o	h	o	h	o	h	o	h
EXPERIMENT II-1	Amplitude, A (ft)	0.10		0.21		0.30		0.22							
	Phase, α (secs)	6.1		4.5		3.9		1.2							
II-6	Amplitude, A (ft)	0.11		0.04	0.07	0.17	0.01	0.25							
	Phase, α (secs)	0.9		8.5	5.5	8.5	5.7	7.6							
	$\alpha_h - \alpha_o$			−3.0		−2.8									
V-1	Amplitude, A (ft)	0.43	0.045	0.34	0.018	0.21	0.025	0.15	0.052	0.13	0.023	0.34	0.011	0.08	0.018
	Phase, α (secs)	4.1	4.7	3.5	1.4	2.4	7.9	1.6	7.8	2.2	8.7	1.9	3.4	0.4	3.7
	$\alpha_h - \alpha_o$	0.6		6.9		5.5		6.2		6.5		1.5		3.3	
V-2	Amplitude, A (ft)	0.09	0.018	0.15	0.058	0.06	0.001	0.06	0.077	0.15	0.024	0.07	0.028	0.11	0.018
	Phase, α (secs)	7.6	0.0	6.6	3.2	6.1	2.4	8.5	4.4	9.1	4.4	7.8	9.6	7.2	11.2
	$\alpha_h - \alpha_o$	0.9		5.6		5.8		5.9		5.8		1.8		4.0	
V-5	Amplitude, A (ft)	0.11	0.066	0.09	0.003	0.09	0.090	0.20	0.056	0.27	0.031	0.13	0.033	0.26	0.007
	Phase, α (secs)	7.5	7.6	6.4	3.8	0.8	4.1	0.0	6.3	9.0	4.4	8.6	10.7	8.6	0.8
	$\alpha_h - \alpha_o$	0.1		6.4		6.2		6.3		5.9		2.1		4.2	

TABLE 5

PERIOD SECONDS	EXPERIMENT II	EXPERIMENT V
4.0	1.0%	
4.5	14.9%	21.3%
5.0	8.0%	
6.0	24.4%	
6.5		1.7%
7.0	3.5%	
7.5	14.9%	13.0%
8.0	16.4%	
8.5	0.5%	14.8%
9.0	8.5%	10.9%
9.5	6.5%	6.5%
10.0	1.5%	6.1%
10.5		6.1%
11.0		11.7%
12.0		7.8%

Relative time changes in amplitudes of individual wave periods at the surface; Experiment II, 1034 to 1122, 27 June 1946; Experiment V, 1453 to 1512, 25 October 1946.

V. CONDITIONS NEAR THE SEA BOTTOM

1. INTERFERENCE PATTERNS

Interference patterns of bottom pressure fluctuations, computed from scaled values of the underwater records for the Cuttyhunk and Bermuda experiments are tabulated in Table 2, and average values are shown in Table 6[8]. The longer average periods of the Bermuda record are influenced by the greater depth of instrumentation; their relationships to simultaneous surface wave patterns are discussed later.

TABLE 6

	EXPERIMENT I	EXPERIMENT II	EXPERIMENT V
Location	Cuttyhunk	Cuttyhunk	Bermuda
Depth	75 ft.	75 ft.	120 ft.
Average Height	0.49	0.32	0.30
Average Period	7.85	7.40	9.37

Average values of bottom interference patterns computed from data in Table 2.

2. PERIODOGRAM AND HARMONIC ANALYSES

a. Periodogram Analysis

The method of periodogram analysis of bottom pressure records differed from that used for surface records (see Footnote 6) in that the analyses were made mechanically on the "Ocean Wave Analyzer"[9]. For this analysis, a suitable length of record (Figure 5)

---

[8] Underwater pressure records were evaluated from calibration data provided by Mr. Klebba. Average interference patterns were computed for the surface from measurements of crest heights and time intervals between crests.

[9] Descriptions of the "Ocean Wave Analyzer" are on file at this Institution.

is wound around the twenty-inch wheel of the analyzer which is then rotated to a high speed. As the wheel slowly decelerates, waves of successively shorter periods on the tape pattern are brought in tune with an electrical circuit. The output for each frequency (between 4 and 40 seconds) is registered by a pen moving over a chart (Figures 17 and 18), the horizontal movement of which is synchronized with the decelerating speed of the analyzer wheel. As an example of the selective tuning, when the wheel rotates at 240 RPM, forty second waves on the tape are selected and at 24 RPM, four second waves on the tape are brought in tune with the analyzer circuit. The relative output for each frequency is recorded as a displacement of the ordinate as the chart is moved to proper period position by a speed indicator.

In view of this type of analysis, as compared with somewhat similar statistical procedures, and because distances of the ordinates from the base line indicate only relative significance of the periods in the frequency band, it is here referred to as a mechanical periodogram analysis.

Comparative studies by the designer indicate that with present adjustments, a maximum accuracy of plus or minus one-half second is obtained for the period band between six and twenty seconds; in the 30 second region this diminishes to plus or minus two seconds.

In Table 7, the limits of the principal spectrum band, and the three principal periods, from the analyzer records, are tabulated for Experiments II and V. The average period of the bottom interference pattern (Table 2) lies within the principal band.

TABLE 7

	CUTTYHUNK, EXP. II			BERMUDA, EXP. V	
Photo Run	1, 2, 3	5	6	1	2 5
Analysis	II-A	II-B	II-C	B-12-c	B-12-d
Principal Spectrum	7.0–10.0	7.0–10.0	7.0–10.0	7.5–15.0	7.5–14.0
Principal Period	9.5	8.5	7.5	10.0	10.5
Second Period	7.5	9.0	8.0	9.5	11.0
Third Period	8.5	8.0	9.0	11.0	10.0

Scaled data from mechanical periodogram analysis of indicated bottom pressure records (Figures 17 and 18).

b. *Harmonic Analysis*

Data for the harmonic analysis of bottom pressure records of Experiments II-6, V-1, V-2, and V-5 were obtained by scaling approximately one minute sections of the records corresponding with the times of surface observations. Replottings of data for Experiments I and II (in one-half second intervals) are illustrated by Figures 6 to 14. These data are also used later for determining quantitative relationships between bottom and surface pressure fluctuations (Figure 19).

The Fourier amplitudes[10] computed by the method of least squares have a probable error of approximately 0.01 feet (Table 4). The relative magnitudes of the amplitudes (Table 8) confirm the sequences brought out by the periodogram analyses (Figures 17 and 18), and tabulated in Table 7. Thus, in three of the four cases, wave periods having

---

[10] See Footnotes 6 and 7.

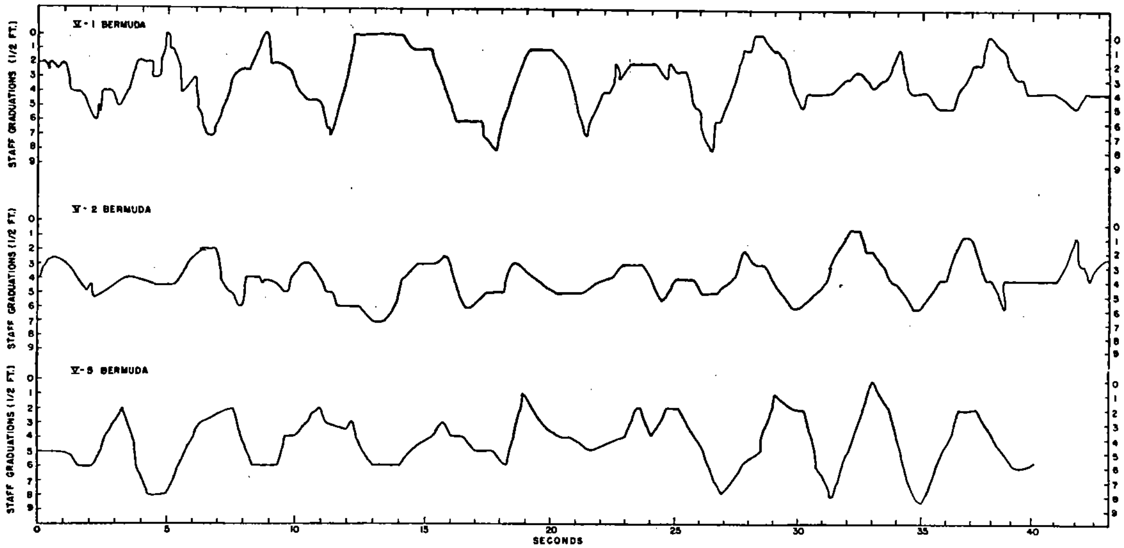

FIG. 15. Sea surface records for Experiment V, photo runs, 1, 2, and 5, off Bermuda, 25 October 1946. Starting times 14 53, 14 57 and 15 12 EST, respectively.

maximum Fourier amplitudes were identical with those shown to have maximum amplitudes by the mechanical analysis. This agreement is significant in that results of the mechanical wave analyses appear to (within limits indicated above) indicate individual waves dominating bottom pressure records.

TABLE 8

	EXPER. II-6		EXPER. V-1		EXPER. V-2		EXPER. V-5	
	T (SEC)	A (FT)	T (SEC)	A (FT)	T (SEC)	A (FT)	T (SEC)	A (FT)
First Period	7.5	0.09	10.0	0.052	10.0	0.077	9.5	0.090
Second Period	7.0	0.09	8.5	0.045	9.0	0.058	8.5	0.066
Third Period	8.0	0.07	9.5	0.025	7.5	0.032	10.0	0.056

First three principal periods (T) and amplitudes (A) by harmonic analysis of bottom pressure records.

VI. RELATIONSHIP OF BOTTOM TO SURFACE PRESSURE FLUCTUATIONS

1. THEORETICAL CONSIDERATIONS

Considerations of wave motion in an incompressible fluid of constant density, ρ, where the depth, h, is greater than the wave length, λ, provide the theoretical basis for investigations of ocean waves. In the following, it is assumed that motion has started from rest by natural forces; the viscosity influence is neglected. Since the motion is irrotational, a velocity potential, ϕ, is assumed. The usual boundary conditions of hydrodynamics, including the Laplace equation of continuity, are to be satisfied.

In the following, the z axis is taken vertically upwards, the xy plane in the undisturbed free surface, and motion is assumed in the unlimited x direction. Thus, a solution to the Laplace equation of continuity:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{1}$$

representing a progressive wave with velocity c, is:

$$\phi = (Ae^{kz} + Be^{-kz}) \operatorname{Cos} k\,(x\text{-}ct) \tag{2}$$

At the bottom (z = −h) the condition to be satisfied is:

$$\frac{\partial\phi}{\partial z} = 0 \tag{3}$$

Thus

$$Ae^{-kh} - Be^{kh} = 0 \tag{4}$$

and, let

$$Ae^{-kh} = Be^{kh} = R/2 \tag{5}$$

Hence

$$\phi = R \operatorname{Cosh} k\,(z + h) \operatorname{Cos} k\,(x\text{-}ct) \tag{6}$$

a. The Surface Wave Velocity

The condition at the free sea surface (z = 0) is:

$$\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} = 0 \tag{7}$$

Hence, from (6)

$$c^2k \operatorname{Cosh} kh \operatorname{Cos} k\,(x\text{-}ct) = g \operatorname{Sinh} kh \tag{8}$$

and

$$c^2 = \frac{g}{k} \operatorname{Tanh} kh \tag{9}$$

Since

$$k = \frac{2\pi}{\lambda} \tag{10}$$

we have

$$c^2 = \frac{g\lambda}{2\pi} \operatorname{Tanh} \frac{2\pi h}{\lambda} \tag{11}$$

If the water is very shallow, so that

$$\operatorname{Tanh} \frac{2\pi h}{\lambda} = \frac{2\pi h}{\lambda}$$

then

$$c^2 = gh \tag{12}$$

In this case we have as examples of $\frac{h}{\lambda}$ limits:

$\frac{2\pi h}{\lambda}$	$\operatorname{Tanh}\frac{2\pi h}{\lambda}$	$\frac{h}{\lambda}$
0.050	0.04996	0.0080
0.100	0.09967	0.0160
0.250	0.24492	0.0398
0.500	0.46212	0.0796

If the water is sufficiently deep so that

$$\operatorname{Tanh}\frac{2\pi h}{\lambda} = 1$$

then

$$c^2 = \frac{g\lambda}{2\pi} \tag{13}$$

In this case $\frac{h}{\lambda}$ limits are as follows:

$\frac{2\pi h}{\lambda}$	$\operatorname{Tanh}\frac{2\pi h}{\lambda}$	$\frac{h}{\lambda}$
1.832	0.950	0.292
2.647	0.990	0.421
3.800	0.999	0.605

Other relationships between velocity, c, wave length, λ, period, T, wave number, k, and frequency, σ, of deep water waves, are:

$$c = \sqrt{\frac{g\lambda}{2\pi}} = \frac{\lambda}{T} = \frac{gT}{2\pi}, \tag{14}$$

$$\lambda = \frac{2\pi}{g}c^2 = \frac{g}{2\pi}T^2, \tag{15}$$

$$T = \sqrt{\frac{2\pi}{g}\lambda} = \frac{2\pi}{g}c, \tag{16}$$

$$k = \frac{2\pi}{\lambda} \tag{17}$$

and

$$\sigma = \frac{2\pi}{T} \tag{18}$$

b. Pressure Fluctuations at Surface and Bottom

The elevation of water, ζ, at the free surface above the point x, y, 0 is:

$$\zeta = \frac{1}{g}\frac{\partial\phi}{\partial t} \tag{19}$$

Hence, from (6)

$$\zeta = \frac{kcr}{g}\operatorname{Cosh} kh \operatorname{Sin} k\,(x\text{-}ct) \tag{20}$$

Letting a be the amplitude of ζ,

$$a = \frac{kcr}{g}\operatorname{Cosh} kh, \tag{21}$$

then

$$\zeta = a \operatorname{Sin} k\,(x\text{-}ct) \tag{22}$$

thence from (6)

$$\phi = \frac{ga}{kc} \frac{\text{Cosh } k\,(z + h)}{\text{Cosh } kh} \text{Cos } k\,(x\text{-}ct) \tag{23}$$

The pressure, P, at any depth is:

$$P = \rho g\,(\zeta - z) \tag{24}$$

and the pressure fluctuation, ΔP, at any depth is:

$$\Delta P = \rho ga \frac{\text{Cosh } k\,(z + h)}{\text{Cosh } kh} \text{Sin } k\,(x\text{-}ct) \tag{25}$$

At the bottom, $z = -h$, and

$$\text{Cosh } k\,(z + h) = 1 \tag{26}$$

Hence the pressure fluctuation at the bottom, ΔP_h is

$$\Delta P_h = \rho ga \frac{\text{Sin } k\,(x\text{-}ct)}{\text{Cosh } kh} \tag{27}$$

At the surface, $z = 0$, and

$$\Delta P_s = \rho ga \text{ Sin } k\,(x\text{-}ct) \tag{28}$$

Thus, the ratio of instantaneous pressure fluctuations at the bottom to those at the surface[11] is

$$\frac{\Delta P_h}{\Delta P_s} = \frac{1}{\text{Cosh } kh} \tag{29}$$

2. Bottom-surface Relations of Observed Fluctuations

It may be noted here that some opposition has been offered to the following method of analysis in that it does not fall entirely within the frame work of customary shallow water wave theory. In the case of certain special wave analyses this criticism is reasonable. However, where we deal with complex wave patterns, composed of both shallow and deep water phenomena, and where the effects of wave transformation are imperfectly known and require empirical correction in each special case, it has appeared advisable to treat the problem in the most direct manner; namely, to approach it from established relationships for deep water waves. Hence, any alterations resulting from shallow water transformations are included in the deduced empirical factors. The very consistency of the results obtained, appear to justify the method of approach in that they serve as a means to an end. In the future, when more observational data is available, and when the phenomena of shallow water wave transformations are better understood, some refinement to this approach may be desirable.

In connection with the above, a comparison of surface and bottom interference patterns (Table 2) shows that at Cuttyhunk the bottom wave period ranged 1.6 to 3.0 times that at the surface and at Bermuda it was 2.6 to 3.8 times the surface. Hence in directly comparing surface and bottom wave profiles, short period waves play a prominent role in surface displacements which probably are not reflected in pressure changes at instrument depths for either locality.

---

[11] This relationship has been extensively used by earlier investigators. It is derived in Horace Lamb's Hydrodynamics.

Surface wave heights (ΔH_s) and wave periods (ΔT_s) were obtained directly by observation. Bottom pressure recordings were evaluated in terms of bottom wave heights (ΔH_h) and bottom wave periods (ΔT_h) from instrument factors. Comparison of surface and bottom wave profiles at Cuttyhunk and Bermuda (Figures 6 to 15) permitted good identification of more than 50 prominent pairs of wave crests. From these, the ratios, $\Delta H_h / \Delta H_s$ were computed. To this series of ratios were added those computed from the interference patterns of Table 2.

The ratios $\Delta H_h / \Delta H_s$ are plotted against bottom wave periods (ΔT_h) for comparison with theoretical values, from Equation 29, in Figure 19. It is apparent that the computed ratios are generally lower than to be expected from theory, as expressed by Equation 29. This being the case, surface wave height computed from bottom pressure records by application of Equation 29 will be too low.

It is essential to examine the extent of the departures of ratios computed from observation, from that given by theory (Equation 29), and, if possible, to establish from our experiments an empirical factor which when applied to bottom pressure records gives reasonable estimations of the causative sea surface wave patterns. Measurements of underwater pressure fluctuations are highly significant to research on sea surface waves, in that they should provide reasonably accurate state of the sea estimations. In fact, estimations of the state of the sea surface from bottom pressure fluctuations are to be preferred over those of direct observation, in that effects of very short period waves, which confuse the surface picture, are eliminated. However, application of the hyperbolic cosine relation of Equation 29

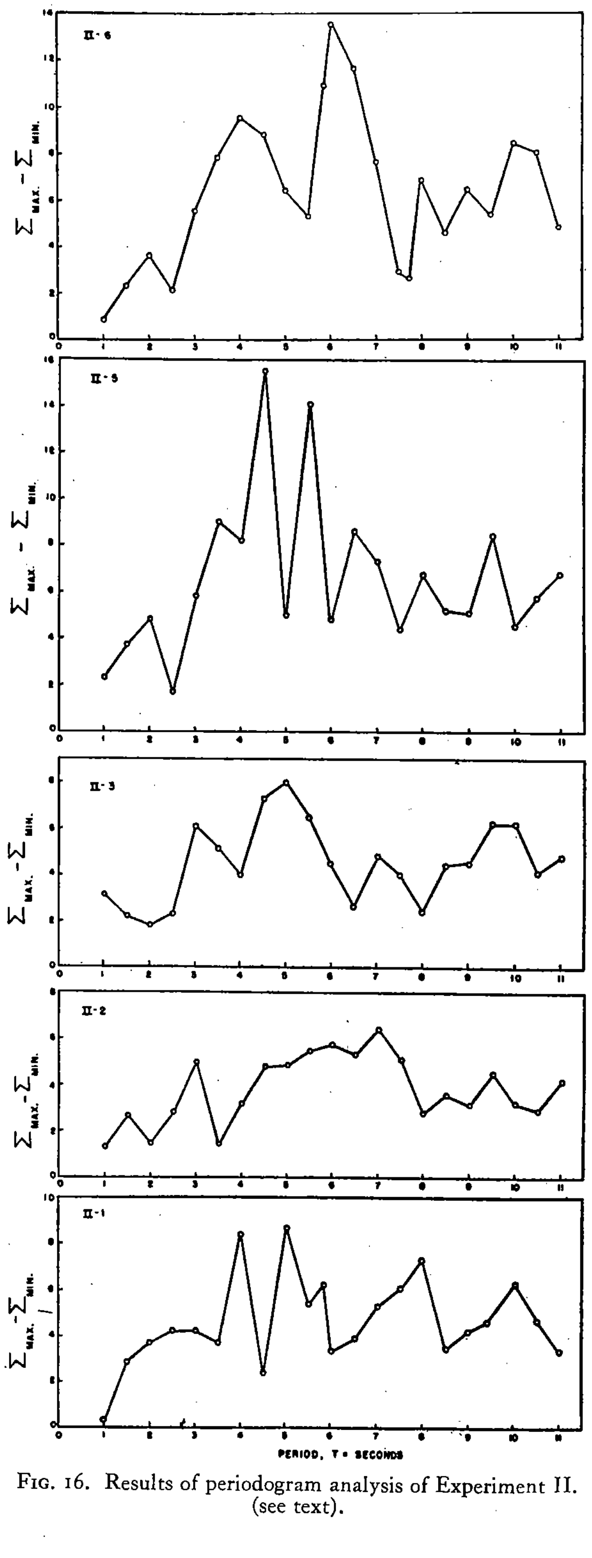

FIG. 16. Results of periodogram analysis of Experiment II. (see text).