

*Journal of Geophysical Research, Biogeosciences*

Supporting Information for

**A Phytoplankton Model for the Allocation of Gross Photosynthetic Energy Including the Trade-offs of Diazotrophy**

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**Introduction**

Three supplemental sections are included. The first (**Text S1)** details how nutrient uptake kinetics are treated in PCAM, including allometric scaling of uptake as illustrated in **Figure S1**. **Text S2** outlines an additional validation of PCAM against observations from the California Cooperative Oceanic Fisheries Investigations (CalCOFI) program, with results shown in **Figure S2**. A full derivation for the optimal allocation solution to PCAM as described in Section 2.8 is included in **Text S3**. Supporting environmental and resource fields for the comparison to GEOTRACES as described in section 3.2.2 is shown in **Figure S3**.

Text S1. Nutrient Uptake Kinetics

Nutrient uptake kinetics were modeled based on existing theory such that cells can acclimate by adjusting the number of uptake sites (*n*) (up to a maximum density) in order to improve uptake affinity at low nutrient concentrations. Each uptake site has a fixed surface area (*A*) and fixed handling time for uptake (*h*). Following Aksnes and Egge (1991) the uptake rate in mol s-1 cell-1 ( is expressed as:

(S1-1)

where transfer velocity, *k,* is assumed equal to *D/r*, where *D* is diffusivity and *r* is cell radius. Temperature-dependent diffusivities are used for nitrate and phosphate (Yuan-Hui and Gregory, 1974). We specify that the maximum number of uptake sites is equal to a fixed fraction of cell surface area () such that:

(S1-2)

where is cell surface area assuming spherical geometry. The maximum fraction devoted to uptake, , is constant in PCAM.

Compared to a traditional Michaelis-Menten nutrient uptake parameterization where uptake *VS* dependents only on seawater nutrient concentration, a cell can alter *n,* which in turn allows *VS* to vary depending on cell acclimation state as well as on ambient nutrient concentration. Such flexibility allows for processes such as luxury uptake. In oligotrophic conditions when *S* is low (denoted with the subscript ‘stv’), uptake rate in (5) reduces to:

(S1-3)

Normalized nutrient affinity (*αS*) is a better metric of the ability of phytoplankton to compete for nutrients, rather than either *KS* (or *VS*) alone (Button, 1978; Fiksen et al., 2013). Here, *αS* is the initial slope of *S* vs *VS* and is defined as:

(S1-4)

Thus, affinity is proportional to *r* and volume-normalized nutrient affinity scales as *r*−2 and is maximum when *n* = *nmax*, illustrating the competitive advantage of smaller cells under low nutrient conditions.

When S is large (replete conditions), (5) simplifies to uptake that is limited by ion handling time:

(S1-5)

where the subscript ‘*rep*’ denotes nutrient replete conditions and *nrep* is the number of uptake sites required such that satisfies the maximum biosynthetic rate ().

For nutrient uptake under dynamic conditions environmental history and preconditioning influence uptake kinetics. A key observation has been that nutrient starved phytoplankton, when re-exposed to replete conditions exhibit a maximum uptake rate much higher than phytoplankton that have been acclimated to replete conditions (Harrison et al., 1989). Equations (8) and (10) describe uptake conditions when acclimated to nutrient starved and replete conditions, respectively. The range of uptake site number from *nrep* to *nmax* represents the range of acclimation possible for a given phytoplankton cell (Fig 2).

For a given *n*, can be rearranged to a familiar Michaelis-Menten form (Aksnes and Egge, 1991) such that:

(S1-6)

with a maximum uptake rate and half saturation for uptake of *KS* = 1/(*Akh*).

Under balanced growth conditions, nutrient uptake is equal to the product of growth rate and cell quota () (Droop, 1968, 1973, 1974). The quota term, *QS*, in PCAM can vary from a maximum value (at full allocation to PSA and RIB to a theoretical minimum value, with zero allocation to PSA and RIB (. Morel (1987) stressed that acclimation in which *QS* decreases with decreasing growth rate has the implication that the half saturation constant for growth () is significantly lower than the half saturation for instantaneous uptake (*KS*). Morel derived a general relationship between *KS* and such that:

(S1-7)

where the subscripts *stv* and *rep* refer to nutrient starved and nutrient replete conditions. The above equation is linked to the transporter model through recognition that = *nmax/h* and = *nrep/h*. Thus, with increasing nutrient limitation, the number of transporters increases towards *nmax*, quota is reduced and affinity increases to its maximum value (*nmaxk*). The steady-state number of uptake sites for nutrient S is:

(S1-8)

The resulting relationships between nutrient uptake parameters, cell quota and growth shown in Fig 2 are entirely consistent with the Aksnes and Egge transporter model when we assume that plankton can vary *n* during acclimation. Increasing *n* under nutrient starvation increases , preparing phytoplankton to take advantage of any pulses in nutrient availability. The result from Figure 2 shows that instantaneous uptake for phytoplankton adapted to starved conditions () and for phytoplankton adapted to replete conditions ().

The allocatable transporter approach enables a dynamic uptake response to changes in nutrient concentration. For example, phytoplankton acclimated to low nutrient conditions can respond with surge uptake when a pulse of nutrients is introduced. Under balanced conditions, nutrient uptake reduces to a familiar Michaelis-Menten formulation with uptake parameters that scale allometrically with cell size (Aksnes and Egge, 1991; Litchman et al., 2007). Nutrient affinity (, a critical parameter for competitiveness for low nutrient concentrations is:

(S1-9)

The theoretical approach leads to allometric scaling such that , and . Nutrient affinity normalized to cell carbon thus scales as *r−2r1−γ* or

Text S2. CalCOFI Productivity

The CalCOFI sampling area spans a range of ocean biomes, from productive, near coastal conditions in an eastern boundary upwelling system to fairly oligotrophic offshore waters (Bograd et al., 2003). Program measurements include 6-hr PP(14C), nitrate, phosphate, percent PAR relative to surface and chlorophyll, all the parameters necessary to estimate and validate *NC* as calculated using PCAM and Eq. 18. CalCOFI 6-hr PP(14C) is converted to an estimate of by multiplying by the ratio 1.81 as determined from a local 14C intercomparison (Eppley, 1992). Analysis of CalCOFI data included all observations from 1985 through January 2012, which, when filtered for samples including all of the necessary variables, resulted in 7848 data points. Only samples with nutrient concentration above the limit of detection and within the upper 120 m were considered. PAR was calculated from climatological surface PAR from MODIS Aqua and percent PAR as measured for each sample by the CalCOFI program. PCAM-predicted *NC* agreed well with observed *NC* with no major biases apparent as a function of input variables such as PAR and nitrate (Fig. S2). PCAM does appear to overestimate some observations where PAR is very high (>500 μmol photons m-2 s-1) and nutrients low. This may be a product of photoinhibition, which is not included in the PCAM model.

Text S3. Derivation of Optimal Allocation for Macronutrients and Light

(S3-1)

(S3-2)

(S3-3)

Substituting (S3-2) and (S3-3) into (S3-1) yields:

(S3-4)

which can be rearranged using the relationship to equal:

(S3-5)

which can be arranged in a quadratic form :

(S3-6)

Further, we note that the ratio is a constant and can be solved using the ATP balance and the relationship :

(S3-7)

substituting yields:

(S3-8)

(S3-9)

The solution for is therefore

(S3-10)

where:

(S3-11)

can subsequently be solved from (S3-9) and by conservation of mass:

(S3-12)



Figure S1. PCAM nutrient kinetics. Above: relative growth rate (red) and relative number of uptake sites (black). Below shows steady-state nutrient uptake (black) with uptake parameters as described in Eqs. (S1-5), (S1-6) and (S1-7) shown in blue. The nitrate concentration corresponding to half saturation for instantaneous uptake (K), the half saturation for the product of growth and quota (KμQ) and the half saturation for steady-state growth rate (Kμ) are indicated by the vertical red lines.



Figure S2. CalCOFI net primary productivity (NPP) from 14C versus PCAM-inferred NPP (left) and chlorophyll (right) for sampling locations in the upper 120 m from 1985-2012 (n = 7848). Color scale shows log10(PAR) (μmol photons m-2 s-1). Black line is a 1:1 line. The dashed line is the best-fit line log10(PCAM NPP) = 1.031 log10(CalCOFI NPP) − 0.153.



**Figure S3.** Properties for GEOTRACES ga02 and ga03 Sections. PAR is in units of μmol photons m-2 s-1 and Fe, P and N are in mol m-3.