WHOI-87-2

**King's Bay/Cumberland Sound, Georgia
Part II: Numerical Modeling**

by

David G. Aubrey
Virginia A. Fry

Woods Hole Oceanographic Institution
Woods Hole, Massachusetts 02543

Daniel R. Lynch

Dartmouth College
Hanover, New Hampshire 03755

March 1987

Technical Report

Funding was provided by the Naval Facilities Command through the Naval Civil Engineering Laboratory. Partial support came from the Woods Hole Oceanographic Institution's Coastal Research Center.

Reproduction in whole or in part is permitted for any purpose of the United States Government. This report should be cited as:
Wood Hole Oceanog. Inst. Tech. Rept., WHOI-87-2. CRC-87-1.

Approved for publication; distribution unlimited.

Approved for Distribution:

David A. Ross, Chairman
Department of Geology and Geophysics

KINGS BAY/CUMBERLAND SOUND, GEORGIA

PART II: NUMERICAL MODELING

DAVID G. AUBREY

VIRGINIA A. FRY

WOODS HOLE OCEANOGRAPHIC INSTITUTION

and

DANIEL R. LYNCH

DARTMOUTH COLLEGE

INTRODUCTION

The decision to locate a Trident submarine base in Kings Bay has major implications for navy maintenance dredging requirements due to high sediment loads in this environment. Kings Bay Naval Base is located approximately 10 km from the entrance to Kings Bay/Cumberland Sound (KBCS). These 10 km of channel must be maintained at project depth to allow free traverse of navy vessels. To construct a viable naval harbor at this location, the navigation channels, berthing areas, and turning basins must be designed from the natural channel configuration. This alteration of the natural configuration will alter the historical patterns of sediment accumulation. Prediction of future shoaling rates and patterns requires numerical modeling to forecast the effects of these channel expansions. Based on these numerical results, a maintenance dredging program must be designed, funded, and implemented to allow the harbor to remain viable for navy purposes.

The present study represents the second part of a two-part study undertaken by the Naval Civil Engineering Laboratory under the guidance of Dr. James Bailard and the Woods Hole Oceanographic Institution under the guidance of Dr. David Aubrey. The first phase of the study was a month-long intensive field

study of KBCS, including deployment of 10 sets of velocity, sea surface, transmission, conductivity, and temperature sensors. This initial phase is reported in a companion report--Part I: Field Experiment. This present report details the numerical modeling efforts undertaken as the second phase of this study. It is complementary to the numerical modeling efforts carried out by the Waterways Experiment Station (WES) for the Kings Bay program. Whereas the WES modeling is predictive in nature, the WHOI modeling is diagnostic. The predictive modeling attempts to model the two-dimensional sediment transport patterns using a single frequency tide as input to bathymetry representative of some test scenario. The diagnostic modeling is designed to examine the sensitivity of the bay's response to the incident spectral tide in a system response sense. It is used to examine how sensitive the tidal response is to geometry of the dredged channels, to various systems of dikes and tide gates, and to various dredging alternatives, for instance. Diagnostic modeling will not provide the horizontal resolution of the predictive modeling and is not as useful in planning specific engineering structures. However diagnostic modeling will increase the understanding of the response of the system as a whole, indicating which parameters (extent of dredging, side slope of dredge cuts, width of dredged channel, extent of diked area, etc.) most severely influence the behavior of the estuary. This sensitivity analysis allows better focusing of modeling and design efforts towards those areas where more of an improvement might be expected.

The diagnostic modeling relies on the principle that sedimentation in an estuary is controlled primarily by tidal circulation and storm response. The distortion of the tide co-oscillating within the estuary and the adjacent ocean is determined by the estuarine geometry. The geometry of the estuary and its dynamical characteristics dictate whether the tide will become distorted such as to have a longer falling or rising tide. This concept leads to the idea of

ebb and flood dominance. Ebb dominant estuaries are those where the cross-sectional and depth-averaged ebb flow is stronger than corresponding flood flow (flood flow is longer, but weaker). Flood dominant estuaries are those where the cross-sectional and depth-averaged flood flow is stronger than corresponding ebb flow (ebb flow is longer, but weaker). In the case of flood dominance, coarse sediment transport may be greater during flood tide, thus the estuary might infill more rapidly than otherwise. In the case of ebb dominance, the estuary might be more self-flushing, and maintain its channel better. These impacts on sedimentation through self-flushing or accumulation will control maintenance dredging requirements in part. Definition of the various channel lengths having different behaviors will help design mitigation measures to reduce these dredging requirements. Although there will be lateral and vertical variations in this flood and ebb dominance, the general classification is still valid. Salt intrusion and flocculation problems increase the complexity of these systems; however salinity and chemical effects can be included in these diagnostic models to improve their utility.

Aubrey and Speer (1985), Speer and Aubrey (1985) and Aubrey (1986) investigated the concept of flood and ebb dominance using a one-dimensional numerical model (depth- and cross-sectionally averaged). Their results indicated that the geometry of the navigation channel and the extent of tidal flats (or alternative storage areas) influence the response of the estuary to the tide. Regions having a larger time-variable cross-sectional channel area experienced stronger flood dominance. Increases in tidal amplitude-to-channel depth ratios also increased the tendency towards flood dominance. Decreased amplitude-to-channel depth ratios increase the tendency towards ebb dominance, as do increases in extent of tidal flats. These factors arise from the nonlinear nature of the equations of motion governing fluid flow in shallow estuaries. These nonlinear terms (specifically the kinematic nonlinearity,

the advective nonlinearity, and the quadratic friction term) result in distortion of the tide, often represented in a Fourier sense as a parent tide and a series of harmonics. Harmonics generated by interaction of a tidal constituent with itself is termed an overtide, whereas interaction of two tidal constituents having different frequencies generates compound tides.

Distortion of the surface tide does not translate directly into a distortion in direction of net sedimentation. The sediment transport equation has a nonlinear relation to velocity, and thus additional distortions in net sedimentation could occur. However, the distortion in the surface tide is a first order indicator of the direction of net transport for coarse sediments; for fine suspended matter the relationship is much more complex. For the case of coarse sediment transport using a simple one-dimensional hydrodynamical model, the magnitude of net sedimentation (the ratio of flood to ebb sediment transport) is derived. For magnitudes of distortion typical of shallow estuaries, the net sediment transport can be a significant fraction (40%) of the gross sediment transport.

The present study builds on the concept of flood- and ebb-dominance, and introduces various types of modeling to achieve the final goal of developing a model useful for examining a broad suite of dredging/engineering alternatives for modifying estuarine and lagoonal environments. It makes use of one- and two-dimensional models of estuarine flows, coupled with sediment transport equations. The one-dimensional model is essentially that of Speer and Aubrey (1985), while the two-dimensional model is that of Lynch and Gray (1979,1980).

Modeling of estuarine environments requires several considerations unique to these problems. First, the tidal amplitude can be a significant fraction of the total water depth, so the full nonlinear terms in the equations of motion must be maintained. Second, the spatial scales of variability are non-uniform, so gridding of the estuary must be done with care. If the smallest

scale of variability is maintained within the estuary, the grid size becomes excessively large and the computational requirements increase unacceptably. A flexible grid system would enable those areas having more stringent resolution requirements to be resolved more adequately. Third, the geometries of estuaries are complex, yielding a difficult gridding puzzle. A uniform or rectangular grid might introduce edge effects or boundary effects severely influencing the solution (in part by propagating numerical instabilities). Fourth, some estuaries have extensive tidal flats that are dynamically important to estuarine fluid and sediment transport. These tidal flats flood and uncover with the tide. Proper representation of tidal flat effects is essential in many problems. Often tidal flats are approximated as being storage areas, having no dynamical contribution. Alternatively the effects of tidal flats can be included in both momentum and continuity to model more properly the sediment transport in these regions.

These and other considerations mandate a careful selection of numerical modeling techniques to represent correctly the physics of these systems. Some models may be valid for certain estuaries and not others; the goal is to find a flexible model that has reasonable computational speed while retaining general applicability. The use of three-dimensional models has not been addressed in great detail in this introduction. Eventually a three-dimensional model will be applied to estuarine physics. However inadequacies in our understanding of the physics of three-dimensional estuarine circulation preclude use of three-dimensional models for diagnostic purposes. Instead, the vertical behavior of the fluid is decoupled from the two horizontal dimensions. A depth-integrated numerical model is applied to the two horizontal dimensions. These results then are used to calculate sediment transport using decoupled vertical equations and sediment transport equations. Although a limitation in

the validity of these models, much can be learned about estuarine physics using this approach. Once we improve our understanding of these physics, the complexity of the models can be increased.

In the following sections are discussed the results from one- and two-dimensional numerical estuarine circulation models.

ONE-DIMENSIONAL MODELS

The one-dimensional numerical model used here is the Speer and Aubrey (1985, SA) model, which is depth- and cross-sectionally averaged. The equations of motion used for this model are:

Momentum Equation:

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x} \frac{U^2}{A} = -g A \frac{\partial \zeta}{\partial x} - \frac{\tau_b}{\rho} P$$

Continuity Equation:

$$\frac{\partial \zeta}{\partial t} + \frac{1}{b} \frac{\partial U}{\partial x} = 0$$

where $\zeta(x,t)$ is the sea surface elevation, t is time, b is the channel width, $U(x,t)$ is the cross-sectional flux, τ_b is the average shear stress on the solid boundaries, P is the wetted channel perimeter, A is the channel cross-sectional area, and ρ is the water density. The problem is closed mathematically by a quadratic friction:

$$\tau_b = f |U| U / (A \cdot A)$$

where f is a dimensionless friction factor. Principal nonlinear effects enter through quadratic friction, the advection terms in the momentum equation, and tidal interactions with estuarine geometry in the continuity equation (kinematic nonlinearity).

The model uses a trapezoidal geometry (figure 1), having two distinct elements: a trapezoidal channel transporting all momentum in the system, and shallow, sloping tidal flats acting in a storage capacity only. These one-

dimensional equations assume a small horizontal aspect ratio ($h/b \ll 1$), long narrow channels ($b/L \ll 1$), a well-mixed water column, and little freshwater runoff.

The two scaling parameters central to describing the system response to an incident tide are the nonlinear parameter (a/h ; tidal amplitude divided by water depth) and a geometric parameter ($\Delta b/b_o$). The nonlinear parameter (a/h) becomes dynamically significant when it is large, at which point the size of the geometric parameter becomes important. The model can be run for different geometries, including different depths and different extent of tidal flats. Tidal flat extent is characterized by the ratio A_f/A_c (ratio of tidal flat area to channel area). By varying A_f/A_c , $\Delta b/b_o$, a/h , b_o/h , and other parameters, the nonlinear tidal response can be determined. Results are presented in terms of two indicators of the nonlinear transfer: M_4/M_2 which is the ratio of the overtide to the parent tide, and M_2 - M_4 phase, defined as:

$$\Delta\phi = 2\phi_{M_2} - \phi_{M_4}$$

Both of these factors are diagnostics of the magnitude of nonlinear interaction (related to interaction coefficients); the relative phase provides insight into which of the three dominant nonlinear mechanisms is the most important in effecting these nonlinear transfers.

As an example of the results obtained by such modeling is a channel 7 km long having the channel cross-sectional shape proportional to that found at Murrells Inlet, South Carolina (Aubrey et al., submitted): Channel width (b_o) is 58 times channel depth (h_1), and $\Delta b/b_o = 0.45$. Tidal flats in the system drain completely at low tides, and the offshore tidal amplitude is constant at one meter. Inshore, the nonlinear distortion parameters (M_4/M_2 , M_4 - M_2 relative phase) are measured at 6 km along the channel. Model

runs indicate stability fields for these geometries (figure 2). As a/h increases, the M_4/M_2 ratio increases. As A_f/A_c increases, M_4/M_2 increases for low a/h . However, for larger a/h as A_f/A_c increases, M_4/M_2 ratio decreases. The relative phase in sea surface exhibits a characteristic behavior. As a/h increases, M_2-M_4 phase decreases rapidly; this represents a change from ebb dominance to flood dominance as a/h increases. The ratio A_f/A_c exerts a more minor control on the behavior of the M_4-M_2 phase. In general, its effects are much less significant than the a/h ratio.

The above example (figure 2) illustrates the dynamical behavior of these systems under idealized geometries. The dominant control of a/h is exhibited, as is the influence of A_f/A_c . The geometric term $\Delta b/b_o$ exerts similar control on nonlinear behavior when a/h is large; its effects are not demonstrated in this example since $\Delta b/b_o$ was kept constant.

Another example can illustrate these effects in a different way. Using the model parameters from the previous example, the stability field can be represented in a two-dimensional plot. The contours of M_4-M_2 relative phase (figure 3) are a function of a/h and A_f/A_c . For small a/h , all estuaries will be ebb dominant, regardless of extent of tidal flats. For large enough a/h (0.25 and greater) all estuaries will be flood dominant, regardless of extent of tidal flat. For estuaries having a/h between 0.05 and 0.25, the estuary can be either flood or ebb dominant, depending on the ratio A_f/A_c .

For the same geometry, a two-dimensional stability field can be derived for the M_4/M_2 ratio (figure 4). Here the impact of the nonlinearity is shown as a function of a/h and A_f/A_c , as in the previous examples. In general, large a/h forces increased nonlinear behavior, while small a/h reduces the nonlinearity. A_f/A_c has only a weak control on magnitude of asymmetry.

At low values of a/h , the tide is weakly nonlinear and increases in nonlinearity as A_f/A_c increases. For larger a/h , increasing A_f/A_c decreases the nonlinearity (although weakly).

Friedrichs (1986) and Aubrey et al. (submitted) present data from 11 estuaries and assess their behavior in relation to the arguments provided above. In general the natural estuarine behavior is in agreement with the model results. The trends of varying M_4-M_2 phase and M_4/M_2 ratio parallel those found in natural estuaries (figures 3 and 4), suggesting that this one-dimensional modeling provides significant insight into estuarine processes (neglecting freshwater inflow, of course). The impact of freshwater discharge is to increase the net discharge of water from the estuary, increasing the asymmetry of the tide towards an ebb dominant case. These effects can be included in the one-dimensional modeling simply, but were not included in the initial calculations to allow us to understand better the physics of estuarine tidal response. The inclusion of riverine influence is being incorporated into these models at the present time.

King's Bay/Cumberland Sound was one of the estuaries presented in the analysis by Friedrichs (1986) and Aubrey et al. (submitted). Its offshore tidal amplitude is approximately 0.9 m (M_2 tide, not total tide), with a mean depth of approximately 13 m, an a/h ratio of 0.07, A_f/A_c ratio of 0.28, an average M_4/M_2 ratio of 0.029, and an average M_4-M_2 phase of 261° . According to the stability diagrams for a 7 km channel (figures 3 and 4) given the A_f/A_c and a/h ratios, the relative phase according to the Speer/Aubrey model would be between 200° and 250° , close to the actual measurement of 261° from the bay. The M_4/M_2 ratio for these conditions would be approximately 0.001, much less than the actual ratio of 0.029. King's Bay therefore is more ebb dominant than predicted by the simple model. Part of the rationale for

this difference is that the model does not include M_4 forcing at the mouth. Another reason for this difference is that the model ignores freshwater inflow. As shown by Parker (1984) and others, freshwater inflow creates other nonlinear interactions which increase the nonlinear ratios. These must be investigated further within the framework presented above to evaluate quantitatively the impact of freshwater inflow on hydrodynamic distortion of the tide.

The data from this model suggest that small changes in estuarine geometry will not have major impacts on the flushing rates of Kings Bay. The dominant control here is the depth of water which limits the nonlinear distortion through a/h . According to this model, the most effective sedimentation controls may not be basin wide, but may have to have more local impacts on the Kings Bay branch, in particular. Options might include diking, tide gates, etc. to increase artificially the local discharge during ebb while limiting the flood flow. There is a trade-off in this option, however. While the increased ebb discharge would improve the hydrodynamic conditions for flushing sediment, more sediment would be arriving from the Crooked River and upestuary reaches. If the salt wedge were to form within Kings Bay as a result of the tide gate or dike, increased sedimentation might result locally within the bay. To assess these tradeoffs, a numerical model including buoyancy effects and coupled to a suspended sediment transport model, must be run.

TWO-DIMENSIONAL NUMERICAL MODELING

The requirements for accurate modeling of complex estuarine systems dictate a need for more sophisticated numerical techniques to resolve the fine scale detail and the lateral variability within these systems. The one-dimensional modeling presented above is a useful tool for first order understanding of the complexities of these estuaries. The overall system response of the estuaries

can be defined with these models to provide guidance for more sophisticated modeling, while keeping expense and computational time at a minimum. However the one-dimensional model does not address lateral variability in estuaries nor vertical structure in suspended sediment transport. Tidal flat problems are incorporated implicitly by formulating the time-varying channel cross-sectional area in a suitable manner. The assumption is made in the one-dimensional models that tidal flats serve as water storage areas, and do not contribute to momentum considerations. This assumption also is made in many two-dimensional models, using devices such as the wier solution to allow storage but prevent momentum transfer.

The major elements that a two-dimensional model of estuarine circulation must address include:

- Time-varying channel cross-sectional area
- Tidal flat covering and uncovering (movable boundaries)
- Full nonlinear terms in the equations of motion
- A sophisticated gridding scheme to minimize the number of computational nodes, providing high resolution where needed.

Although there are many techniques available for addressing these problems, each has their drawbacks. Some examples include the models of Thacker (1977, 1978a, b), which incorporate a non-uniform grid using finite difference techniques; the drawback of this model includes poor stability. Another example is the boundary-fitted coordinate method (Johnson, 1982; Thompson et al., 1977; Spaulding, 1984), where the numerical grid is transformed into a rectangular grid using a coordinate transformation. Although useful for a variety of problems, there are difficulties in extremely complex domains. The method does not take advantage of the sparse gridding opportunities of other techniques.

For this modeling we have selected a finite element technique, which has several advantages. First, it allows for variable grid spacing so nodes can be concentrated in areas requiring high resolution, and used more sparsely in other areas. Second, it has been modified to include a movable boundary scheme, allowing the tide flat problem to be addressed. Third, it has been evaluated against analytical solutions and has compared favorably. Disadvantages include its relative newness in hydrodynamic and oceanographic studies compared to the better known finite difference techniques. Although Connor and Wang (1973), Wang and Connor (1975), Lynch and Gray (1979, 1980) and others have applied these techniques to circulation problems, they still are not as well understood as the finite difference techniques. Finite element techniques are slower computationally than finite difference techniques, for an equal number of computational nodes. However finite element grids can be more sparse than finite difference grids, resulting in tremendous computational savings. These savings can offset the increased computational time, making them comparable in speed to finite difference techniques.

The model chosen is that of Lynch and Gray (1979, 1980) in which the shallow water wave equation along with the primitive momentum equation forms the basis for the hydrodynamic modeling. Use of this scheme, rather than the primitive shallow water equations, is predicted both to damp and to propagate short wavelength noise while maintaining high accuracy for the predominant tidal frequencies.

The derivation of the wave equation from the shallow water equations proceeds from the primitive equations:

Continuity:

$$\frac{\partial H}{\partial t} + \nabla \cdot (H \underline{v}) = 0$$

Momentum:

$$\frac{\partial H \underline{v}}{\partial t} + \nabla \cdot (H \underline{v} \underline{v}) + g H \nabla \zeta + f \times H \underline{v} + \tau H \underline{v} = H \underline{\psi}$$

where H = total fluid depth
 ζ = elevation of free surface
 $\underline{v}$ = vertically averaged velocity
 f = Coriolis parameter
 τ = friction parameter
 ψ = accounts for wind stress and atmospheric pressure variations
 ∇ = gradient operator in horizontal plane

The continuity equation is differentiated with respect to time and the conservative form of the momentum equation is substituted into the differentiated continuity equation, yielding the nonlinear shallow water wave equation:

$$\frac{\partial^2 H}{\partial t^2} + \tau \frac{\partial H}{\partial t} - \nabla \cdot [\nabla \cdot (H \underline{v} \underline{v}) + g H \nabla \zeta + f \times H \underline{v} - H \psi] - H \underline{v} \cdot \nabla \tau = 0$$

The finite element method discretizes the primitive momentum equation and the shallow water wave equation in space. Finite difference techniques are then used to discretize in the time domain. Details of this approach are provided in Lynch and Gray (1979).

The moving boundary formulation is provided in Lynch and Gray (1980). Here, the moving boundary problem is solved using a deforming numerical grid, accomplished such that node points at the perimeter of the numerical domain always coincide with the moving boundary. Although more appealing intuitively than a formulation having densely spaced fixed grid points, there are disadvantages (Lynch and Gray, 1980) involving the practical and theoretical problems of ensuring that boundary nodes track the moving boundary and account accurately for the effects of grid deformation. Lynch and Gray (1980) develop a general moving boundary formulation using Galerkin techniques applicable to both parabolic and hyperbolic problems, usable in three dimensions, and valid for higher-order triangular and quadrilateral finite elements. They apply

these techniques to the hyperbolic wave equation developed above, coupling the node motion to the state of the fluid by means of a boundary condition. Lynch and Gray (1980) apply this method to a channel and a harbor test case indicating these solutions are reasonable and well-behaved, introducing no new instabilities to the basic finite element formulation of the problem. The model is a lumped-explicit wave equation model, which means that depth as well as velocity is obtained without matrix inversion. Matrix inversion can be a computationally slow process. The bottom friction is characterized by the Manning-Chezy formula:

$$\tau(x,y) = \frac{g|V|V}{C^2H}$$

$$\text{where } C = \frac{1.49 H_1^{1/6}}{n} \quad n = .035$$

τ is finite throughout the interior of the model but approaches infinity as $H \rightarrow 0$ at the moving boundary. Basic research is needed to solve this problem. For the present, numerical experiments have shown good results when the friction depth is based on an average depth in the boundary elements. "Depth for friction at shore" is the terminology given to the depth that determines the friction at the moving boundaries.

The stability of this model is dependent on the gravity-wave Courant number, which is governed by

$$gH \frac{\Delta t}{\Delta x}^2 \leq \frac{1}{3}$$

The moving boundary model was applied to several situations of interest to simulation of shallow bays and estuaries. Both one- and two-dimensional model runs were made, using various channel lengths, water depths, and node spacing.

Interpretation of the two-dimensional modeling will begin by looking at the data with insight from knowledge attained from the 1-d model. In the 1-d modeling by Aubrey and Speer, diagnostic testing found that a/h , A_f/A_c and

$\Delta b/b$ were important parameters in determining the ebb or flood dominance of an estuary. The type of dominance is a first order indication of the sediment transport of the area. Two-dimensional modeling of sea surface elevation and circulation of an estuary should raise this first order approximation to a more detailed understanding. Initial interpretations will examine the effect of variations in a/h . More diagnostic testing must be done in order to realize fully the sensitivity of sea surface, U , and V velocity when varying different parameters. Knowledge of the terms in the equations that are responsible for the data response will increase further our understanding of the dynamics of estuarine circulation.

The velocities in an estuary provide a closer link to sediment transport than does sea surface elevation. Understanding the sensitivity of the velocity in the cross-channel direction to varying channel geometry will clarify understanding of sediment transport across the channel. Along-channel velocity and the extent of the nonlinear distortion will determine the amount of transport along channel.

1) 1-dimensional runs

As a first quantitative test of the moving boundary model, one-dimensional simulations of long waves on a uniformly sloping beach, for which there is an old analytic solution (Carrier and Greenspan, 1958), were run. The length of the system reflects the early orientation of the model developers toward storm surge calculations: a distance of 300 km separates the ocean end, where a periodic elevation is imposed, from the landward end, where the moving boundary condition is imposed. The unlumped, explicit model was employed with 10 equal-length elements. A 1.0 meter (amplitude) M_2 tide was prescribed at the ocean end. Since the analytic solution has no dissipation, neither does the simulation. Exact initial conditions were used to initialize the system, and the

explicit model was run for 10 M_2 periods at 200 time steps per period. The gravity-wave Courant number for this simulation was 0.4 at the ocean end, close to the linear stability limit of $\frac{1}{\sqrt{3}}$. Results were stable throughout the 10-period simulation even in the absence of any dissipative terms. Figures 5, 6, and 7 demonstrate the accuracy of the solution. Figures 5 and 6 show the time variations of elevation at the middle and at the landward end of the system, with "A" indicating the analytic solution and "N" indicating the numerical during the first (1C) and/or the tenth (10C) cycle. Figure 7 shows elevation versus distance one-half way through the tenth cycle, with the ocean end at $x=0$. The solution shows a standing wave with the landward elevation amplitude (the antinode) being more than double that at the forcing end. The agreement with the analytical is very good and frankly surprising, as one always fears lossless test cases in nonlinear simulations.

2) Two-dimensional Runs: Large length Scale

Next some two-dimensional runs were made using EST 20-T, Lynch's original explicit lumped model as reported in Lynch & Gray (1980) without modification. All of the two-dimensional runs used the simple rectangular grid in figure 3b of Lynch & Gray (1980--see figure 8), with different length scales and bathymetric variation. In all cases the bathymetry varied linearly with y (not x), and the moving boundary condition was implemented at nodes 5, 10, 15, ... 50. Bottom stress was computed according to Manning's formula with $n=.035$.

The first runs were physically one-dimensional: a uniform M_2 tide was specified along $y=0$ (nodes 1, 6, 11, ... 46). Lateral boundaries (nodes 1, 2, 3, 4, 5 and 46, 47, 48, 49, 50) were impermeable. The mesh spacing was 50,000 ft., and there were 200 time steps per M_2 period. Runs DAVE1, DAVE2, and DAVE3 contain the results for the following input parameters:

DAVE1:	Maximum depth	30. ft.
	Tidal amplitude	1. ft.
	Depth for friction at shore	5. ft.
	Output	Elevation at nodes 1, 2, 3, 4, 5.
DAVE2:	Maximum depth	30. ft.
	Tidal amplitude	1. ft.
	Depth for friction at shore	2. ft.
	Output	Elevation at nodes 1, 2, 3, 4, 5.
DAVE3:	Maximum depth	10. ft.
	Tidal amplitude	1. ft.
	Depth for friction at shore	2. ft.
	Output	Elevation at nodes 1, 2, 3, 4, 5.

Next more realistic simulations were run: a uniform M_2 tide was specified along $x=0$ (nodes 1, 2, 3, 4, 5) with a symmetry boundary along $y=0$ (nodes 1, 6, 11, ...) and a closed end at nodes 46-50 (figure 8). The symmetry boundary in effect generated a triangular cross-section channel. The mesh length again was 50,000 ft., with 200 time steps per M_2 period. DAVE4 and DAVE5 contain the results for the input:

DAVE4:	Max. depth	30. ft.
	Tidal amplitude	1. ft.
	Depth for friction at shore	5. ft.
	Output	Elevation at nodes 10, 20, 30, 40, 50.
DAVE5A:	Max. depth	10. ft.
	Tidal amplitude	.5 ft.
	Depth for friction at shore	2. ft.
	Output	Elevation at nodes 10, 20, 30, 40, 50.

DAVE5B: Max. depth	10. ft.
Tidal amplitude	.5 ft.
Depth for friction at shore	2. ft.
Output	Elevation at nodes 5, 15, 25, 35, 45.

For reference, the Courant number in these runs is 0.14 based on a 30 ft. depth. No stability problems were found.

Results from these runs were analyzed using least-squares harmonic analysis (tables 1 and 2), to extract the amplitudes and phases for M_2 , M_4 , and M_6 (table 1). The resulting amplitude ratios (M_4/M_2 , M_6/M_2) were determined, as well as the relative phase between M_2 and M_4 (table 2). Figure 8 shows the model geometry of the estuary for the various runs. The nonlinear parameter a/h (for h at the tidal forcing boundary) is 0.033 for DAVE 1, 2 and 0.1 for DAVE 3. Down channel the a/h value will increase. The mid-channel a/h value for DAVE 4 is .033, and .05 for DAVE 5A, B. In 4, 5A, B models the a/h value will increase toward the lateral boundaries.

Results (table 1) show that there may be two factors controlling the amplification of M_2 , M_4 , M_6 tidal constituents. Initial amplification occurs downchannel due to conservation of mass followed by frictional damping. DAVE 1 and 2 show the greatest extent of amplification at grid point 4. In DAVE 3, with the greater nonlinear ratio and lower initial starting depth, M_2 decays monotonically with no initial amplification. DAVE 4, 5A and B with grid points at shallower depths show frictional damping being the overriding signal. DAVE 4 decays monotonically until it reaches the downstream reflective boundary, where a small node causes local amplification. The overall trend in the ratio of harmonics to parent tide (table 2) shows an increase to either grid point 3 or 4 followed by a decrease to grid point 5. Frictional damping

acts as a low pass filter preferentially damping higher frequencies. The M_4/M_2 ratio of sea surface varies from near 0.0% to a maximum of 6.9%. This maximum ratio occurs as would be predicted in DAVE 3, the most strongly nonlinear case (highest a/h ratio). Amplitude ratios of M_6/M_2 in sea surface show DAVE 3 again exhibits the largest nonlinear transfer, ranging up to 13.5%, higher than the M_4/M_2 ratio. This suggests the dynamics in this system are increasing the odd harmonics more than the even harmonics. It is interesting to note that M_4 distortion of an M_2 tide will cause a flood or ebb dominant case whereas M_6 will not. The amplification of the second harmonic (M_6) over the first harmonic (M_4) indicates the friction terms dominate in the equations of motion. The time-variable geometry of the kinematic nonlinearity evidently does not exert the same degree of dominance as the frictional term. This factor needs more examination from both numerical and analytical approaches.

3) 2-D Runs: Small Length Scale

The next series of runs was basically the same setup as DAVE4 and 5, but at a much smaller length scale: 75. ft between nodes in the y (cross-channel) direction, and 3333. ft in the x (long-channel) direction. The mid-channel depth was 45. ft. These length scales were suggested to approximate the length scales found at King's Bay/Cumberland Sound and other estuaries of similar size. Satisfaction of the Courant (gravity wave) condition required excessively small time steps for this length scale: 120,000 steps per M_2 period was stable, but after 20 minutes of VAX 785 CPU time the simulation had completed only 10,000 steps, one-twelfth of a period. At this point the cross-channel length and the time step were arbitrarily increased by a factor of 10 to speed up the model runs. DAVE8 results have the following input parameters:

DAVE8: Max. depth 45. ft.
 Tidal amplitude 3. ft.
 Depth for friction at shore 2. ft.
 Output Elevation at nodes 5, 15, 25, 35,
 45.

There is almost no nonlinear transfer in this simulation: all the results are superimposed on top of one another. This simulation required 91 CPU minutes on the 785, for 48,000 time steps. No harmonic analyses of these results were run. There were too few test runs available to interpret these runs with confidence.

4.) 2-D Runs: Intermediate Length Scales

At this point the length scale was increased to obtain some representative results more efficiently. DAVE11, 12, and 13 contain results on the same grid with same boundary conditions, with x- and y-mesh spacings equal to 7500. ft. There were 1200 time steps per M_2 period. Geometry of the runs is shown as figure 8.

DAVE11: Max. depth 45. ft.
 Tidal amplitude 3. ft.
 Depth for friction at shore 2. ft.

DAVE12: Max. depth 45. ft.
 Tidal amplitude 6. ft.
 Depth for friction at shore 2. ft.

DAVE13: Max. depth 25. ft.
 Tidal amplitude 3. ft.
 Depth for friction at shore 2. ft.

These simulations achieved the goal of quick turnaround: 3600 steps took 5 minutes of CPU time on a VAX 11/785. The runs exhibit varying degrees of non-linearity. a/h ratio for DAVE11 is 0.067, for DAVE12 it is 0.133, while for DAVE13 it is 0.12. DAVE12 is thus the run having the largest expected degree of nonlinear transfer. Some observations can be made based on these runs:

-There is a visible short-wavelength parasite in the solution that is less pronounced in the interior than on the boundary, and more pronounced in the harmonics than in the fundamental. We believe this is induced by the generation mechanism that is present everywhere, and exacerbated by the tidal boundary conditions in which no harmonics are admitted.

-The phases of the harmonics are arbitrary at the tidal end. Also arbitrary are the phases of normal velocity at the impermeable and symmetry boundaries.

-There is a weakness in the solution at the tidal boundary related to the enforcement of the boundary conditions. This problem and its cure have been shown in several fixed-boundary studies but they have not been corrected in these simulations.

Harmonic analysis was performed on these model simulations (tables 3 through 8), incorporating sea surface, u - and v -velocities. In this coordinate system (figure 8), u -velocity is along-channel while v -velocity is cross-channel.

SEA SURFACE:

M_2 , M_4 and M_6 are amplified downchannel in all three test cases (table 3). The continuity equation along with the boundary conditions causes the water to act like a standing wave with an antinode at the closed boundary. Weakest growth is along points at the lateral moving boundary (row A), while strongest is in the intermediate position (row B), along the channel

(figure 8). Friction will be the dominant term when h is very small accounting for the weak growth along row A. The nonlinear parameter a/h in row B is larger than in row C and thus nonlinearities here will be more developed. The extent of M_6 amplification over M_4 is dependent on h which determines the wavelength of the tidal signal. DAVE 11A, B, C, 12A, 13A where h is small has an M_6/M_4 ratio of approximately 5 to 6. DAVE 12B, C, 13B, C where a/h is larger may be more dependent on the kinematic nonlinear term in the continuity equation.

Nonlinear ratios (table 4) in sea surface show similar results. The nonlinear ratios for M_6/M_2 are larger than those for M_4/M_2 in all cases except for runs DAVE13B and C. Nonlinear ratios are on the order of 5% for most of these cases at the downstream boundary. Relative M_4 - M_2 phases are varied for these different cases. For DAVE11B and C, the relative phase is near 180° , which is neither ebb nor flood dominant. For DAVE12A, near the lateral boundary, the flow is ebb-dominant. For DAVE12B and C, the flows are flood-dominant, having relative phases near 50° . DAVE13A, B, and C are similar to DAVE12B and C: flood-dominant with relative phases near 50 - 60° . These are the most nonlinear of the runs. Clearly more numerical runs are necessary to distinguish the sources of the various nonlinear responses, and to identify specific nonlinear terms dominating any specific simulation.

U-VELOCITY:

M_2 velocities decrease rapidly downchannel, from a maximum at the ocean boundary to a minimum at the landward boundary (forced by the boundary condition). M_2 , M_4 velocities are greatest in mid-channel as is expected for this geometry. In velocity, the M_4 tide is largest for runs 12B and C (table 5), which has the largest a/h of the three cases. M_4 overtides exceed the M_6 overtides in velocity (table 5) for the DAVE12 simulation having the

highest degree of nonlinearity, exceeding M_6 by a factor of two. For DAVE11 and 13, the magnitudes of the two overtides of velocity (M_4 and M_6) are approximately equal.

Nonlinear velocity ratios (M_4/M_2 and M_6/M_2) demonstrate the same results as the magnitude of the terms. The most nonlinear response is that of DAVE12, with lowest degree of nonlinearity in DAVE11. Relative M_4 - M_2 phase in velocity shows interesting results. All relative velocity phases are between 180° and 360° , which along with the mean values showing outward flow indicate ebb dominance in velocity. None of these cases resulted in a flood-dominant situation for velocity.

V-VELOCITY:

Results for the cross-channel velocity (tables 7 and 8) are generally similar to those for along-channel velocity. M_2 v-velocity increases down-channel as the end of channel boundary condition exerts more control on the flow. Nonlinear terms grow with distance downchannel with no exceptions. Highest M_4/M_2 ratios near the landward boundary for DAVE12 and 13 are around 20-25%, while for 11B the ratio is smaller (about 9%). M_4/M_2 ratios are larger than the M_6/M_2 ratios, so the even harmonics dominate in cross-channel velocity. Relative M_4 - M_2 phase is between 180 - 360° in velocity, indicating that velocities of higher magnitude flow toward the mid-channel. A mean flow is also in the direction into the channel. More diagnostic testing must be run to understand the relation between the sea surface and the U and V-velocity.

B) FUTURE WORK

Two different lines of development are required to improve the model and make it more amenable for application to problems of direct Navy relevance. The first involves bringing the present codes up to date with developments researched only in the simpler fixed-boundary context. These include:

-Implementation of a semi-implicit algorithm for removing the first (gravity wave) Courant constraint on the time stepping. This is well thought out and works well in the fixed boundary models; no obstacles are anticipated for its implementation in the present context. This method is called for even in large length scale simulations, where significant economies have been realized in going to 10-to-30 minute time steps in tidal simulations.

-Implementation of mass-conserving boundary conditions at tidal boundaries. This is a newer idea that explains the observable weakness in many of the early Lynch and Gray computed solutions near boundaries where depth is specified. This solution has been worked out thoroughly, and involves use of the wave equation instead of the momentum equation for computing the boundary flux (Lynch, 1985). This requires some systematic recoding of the input/output files and the enforcement of the boundary conditions, but poses no conceptual barriers.

-Implementation of a two-level momentum equation. Early work by Lynch and Gray used a three-level leapfrog scheme for the momentum equation. Subsequently, Kinnmark suggested that a two-level scheme is preferable and eliminates some nonlinear instabilities that develop at long times. This finding has been confirmed and now this approach is used in the fixed boundary codes; again, incorporation into the present codes is not a conceptual problem.

The more substantive work involves development of the deforming element concept for the space scales relevant to estuarine questions. There are two fundamental issues here: basic algorithm development and application; and interactive graphics.

The algorithm development basically involves rendering the advective terms implicit by iteration. The research proposed will search for a robust and cost-effective way to remove the advective Courant constraint. It is perfectly

clear that this is a fundamental barrier to any meaningful use of the present simulations at estuarine length scales. The codes developed will be tested in the context of Thacker's analytic solution for a parabolic basin (Thacker, 1981) and then applied to diagnostic studies of tidal distortion.

The graphical part of the proposed work centers around the control of mesh motion both on the boundary and in the interior. Early simulations by Lynch and Gray used moving nodes only on the boundary, which is inadequate for the estuarine situation. Since then some very complex problems have been solved with all nodes in motion using a generic "mesh stretching" approach. The simulations in Sullivan et al. (submitted) give a state-of-the-art demonstration. Mesh motions at this relative scale are a reasonable target for the near-term hydrodynamic work. To make this possible, a tandem processing approach is necessary, with one processor (a VAX 785 or MicroVAX) doing the simulation and a second workstation (MicroVAX) doing the interactive graphics: continuously updating the display of the current mesh, elevation contours and velocity plots; and providing menu-driven options to intervene in the simulation to remesh or adjust the nodal structure as warranted. This effort can be directed toward the development of an "Interactive Graphical Controller" (IGC) which can be implemented on a MicroVAX, with special tuning for hydrodynamic problems.

While the numerical improvements are implemented, better understanding of the nonlinear transfers must be achieved. Knowledge of the terms in the shallow water equations that cause the nonlinear distortion is essential to our understanding of the sensitivity of an estuary to varying estuarine parameters (i.e., geometry, tidal flats, bottom stress etc.). Nonlinear distortion can be important in the determination of the stability of an estuary. As indicated in the section on one-dimensional modeling, progress has been made

in deriving stability fields for nonlinear estuarine transfer processes. These are understood only for the one-dimensional case having no significant fresh-water inflow. This analysis will have to be extended to the case where fresh-water inflow is significant, and to two- and three-dimensional situations.

Because of the relative complexity of these issues, it is suggested that these models be developed and tested in a sequential fashion so the understanding of the physical mechanisms can be understood well. This is the reason that well-mixed one-dimensional analysis was first implemented to complement the existing analytical work. The next step would be a two-dimensional analysis similar to what is described in the section on two-dimensional modeling. The extreme complexity of these environments mandates this step-by-step progression to clarify the physics at each level of complexity.

Another major thrust would be improvement of the coupling between sediment transport models and circulation models. Although some of the models described above have examined the coupling between the sediment and water motion, none of these calculations are in final form. The interaction between a movable bed, a stabilizing suspended sediment load concentrated near the bed, and the overlying fluid flow must be implemented carefully. This would be a major focus of the future work, to be developed in parallel with the circulation models discussed above. A recent review of the fine particle sedimentation in harbors (Mehta, 1986) demonstrates the uncertainties in modeling estuarine sediment transport. This does not argue against development of a sediment transport model; rather it suggests the transport model must be modular and easily modified to reflect improvements in our understanding of the physical, chemical, and biological interactions in estuarine environments.

ACKNOWLEDGEMENTS

This work was funded by the Naval Facilities Command through the Naval Civil Engineering Laboratory as a grant to the Woods Hole Oceanographic Institution. Partial support for preparation of this report came from the Woods Hole Oceanographic Institution's Coastal Research Center. Dr. James Bailard provided comments on this report.

REFERENCES

- Aubrey, D.G. and P.E. Speer, 1985. A study of non-linear tidal propagation in shallow inlet/estuarine systems. Part I: Observations. *Estu. Coast. Shelf Sci.*, 21, p. 185-205.
- Aubrey, D.G., 1986. Hydrodynamic controls on sediment transport in well-mixed bays and estuaries. Physics of Shallow Bays and Estuaries, J. van de Kreeke (ed.), Springer-Verlag, p. 245-258.
- Aubrey, D.G. and C.T. Friedrichs, submitted. Seasonal climatology of tidal nonlinearities in a shallow estuary.
- Carrier, G.F. and H.P. Greenspan, 1958. Water waves of finite amplitude on a sloping beach. *J. Fluid Mechanics*, v. 4, p. 97-109.
- Connor, J.J. and J.D. Wang, 1973. Mathematical models of the Massachusetts Bay, Part I: Finite element modeling of two-dimensional hydrodynamic circulation. Tech. Report no. 172, Ralph M. Parsons Laboratory.
- Friedrichs, C.T., 1986. Parameters which determine tidal distortion in well-mixed estuaries: An observational test of the Speer-Aubrey one-dimensional numerical model for non-linear tidal propagation. Amherst College Senior's thesis, 89 pp.
- Friedrichs, C.T. and D.G. Aubrey, submitted. Tidal distortion in shallow, well-mixed estuaries.
- Johnson, B.H., 1982. Numerical modeling of estuarine hydrodynamics on a boundary fitted coordinate system. *Numerical Grid Generation*, J. Thompson (ed.), Elsevier, p. 409-436.
- Lynch, D.R., 1985. Mass balance in shallow water simulations. *Comm. in Applied Numerical Methods*, v. 1, p. 153-159.
- Lynch, D.R. and W.G. Gray, 1979. A Wave Equation Model for Finite Element Tidal Computations. *Computers and Fluids*, v. 7, p. 207-228.
- Lynch, D.R. and W.G. Gray, 1980. Finite Element Simulation of Flow in Deforming Regions. *Jour. Computational Physics*, v. 36, No. 2, p. 135-153.

- Mehta, A.J. (ed.), 1986. Estuarine cohesive sediment dynamics. Lecture Notes on Coastal and Estuarine Studies, Springer-Verlag, New York.
- Parker, B.B., 1984. Frictional effects on the tidal dynamics of a shallow estuary. Ph.D. thesis, John Hopkins University.
- Spaulding, M.L., 1984. A vertically averaged circulation model using boundary-fitted coordinates. J. Phys. Oceanogr., v. 14, p. 973-982.
- Speer, P.E. and D.G. Aubrey, 1985. A study of non-linear propagation in shallow inlet/estuarine systems. Part II: Theory. Estu. Coast. Shelf Sci., 21, p. 207-224.
- Sullivan, J.M., Jr., D.R. Lynch and K. O'Neill, submitted. Finite element simulation of planar instabilities during solidification of an undercooled melt.
- Thacker, W.C., 1977. Irregular grid finite-difference techniques: simulations of oscillations in shallow circular basins. J. Phys. Oceanogr., v. 7, p. 284-292.
- Thacker, W.C., 1978a. Comparison of finite-element and finite-difference schemes. Part I: One-dimensional gravity wave motion. J. Phys. Oceanogr., v. 8, p. 284-292.
- Thacker, W.C., 1978b. Comparison of finite-element and finite-difference schemes. Part II: Two-dimensional gravity wave motion. J. Phys. Oceanogr., v. 8, p. 680-689.
- Thacker, W.C., 1981. Some exact solutions to the nonlinear shallow water wave equations. J. Fluid Mechanics, v. 107, p. 499-508.
- Thompson, J.F., F.C. Thames and C.W. Mastin, 1977. TOMCAT--a code for numerical generation of boundary-fitted curvilinear coordinate systems on fields containing any number of arbitrary two-dimensional bodies. Jour. Computational Physics, v. 24, p. 274-302.
- Wang, J.D. and J.J. Connor, 1975. Mathematical modelling of near coastal circulation. MIT Parsons Laboratory Report No. 200.

Figure 1. Channel geometry with tidal flats included.

Figure 2. Speer-Aubrey one-dimensional numerical model predictions for the non-linear tidal distortion parameters under conditions of varying a/h and A_f/A_c . For all model runs $L = 7$ km; $b_0 = 58 \times h_1$, $b/b_0 = 0.45$, and results are recorded 6 km into channel:
 (a) $M_2 - M_4$ relative phase; (b) M_4/M_2 ratio.

EFFECT OF a/h AND Af/Ac ON M2-M4

Figure 3. Effects of a/h and Af/Ac on M2-M4 phase: (a) results for eleven naturally occurring estuarine systems (* Af/Ac for Price = 6.38); (b) contour plot of results for 55 model systems.

EFFECT OF a/h AND Af/Ac ON M4/M2

Figure 4. Effects of a/h and Af/Ac on M2-M4 ratio: (a) results for eleven naturally occurring estuarine systems (* Af/Ac for Price = 6.38); (b) contour plot of results for 55 model systems.

Figure 5. Comparison of analytical (A) versus numerical (N) solutions for the tidal height at mid-point of channel.

Figure 6. Comparison of analytical (A) versus numerical (N) solutions for the tidal height at landward end of channel.

Figure 7. Comparison of analytical (A) versus numerical (N) solution for the tidal elevation one-half way through the tenth cycle.

DAVE 1,2,3

DAVE 4,5A,5B

DAVE 11,12,13

Figure 8. Grid layout for 2-d numerical modeling.

TABLE 1: SEA SURFACE

		M_2 (ft)				
		1	2	3	4	5
DAVE 1	1	.972	1.02	.986	1.12	.567
	2	.972	1.03	1.06	1.05	.368
	3	.972	.637	.388	.172	.048
DAVE 4	10		20	30	40	50
	4	.717	.375	.229	.273	.308
5A		.120	.039	.025	.019	.024
DAVE 5B	5		15	25	35	45
	5B	.486	.020	.028	0.19	0.23

		M_4 (ft)				
		1	2	3	4	5
DAVE 1	1	0.00	.008	.013	.034	.011
	2	0.00	.005	.011	0.39	.007
	3	0.00	.032	.026	.012	.002
DAVE 4	10		20	30	40	50
	4	.008	.006	.009	.003	.006
5A		.003	0.00	0.00	0.00	0.00
DAVE 5B	5		15	25	35	45
	5B	0.00	.001	.001	0.00	0.00

		M_6 (ft)				
		1	2	3	4	5
DAVE 1	1	0.00	.018	.026	.092	.042
	2	0.00	.027	.043	.110	.027
	3	0.00	0.67	.052	.017	.003
DAVE 4	10		20	30	40	50
	4	.023	.018	.020	.012	.023
5A		.001	.001	.001	.001	.002
DAVE 5B	5		15	25	35	45
	5B	0.00	.002	.002	.001	0.00

TABLE 2: SEA SURFACE

		M_4/M_2 (%)				
		1	2	3	4	5
DAVE 1	1	-	.8	1.3	4.1	1.9
	2	-	.5	1.0	3.7	1.9
	3	-	5.0	6.7	6.9	3.5
		10	20	30	40	50
DAVE 4	1	1.1	1.6	3.9	1.2	2.1
	5A	2.1	1.1	1.7	1.0	.7
		5	15	25	35	45
DAVE 5B	1	-	7.4	2.4	2.5	.6
	5	-	7.4	2.4	2.5	.6
		M_6/M_2 (%)				
		1	2	3	4	5
DAVE 1	1	-	1.8	2.6	11.0	7.5
	2	-	2.6	4.1	10.5	7.3
	3	-	10.6	13.5	9.9	5.3
		10	20	30	40	50
DAVE 4	1	3.2	4.7	8.8	4.6	7.4
	5A	1.0	3.1	5.5	4.2	6.8
		5	15	25	35	45
DAVE 5B	1	-	12.3	6.7	5.3	4.3
	5	-	12.3	6.7	5.3	4.3
		M_2-M_4 (degrees)				
		1	2	3	4	5
DAVE 1	1	-	104	118	52	119
	2	-	107	82	54	132
	3	-	58	55	32	115
		10	20	30	40	50
DAVE 4	1	113	65	88	105	82
	5A	211	155	68	165	73
		5	15	25	35	45
DAVE 5B	1	-	275	71	67	59
	5	-	275	71	67	59

TABLE 3: SEA SURFACE

M ₂ (ft)				
	M	N	O	P
11A	3.10	3.31	3.37	3.39
B	3.10	3.31	3.37	3.40
C	3.10	3.30	3.35	3.39
12A	6.21	6.64	6.77	6.81
B	6.21	6.69	6.78	6.92
C	6.21	6.67	6.82	6.90
13A	3.10	3.20	3.29	3.33
B	3.10	3.20	3.30	3.36
c	3.10	3.18	3.29	3.35
M ₄ (ft)				
	M	N	O	P
11A	0.000	.023	.028	.028
B	0.000	.013	.018	.022
C	0.000	.014	.020	.023
12A	0.000	.054	.061	.061
B	0.000	.127	.153	.168
C	0.000	.114	.142	.156
13A	0.000	.085	.139	.130
B	0.000	.148	.190	.207
C	0.000	.138	.180	.199
M ₆ (ft)				
	M	N	O	P
11A	0.000	.112	.135	.144
B	0.000	.087	.108	.118
C	0.000	.084	.105	.116
12A	0.000	.279	.319	.328
B	0.000	.215	.251	.276
C	0.000	.209	.254	2.73
13A	0.000	.112	.172	.192
B	0.000	.096	.125	.146
C	0.000	.091	.120	.140

TABLE 4: SEA SURFACE

M_4/M_2 (%)				
	M	N	O	P
11A	0.0	.7	.8	.8
B	0.0	.4	.5	.6
C	0.0	.4	.6	.7
12A	0.0	.8	.9	.9
B	0.0	1.9	2.3	2.4
C	0.0	1.7	2.1	2.3
13A	0.0	2.7	3.4	3.9
B	0.0	4.6	5.8	6.1
C	0.0	1.7	2.1	2.3
M_6/M_2 (%)				
	M	N	O	P
11A	0.0	3.4	4.0	4.3
B	0.0	2.6	3.2	3.5
C	0.0	2.6	3.1	3.4
12A	0.0	4.2	4.7	4.8
B	0.0	3.2	3.7	4.0
C	0.0	3.1	3.7	4.0
13A	0.0	4.3	5.2	5.8
B	0.0	3.0	3.8	4.3
C	0.0	2.9	3.7	4.2
M_2-M_4 (degrees)				
	M	N	O	P
11A	-	232	227	223
B	-	184	191	195
C	-	187	190	192
12A	-	333	343	349
B	-	51	51	50
C	-	50	51	52
13A	-	45	52	60
B	-	59	63	64
C	-	58	62	65

TABLE 5: U-VELOCITY

		M ₂ (ft/sec)			
		M	N	O	P
11B		1.31	.808	.525	
C		1.74	1.09	.659	.089
12B		2.24	1.37	.894	
C		3.24	2.10	1.32	.194
13B		1.94	1.15	.673	
C		2.98	1.89	1.17	.167
		M ₄ (ft/sec)			
		M	N	O	P
11B		.032	.035	.027	
C		.009	.022	.018	.004
12B		.410	.266	.191	
C		.478	.349	.223	.041
13B		.207	.171	.110	
C		.225	.230	.159	.023
		M ₆ (ft/sec)			
		M	N	O	P
11B		.139	.085	.046	
C		.107	.079	.051	.008
12B		.294	.188	.104	
C		.194	.143	.082	.008
13B		.260	.205	.148	
C		.343	.240	.153	.021

TABLE 6: U-VELOCITY

		M_4/M_2 (%)			
		M	N	O	P
11B		2.5	4.3	5.2	
C		.5	2.1	2.7	5.0
12B		18.4	19.3	21.4	
C		14.8	16.6	17.3	21.0
13B		10.7	14.9	16.4	
C		7.5	12.2	13.6	14.0
		M_6/M_2 (%)			
		M	N	O	P
11B		10.6	10.5	8.8	
C		6.2	7.3	7.8	8.6
12B		13.2	13.7	11.6	
C		6.0	6.8	6.3	4.1
13B		13.4	17.9	22.0	
C		11.5	12.7	13.0	12.8
		M_2-M_4 (degrees)			
		M	N	O	P
11B		271	205	185	
C		289	228	212	217
12B		311	277	251	
C		333	314	304	268
13B		325	316	296	
C		351	350	350	336

TABLE 7: V-VELOCITY

M ₂ (ft/sec)				
	M	N	O	P
11B	.153	.280	.300	.329
C				.089
12B	.260	.445	.503	.715
C				.194
13B	.127	.408	.449	.548
C				.167
M ₄ (ft/sec)				
	M	N	O	P
11B	.032	.021	.022	.028
C				.004
12B	.121	.138	.147	.189
C				.041
13B	.058	.084	.111	.128
C				.023
M ₆ (ft/sec)				
	M	N	O	P
11B	.003	.006	.011	.026
C				.008
12B	.034	.027	.032	.047
C				.008
13B	.017	.069	.087	.093
C				.021

TABLE 8: V-VELOCITY

M_4/M_2 (%)				
	M	N	O	P
11B	20.8	7.6	7.5	8.6
C				5.0
12B	46.8	30.9	29.2	26.4
C				21.0
13B	45.3	20.7	24.8	23.3
C				14.0
M_6/M_2 (%)				
	M	N	O	P
11B	1.9	2.3	3.8	8.0
C				8.6
12B	12.9	6.1	6.3	6.6
C				4.1
13B	13.7	16.9	19.4	16.9
C				12.8
M_2-M_4 (degrees)				
	M	N	O	P
11B	223	224	201	199
C				217
12B	252	277	261	241
C				268
13B	212	4	8	342
C				336

REPORT DOCUMENTATION PAGE		1. REPORT NO. WHOI-87-2	2.	3. Recipient's Accession No.
4. Title and Subtitle King's Bay/Cumberland Sound, Georgia Part II: Numerical Modeling		5. Report Date March 1987		
7. Author(s) David G. Aubrey, Virginia A. Fry and Daniel R. Lynch		8. Performing Organization Rept. No. WHOI-87-2		
9. Performing Organization Name and Address Woods Hole Oceanographic Institution Woods Hole, Massachusetts 02543		10. Project/Task/Work Unit No.		
		11. Contract(C) or Grant(G) No. (C) (G)		
12. Sponsoring Organization Name and Address Naval Facilities Command		13. Type of Report & Period Covered Technical		
		14.		
15. Supplementary Notes This report should be cited as: Woods Hole Oceanog. Inst. Tech. Rept., WHOI-87-2.				
16. Abstract (Limit: 200 words) As a complement to field measurements of waves, surface tides, currents, and sediment transport, numerical modeling of King's Bay/Cumberland Sound was initiated. Diagnostic numerical models (both 1- and 2-dimensional) were applied to determine their applicability to estuaries of the same scale as King's Bay. One-dimensional models showed the estuarine system to be ebb-dominant, in accord with observations. This model did not reveal any extreme system sensitivity to changes in channel geometry on the scale expected from maintenance dredging. The two-dimensional model (a finite element model having a moving boundary formulation) was run to examine its applicability for diagnostic modeling of these systems. Preliminary results indicate the method is promising, but some model developments are indicated. Suggested model developments include: semi-implicit algorithm to reduce run-time; mass-conserving boundary conditions at tidal boundaries; implementation of a two-level momentum equation; algorithm development to extend the deforming element concept for smaller estuarine space scales; and formulation of a comprehensive interactive graphical package to facilitate model formulation, boundary and domain gridding, and presentation of results. This latter graphical task is essential for successful application of these numerical models. Results from these studies suggest that diagnostic models of shallow estuaries will be a valuable tool to be used in conjunction with more expensive "predictive" models, to understand circulation and transport processes under natural and impacted conditions.				
17. Document Analysis				
a. Descriptors		5. tidal inlet		
1. King's Bay		6. numerical modeling		
2. estuaries				
3. tides				
4. sediment transport				
b. Identifiers/Open-Ended Terms				
c. COSATI Field/Group				
18. Availability Statement: Approved for publication; distribution unlimited.		19. Security Class (This Report) UNCLASSIFIED		21. No. of Pages 45
		20. Security Class (This Page)		22. Price

DOCUMENT LIBRARY

November 21, 1986

Distribution List for Technical Report Exchange

Institute of Marine Sciences Library
University of Alaska
O'Neill Building
905 Koyukuk Ave., North
Fairbanks, AK

Attn: Stella Sanchez-Wade
Documents Section
Scripps Institution of Oceanography
Library, Mail Code C-075C
La Jolla, CA 92093

Hancock Library of Biology & Oceanography
Alan Hancock Laboratory
University of Southern California
University Park
Los Angeles, CA 90089-0371

Gifts & Exchanges
Library
Bedford Institute of Oceanography
P.O. Box 1006
Dartmouth, NS, B2Y 4A2, CANADA

Office of the International
Ice Patrol
c/o Coast Guard R & D Center
Avery Point
Groton, CT 06340

Library
Physical Oceanographic Laboratory
Nova University
8000 N. Ocean Drive
Dania, FL 33304

NOAA/EDIS Miami Library Center
4301 Rickenbacker Causeway
Miami, FL 33149

Library
Skidaway Institute of Oceanography
P.O. Box 13687
Savannah, GA 31416

Institute of Geophysics
University of Hawaii
Library Room 252
2525 Correa Road
Honolulu, HI 96822

Library
Chesapeake Bay Institute
4800 Atwell Road
Shady Side, MD 20876

MIT Libraries
Serial Journal Room 14E-210
Cambridge, MA 02139

Director, Ralph M. Parsons Laboratory
Room 48-311
MIT
Cambridge, MA 02139

Marine Resources Information Center
Bldg. E38-320
MIT
Cambridge, MA 02139

Library
Lamont-Doherty Geological Observatory
Columbia University
Palisades, NY 10964

Library
Serials Department
Oregon State University
Corvallis, OR 97331

Pell Marine Science Library
University of Rhode Island
Narragansett Bay Campus
Narragansett, RI 02882

Working Collection
Texas A&M University
Dept. of Oceanography
College Station, TX 77843

Library
Virginia Institute of Marine Science
Gloucester Point, VA 23062

Fisheries-Oceanography Library
151 Oceanography Teaching Bldg.
University of Washington
Seattle, WA 98195

Library
R.S.M.A.S.
University of Miami
4600 Rickenbacker Causeway
Miami, FL 33149

Maury Oceanographic Library
Naval Oceanographic Office
Bay St. Louis
NSTL, MS 39522-5001
ATTN: Code 4601