INTERACTION OF HIGH FREQUENCY INTERNAL WAVES AND THE BOTTOM
BOUNDARY LAYER ON THE CONTINENTAL SHELF

by

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Submitted to the Massachusetts Institute of Technology -
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ABSTRACT

Intermittent, shoreward propagating packets of high frequency first mode internal waves are common on the continental shelf when the water column is stratified and may induce large fluctuations in near bottom velocity. Simple theoretical considerations here lead to an approximate method for estimating those quantities of most interest for the bottom boundary layer interaction problem. Examination of data from the pilot Coastal Ocean Dynamics Experiment (CODE I) shows that near bottom velocity fluctuations in the high frequency internal wave band were dominated by shoreward propagating, intermittent mode 1 internal events. Predictions of CODE I internal wave characteristics using the above approximate method are shown to be good.

A boundary layer model is developed, which allows for the nonlinear interaction of surface waves, internal waves, and a steady current over a rough bottom. Modeling results suggest that internal waves will significantly enhance the stress felt by the steady current, and can increase the variability and decrease the reliability of boundary layer measurements by the "log profile" technique, when the waves are present. Theoretical dissipation of internal wave energy in the bottom boundary layer is found to be significantly enhanced in the presence of surface waves and currents, and may be important to the overall internal wave energy balance on the shelf.

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BIOGRAPHICAL SKETCH

The author was born in Conneant, Ohio, on September 11, 1955. He grew up in Hatboro, Pennsylvania, a small town north of Philadelphia, where he attended high school. He attended Brown University, receiving an Sc.B. in Mechanical Engineering, magna cum laude, in 1978, and immediately entered the MIT/WHOI Joint Program in Oceanographic Engineering.

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LIST OF PRINCIPAL SYMBOLS

Subscripts "k": i = internal wave

w = surface wave

c = steady current

b = bottom

a	=	Maximum amplitude of isotherm height (η)
A	=	Density structure dependence of solitary waves
b	=	Bottom
c	=	Long wave speed
c_g	=	Internal wave group velocity
C	=	Constant of proportionality in expression for h
C_d	=	Drag coefficient, $= (u_{*k}''/u_{*k})^2$
d	=	Dissipation
D	=	Local water depth
e_i	=	Long wave mode 1 transfer subfunctions
E	=	Steady shear velocity enhancement for 2 components
E_{tot}	=	Depth integrated timeaveraged internal wave energy
E_f	=	Horizontal energy flux
E_η	=	Estimated error in isotherm height
E_θ	=	Absolute error in temperature
f	=	Coriolis parameter, or inertial frequency
f_i	=	Long wave model energy ratio subfunctions
F	=	Additional steady shear velocity enhancement for 3 components
g	=	Acceleration due to gravity = 9.81 m/sec^2
g_i	=	mode 1 long wave speed subfunctions
h	=	Half height of interior region in Appendix B
h_{cwi}	=	Characteristic length scale for eddy viscosity in lowest sublayer
h_{ci}	=	Characteristic length scale for eddy viscosity in intermediate sublayer
h_c	=	Characteristic length scale for eddy viscosity in outer layer

LIST OF PRINCIPAL SYMBOLS (cont)

H	=	Mixed layer depth
H_i	=	Pycnocline center height
k	=	Horizontal wave number
k_b	=	Nikuradse equivalent bottom roughness, $= 30 z_0$
$K(z)$	=	Eddy viscosity
K_m	=	Constant vertical eddy viscosity
L_s	=	Solitary "wavelength"
m	=	Vertical wave number
n	=	Mode number or discretization parameter
N	=	Brunt-Vaisala (buoyancy) frequency, usually depth averaged
N_i	=	Brunt-Vaisala frequency in the pycnocline
p	=	pressure
r_{xy}	=	Correlation coefficient between x and y
R^2	=	Regression coefficient squared
S_{ii}	=	Autospectrum of process i
t	=	Time
T_s	=	Solitary wave "period"
u	=	Onshore velocity component
$u_{"k"}$	=	Velocity (z dependent) of component "k"
u_b	=	Maximum amplitude of surface wave potential solution bottom velocity
u_{bi}	=	Maximum amplitude of internal wave potential solution bottom velocity
$u_{d"k"}$	=	Velocity deficit of component "k"
$u_{*"k"}$	=	Shear velocity for component "k"
u_{*ci}	=	Characteristic shear velocity of intermediate sublayer in 3-component model
u_{*cwi}	=	Characteristic shear velocity of lowest sublayer in 3-component model
v	=	Longshore velocity component
w	=	Vertical velocity

LIST OF PRINCIPAL SYMBOLS (cont)

x	=	Horizontal, onshore positive coordinate
z	=	Vertical, positive up coordinate
z_c	=	Pycnocline center height
z_0	=	The zero velocity intercept of the boundary layer velocity profile
γ	=	In Chapter 4, an initial time phase shift
γ^2	=	Coherence (squared)
η	=	Isotherm (isopycnal) height
θ	=	In Appendix A, temperature
θ_{cw}	=	Angle between surface wave and current
θ_{ci}	=	Angle between internal wave and current
κ	=	von Karman's constant, = 0.4
ρ_0	=	Characteristic density
$\Delta\rho$	=	Change in density over a characteristic depth
$\tau_{"k"}$	=	Shear stress
$\phi(z)$	=	in Appendix B, internal wave vertical structure function
$\phi_{"k"}$	=	Phase angle or rotation angle through boundary layer
χ^2	=	Chi-squared, as in statistical goodness of fit test
$\omega_{"k"}$	=	Radian frequency of component "k"

CHAPTER 1

■ INTRODUCTION

The dynamics of the ocean over the continental shelf in many ways are distinct from the dynamics of the deep ocean. The primary cause of this distinction is topographical; a shallow, sloping bottom and the constraint of the coastline lead to dynamical balances unique to the continental shelf (Allen 1980). One result is the potential first order dynamical importance of bottom friction to continental shelf flows. Csanady (1978), Brink and Allen (1978) and Brink (1982) have developed theoretical models that predict that bottom friction can play a major role in determining the controlling dynamics of low frequency free and wind forced motions, especially over the inner shelf, though there is still considerable uncertainty as to the importance of bottom friction in the longshore momentum balance across the entire shelf.

Recent theoretical predictions and field measurements have indicated that the magnitude of bottom friction on the shelf is considerably larger and more variable in both space and time than traditional wisdom might dictate. Grant (1977), Smith (1977), and Grant and Madsen (1979) have theoretically shown that nonlinear wave-current interaction over a rough bottom can significantly enhance the boundary shear stress felt by a current. Cacchione and Drake (1983), Grant et al. (1984) and Wiberg and Smith (1984) have confirmed predicted behavior in the field. In addition, moveable bed effects associated with sediment transporting processes and biological activity increase the effective physical roughness of the bed (Smith and McLean 1977, Grant and Madsen

1982, Nowell et al. 1981). During storms, near bottom stratification due to suspended sediment may act to inhibit turbulent momentum transfer near the bed, decreasing the boundary stress with respect to its value in the absence of such stratification (Smith and McLean 1977, Glenn 1983, Grant and Glenn 1983). Thus, the interactions of waves, currents, and a moveable bed in the continental shelf bottom boundary layer may combine to influence the dynamics of lower frequency motions, the velocity profile near the boundary, and patterns of sediment transport. This influence may be both space and time dependent.

Another important feature of the ocean over the continental shelf is horizontal density stratification of the water column, or the presence of vertical gradients in density caused by differences in temperature and salinity over the water column. When the water column is stratified the flow field is affected in several ways.

Stratification is important to the general shelf circulation (e.g., Allen 1980), it may affect the dynamics of the surface boundary layer and the outer portion of the bottom boundary layer (e.g., Long 1981, Grant and Glenn 1983), and it makes the existence of internal gravity waves possible. In fact, internal waves are commonly observed on the shelf when the water column is stratified (e.g., review articles by Huthnance 1981, Levine 1983), and large near bottom velocity fluctuations attributed to the passage of high frequency internal wave packets have been implicated as a potentially important forcing for continental shelf bottom boundary layer processes during the summer months (Butman, et al 1979).

Previous studies of the internal wave bottom boundary layer in the deep ocean and on the continental slope have predicted **little** influence for high frequency internal waves. In the deep ocean, **D'Asaro** (1982) used a **simple** slab model of the bottom boundary layer to **qualitatively** explain the observed behavior of near inertial velocities and energy **flux**, but concluded that higher frequency energy is almost completely reflected from the bottom, hence has little influence on the boundary layer. On the continental slope, Wunsch and **Hendry** (1972) considered the internal wave bottom boundary layer and again predicted **little** influence for high frequency internal waves. However, the internal wave boundary layer on the continental shelf may be quite different from either the deep ocean or the continental slope. In addition to the possibility of strong fluctuations in the internal wave high frequency band (as above), interactions with surface waves, currents and a moveable bed may complicate the internal wave boundary layer on the continental shelf. Although techniques necessary to model the internal wave boundary layer under conditions typical of a continental shelf environment are available, the problem has not been addressed formally.

The purpose of the present study is to consider the nature and extent of interaction between internal waves and the bottom boundary layer on the continental shelf, including both the effect of internal waves on the boundary layer and the effect of the boundary layer on internal waves through energy dissipation. Attention is focussed on high frequency (supertidal) internal waves. Characteristics of high frequency internal waves on the continental shelf are investigated,

particularly with respect to their influence on the near bottom flow field and the relationship between total energy flux and bottom horizontal kinetic energy. A simple boundary layer model is developed that incorporates possible interactions between surface waves, internal waves, and "steady" currents, and the model is used to examine the possible influence of internal waves on the bottom boundary layer and the dissipation of internal wave energy in the bottom boundary layer.

Observations and analyses of the vertical and cross-shelf structure of current and temperature fluctuations in the internal wave frequency band have indicated that there are some common statistical characteristics of internal waves on the continental shelf (e.g., Winant and Olson 1976, Gordon 1978, Winant and Bratkovich 1981), but it is not a priori clear what the "instantaneous" nature of fluctuations in the internal wave band should be, for an arbitrary shelf location and season. The shelf internal wave field has been described as a modified continuation of the deep ocean internal wave field (Gordon 1978) and as at least partly consisting of trains of large amplitude, topographically generated, onshore propagating internal solitons (many recent observations; some early ones were reported by Lee 1961, Halpern 1971, and Apel et al., 1975 - see also Levine 1983). In Chapter 2, known characteristics of internal waves on the continental shelf are examined in more detail, and a theoretical mechanism is provided for estimating quantities of direct interest for the boundary layer interaction problem from a knowledge of the nature and characteristic amplitude of internal waves and the background density structure.

The studies referenced in Chapter 2, while they provide a general

picture of possible characteristics of internal waves on the continental shelf, were not carried out from the point of view of interactions in the bottom boundary layer and are thus somewhat lacking for the present purpose. Data collected during the first Coastal Ocean Dynamics Experiment (CODE I) provide an opportunity to examine a shelf internal wave field from the bottom boundary layer point of view, and this is the subject of Chapter 3. Data from CODE II are referenced throughout the thesis, however, so that it seems appropriate to describe the overall experiment at this point.

The CODE experiment is a multi-institution, intensive effort "to identify and study the important dynamical processes which govern the wind driven motion of coastal water over the continental shelf" (Allen et al. 1982). The initial effort consisted of two small scale, densely instrumented field experiments covering four month periods during the spring/summers of 1981 and 1982, on the Northern California continental shelf, and a more lightly instrumented long term, large scale experiment centered on the same location. A map showing the small scale array design is presented in Figure 1.1.

The experiment included a bottom boundary layer/bottom stress component, which was designed to determine the variability of bottom stress and near bottom flow, and which incorporated field experiments and a modelling effort aimed at prediction of stress and the associated near bottom velocity profile as a function of easily measured flow and bottom conditions. The bottom boundary layer field experiment comprised two subcomponents: a long term, sparsely sampled experiment designed to monitor the long term changes in stress, and a high frequency, densely

Figure 1.1: Locations of the principal CODE-1 mooring sites along north (N), central (C) and south (S) cross-shelf transects.

sampled experiment to determine details of boundary layer processes over a shorter, nested sub-period. The high frequency bottom boundary layer experiment of CODE I (Grant et al. 1983) covered a two week portion of the pilot CODE experiment during late May and early June, 1981, and was centered on two separate central and inner shelf sites (C3 and C1, respectively, in Figure 1.1).

Early in the process of analysing data from this high frequency bottom boundary layer experiment, clearly discernable oscillations in bottom velocity and density structure were observed near the high frequency end of the internal wave band. These assumed internal waves occurred sporadically throughout the data records. Curiosity about their nature and pervasiveness at the CODE site, and concern for their possible dynamical influence in the bottom boundary layer and possible contamination of boundary layer measurements provided initial motivation for the present work. The high frequency bottom boundary layer experiment has provided long time series of high quality boundary layer data, and simultaneous measurements of velocity and temperature from nearby moorings have provided a basis for examination of the internal wave field. CODE II was not designed as an internal wave experiment, so the scope of the internal wave data analysis of Chapter 3 is somewhat limited. Nevertheless, the analysis provides an opportunity (1) to examine a shelf internal wave field from the bottom boundary layer point of view, (2) to test some of the theoretical conclusions of Chapter 2, and (3) to provide an additional observational basis for the theoretical boundary layer analysis that follows.

A simple theoretical boundary layer model that accounts for possible

interactions between surface waves, internal waves, and "steady" currents in the boundary layer over a rough bottom is developed in Chapter 4. The model extends the wave-Current interaction treatment of Grant and Madsen (1979) to include a second, lower frequency wave. A number of simplifying assumptions are made, especially for the nature of the internal wave forcing, but it is believed that the results reveal the lowest order features of a bottom boundary layer that may be forced by surface waves at high frequencies, by internal waves at lower frequencies, and by very low frequency ("steady") currents. The model is used to examine possible enhancement of the shear stress felt by the "steady" current due to additional interactions involving internal waves. The intent is not to develop a comprehensive model for unsteady low frequency forcing, but instead to consider the lowest order aspects of the complex interactions which may occur when a third, intermediate frequency forcing is added to the surface wave-current boundary layer problem.

In Chapter 5, the three component boundary layer model is used to predict possible effects that unsteadiness due to internal waves may have on boundary layer velocity profile measurements. A hypothetical example patterned after an internal wave event occurrence during the high frequency bottom boundary layer experiment of CODE I is considered in detail, and input parameters are varied around this "base state" to theoretically model the sensitivity of boundary layer velocity profile experiments to relative current component strength, internal wave frequency, problem geometry, and averaging time. A three hour segment of the CODE I high frequency boundary layer experiment data set known to

contain an internal wave event is presented and shown to exhibit qualitative agreement with predicted trends.

Chapter 6 considers the dissipation of internal wave energy by bottom friction in the continental shelf bottom boundary layer. Possible enhancement of internal wave energy dissipation by boundary layer interactions with surface waves and a steady current is allowed for by using a fully flow dependent internal wave drag coefficient, derived using the boundary layer model of Chapter 4. A simple model is developed to calculate the fate of an internal wave propagating onshore in the presence of surface waves and a steady current, considering only the balance between internal wave shoaling and energy dissipation by bottom friction. The model is used to examine the effects of surface wave and current environment, characteristic internal wave amplitude, and shelf geometry, in terms of local dissipation rate and efficiency of internal wave dissipation. Qualitative comparisons are made to published estimates of internal wave dissipation and/or cross-shelf reduction in energy flux.

Finally, Chapter 7 summarizes and relates the assumptions and conclusions of Chapters 2-6, and offers suggestions for further research.

CHAPTER 2

CHARACTERISTICS OF INTERNAL WAVES ON THE CONTINENTAL SHELF

2.1 Introduction:

The interaction of internal waves and the bottom boundary layer on the continental **shelf** includes both the effect of internal waves on the boundary layer through imposition of an unsteady forcing and the effect of the boundary layer on the internal wave field through dissipation of wave energy. For the boundary layer problem, the nature and intensity of internal wave induced fluctuations in velocity just above the bottom are clearly of primary importance. For the energy dissipation problem, the **relation** of bottom velocity to the total internal wave energy integrated over the water column and the speed at which energy propagates are additionally important. However, internal waves are first and foremost vertical disturbances of a level surface that propagate on the density gradient or density difference across the interior of a stably stratified water column. The transfer function between these interior vertical deflections and induced bottom velocity fluctuations, the vertical distribution of energy, and propagational characteristics of internal waves are determined at lowest order by the background density and velocity structure and by the nature of solutions to an appropriate governing wave equation. Within the context of allowable free wave solutions for a given background structure, the nature of the internal wave field may be described further by the distribution of energy as a function of time, frequency, and vertical mode, by any tendency to anisotropy in propagation direction, and by the

likely importance of nonlinearity.

The purposes of this chapter are (1) to describe the current state of knowledge of the nature of the internal wave field on the continental shelf and (2) to provide a mechanism for estimating the quantities of interest for the bottom boundary layer interaction problem from a minimum knowledge of the density structure and wave height. Several examples of observations of internal waves and background density structures on the continental shelf are summarized and compared, to illustrate the expected range of behavior of the internal wave field on the shelf. Here, as throughout the thesis, attention is focussed on the high frequency end of the internal wave spectrum, and especially on the behavior of the topographically generated packets of high frequency, energetic internal waves that are a feature common to many continental shelves when the water column is stratified. Theoretical sensitivity of internal waves to vertical density structure and assumptions about nonlinearity is then investigated. Of particular interest are the quantities described above - The transfer function between isopycnal deflections and bottom velocity fluctuations, the horizontal energy propagation velocity, and the relationship of bottom horizontal kinetic energy (HKE) to total energy in the water column. A set of simple relationships describing the structure and wave height dependence of these quantities is derived, and example calculations based on approximations to observed density profiles are carried out.

2.2 Known Characteristics:

Descriptions of internal waves on the continental shelf have tended

to be of two kinds. Statistical analyses concentrate on the vertical structure and time history of current and **temperature** auto- and cross-spectra measured with vertical arrays of current meters or temperature sensors. There are few complete statistical descriptions in the literature. Compared here are studies by **Winant** and **Olson** (1976) of the vertical structure of currents at a single location near **San Diego**, California during the late summer of 1974, by **Gordon** (1978) of the vertical, cross-shelf and temporal **distribution** of **internal** wave energy on the Northwest African shelf in early 1974, and by **Winant** and **Bratkovich** (1981) of the vertical and cross-shelf distribution of horizontal kinetic energy (**HKE**) and temperature fluctuations over a period of one year on the Southern California shelf. The second kind of description of **internal** waves on the **shelf** concentrates on **isolated** groups of large **amplitude** **internal** waves, commonly observed travelling on the thermocline near coastlines (**e.g.**, **Huthnance** 1981, **Fu and Holt** 1982). These are generally long crested, isobath parallel, onshore propagating wave trains, whose generation is often attributed to the interaction of the barotropic tide and a topographic feature. They appear to result from the evolution of an initial internal lee wave or hydraulic jump **into** a train of smaller amplitude waves as governed by a **K-dV** type equation (**Maxworthy** 1979, **Farmer and Smith** 1980, **Lee and Beardsley** 1974). Because of modulation of the surface roughness by alternating surface convergence and **divergence** zones accompanying the waves, they are often visible as bands of different scattering intensity in aerial photographs and radar images, and their presence thus has been documented worldwide (**e.g.**, **Fu and Holt** 1982). **Characteristics** of these

waves described here are derived from a compilation of measurements from two locations on the East Coast of the United States, the first in Massachusetts Bay and the second in the mid-Atlantic Bight, just off Delaware Bay.

As a prelude to the discussion of shelf internal wave statistics, and in light of the eventual purpose of this chapter, a summary of the density structures reported in conjunction with the statistics seems appropriate. Winant and Olson (1976) observed a shallow water density structure consisting of a surface mixed layer, a moderately thick pycnocline, and a bottom mixed layer, and concluded that the observed vertical structure of the high frequency currents and the estimated propagation speed of internal waves were consistent with a linear model for first mode internal waves in a vertically symmetrical version of that density structure. Gordon (1978) did not report the details of the density structure, but presented a time history of some average value of the Brunt-Vaisala frequency, N , and carried out energy flux calculations based on linear internal wave theory for a water column with constant surface to bottom stratification. Winant and Bratkovich (1981) reported a marked seasonal change in both the density structure and the high frequency energy. The density was controlled primarily by temperature. They characterized the spring/summer density structure as consisting of a shallow upper mixed layer about 5 m thick, a pycnocline about 20 m thick with $N_1 = 10-15$ cph (N_1 refers to the value in the pycnocline, as opposed to the depth averaged value), and a well mixed lower layer, but later concluded that observed cross-shelf lags were consistent with a linear two layer model consisting of a 10 m thick unstratified upper

layer, a sharp density jump, and a variable depth unstratified lower layer. In the fall/winter, the water column was more nearly isothermal, with $N \sim 3$ cph. The near surface onshore current variance was about a factor of 2-3 higher during the spring/summer months.

The descriptions of the statistics of the internal wave field on the continental shelf in all three studies agree in certain particulars. The high frequency HKE is dominated by cross-shelf velocity fluctuations, especially at the surface and near the bottom of the water column. HKE and temperature spectra tend to show a low, broad peak, or "knee", slightly below the local value of N_i . The phase relationships of the strongest vertical eigenfunctions of the high frequency energy, which account for anywhere between 25-45 percent of the total energy, are consistent with the onshore propagation of mode 1 internal waves. Though the relative strength of the higher modes is either not resolved or not reported for the high frequency energy, Winant and Bratkovich (1981) indicate that the energy contained in each succeeding higher mode of the internal tide is reduced by about a factor of two. For lack of better information it is assumed that the modal distribution of energy at higher frequencies is roughly the same. The near surface onshore current variance measured by Winant and Bratkovich (1981) was roughly constant across the shelf, while the depth averaged energy density calculated by Gordon (1978) decreased uniformly towards the shore. Both of these observations are contrary to the theoretical results of Wunsch (1968,1969), who predicted a general increase in energy density/wave intensity as internal waves propagate into a wedge. Gordon (1978) has suggested that significant energy dissipation thus

occurs across the entire shelf and that the wave field over the shelf is "saturated"; i.e., that in some sense the local wave height is as high as the water depth will allow and the total energy is as high as the local value of N will allow.

There is one notable difference between the observations of Winant and Olson (1976) and Winant and Bratkovich (1981) on the Southern California shelf and those of Gordon (1978) on the Northwest African shelf. From his analysis of cross-shelf energy behavior, Gordon concluded that the deep ocean internal wave field was the likely source of most of the internal wave energy on the shelf. He did not notice (personal communication) any clearly discernable large amplitude fluctuations in his data, such as might result from local topographic generation of internal waves. Winant and Olson (1976) and Winant and Bratkovich (1981), on the other hand, do not define a particular source for the high frequency internal wave energy, but they indirectly associate the observed energy with clearly discernable packets of high frequency waves on the Southern California shelf that often accompany apparently tidally induced internal surges (e.g., Winant 1974).

The occurrence and behavior of large amplitude, tidally generated internal wave packets have been well documented in Massachusetts Bay. Halpern (1971) reported on high frequency temperature and current fluctuations measured in the upper half of an 80 m water column during the summer of 1967 at a site just to the west of a prominent sill (Stellwagen Bank) between Massachusetts Bay and the Gulf of Maine. The vertical structure of the fluctuations was generally consistent with mode 1 internal waves propagating towards shore from Stellwagen Bank,

with crests roughly parallel to the bank. Wave periods were about 7 min, wavelengths about 200 m, and wave speeds between 55-155 cm/sec. Maximum particle velocity (peak to peak) at the surface was about 50 cm/sec. The measured density structure shows a near surface pycnocline with a lower limit at about 30 m depth and a maximum value of $N_1 \sim 25$ cph at about 10 m depth. Thermistor chain data indicate the presence of a shallow surface mixed layer.

Several subsequent studies have dealt with the Massachusetts Bay internal waves in more detail. Lee and Beardsley (1974) investigated the nature of solutions of a $K-dV$ type equation in an attempt to explain propagational characteristics and the generation mechanism for the waves. Haury, et al. (1979) reported additional direct measurements of wave height and evolution of wave packets, using vertical current meters (VCM's) and a 200 KHz acoustic backscatter system. Their measurements clearly show trains of asymmetrical (narrow peak, broad trough) waves of depression with maximum height of about 25 m, and occasional strong breaking events. Trask and Briscoe (1983), using a simple prediction scheme based on an average propagation speed of 50 cm/sec and a single direction of 240° T, were able to correlate predicted wave position with observations from August and September, 1978 SEASAT SAR images. They note the seeming disappearance of the internal waves in October, and suggest that this may be due to the deepening of the thermocline over Stellwagen Bank in the fall.

Multiple internal wave packets have been observed in SAR images over the entire East Coast. One such image, presented in Fu and Holt (1982), shows the area off Delaware Bay on August 31, 1978. Observed

Internal wave crest lengths are on the order of 50 km, maximum wave separation is about 1.3 km, estimated group speeds are on the order of 30 cm/sec, and there are up to 30 waves in a packet. Butman, et al. (1979) report measurements during late June, 1976, of current speed at 1 m above the bottom in 87 m of water in the center of the Fu and Holt image area. They note strong fluctuations in current speed associated with the passage of internal wave packets, with between 1-11 waves per packet. Average peak width was about 12 min, average time between peaks was about 20 min, maximum peak to peak amplitude was about 20 cm/sec, and average packet separation was 12.3 hrs. Maximum observed speeds at 1 m during the summer were due to the superposition of internal waves and tidal flows.

It should be pointed out that topographically generated internal wave trains on the continental shelf produce density and velocity fluctuations that are quite intermittent at any given location. For example, Halpern (1971) notes that internal wave trains passing a mooring in Massachusetts Bay produced temperature fluctuations for about 25 hrs every 12.4 hrs, or were present about 20 percent of the time. Making a rough estimate from the results reported by Butman et al. (1979), with 1-11 waves/packet, a mean peak to peak time interval of 20 min, and a mean packet separation of 12.3 hrs, the waves are estimated to have been present 3-30 percent of the time. In addition, the wave packets may have limited spatial coverage, simply due to the fact that they are of limited crest length and may originate in limited generation areas like Stellwagen Bank in Massachusetts Bay.

In summary, the high frequency internal wave field over the

continental shelf is expected to be energetically dominated by first mode waves propagating towards the shore. Near surface and near bottom velocity fluctuations induced by these waves consequently are strongest in the cross-shelf direction, with vertical amplitude greatest in the vicinity of the pycnocline. Energy spectra often show a clear semidiurnal tidal peak, are "red" at slightly higher frequency, and show a second broad, flat peak, or "knee", slightly below the local value of N . In many cases, this broad high frequency peak may be associated with the presence of packets of topographically generated, large amplitude, long crested, onshore propagating first mode internal waves, which are often observed on the continental shelf. These waves, though intermittent, can produce large, organized fluctuations in near bottom velocity. However, there is no direct evidence linking the general statistical behavior of the high frequency wave field to the presence of such waves. Gordon (1978) has suggested that the Northwest African shelf wave field is an extension of the deep ocean wave field, while Winant and Olson (1976) and Winant and Bratkovich (1981) imply that similar high frequency statistical behavior on the Southern California shelf may be due at least in part to the passage of packets of high frequency waves, possibly linked to internal tidal surges. In any case, dissipation of internal wave energy appears to take place across much of the shelf.

Internal waves on the continental shelf are highly seasonal. They have been observed to effectively disappear on the East Coast during the winter months (e.g., Butman, et al. 1979), and energy levels are much reduced during the winter on the Southern California shelf (Winant and

Bratkovich 1981). Trask and Briscoe (1983) suggest that the changing density structure over the generating topographic feature in Massachusetts Bay may be responsible for seasonal dependence there. Even if the source of the wave energy remains constant, the stratification dependent saturation criterion suggested by Gordon (1978) would imply that lower stratification necessarily leads to lower energy levels.

2.3 Approach to Modeling - Formulation of the Problem:

The internal wave quantities of most interest for the boundary layer **interaction** problem are the magnitude and some characteristic frequency of bottom velocity **fluctuations** induced by **internal** waves, the **relation-**ship of bottom **KE** to total depth integrated energy, and the speed of propagation of a wave packet (more specifically, the propagation speed of the energy in a wave packet). Velocity fluctuations induced by the large **amplitude**, **intermittent** **internal** wave trains common to many continental shelves are especially of interest; for example, **Butman**, et al. (1979) report that maximum near bottom velocities during the summer months on the Mid-Atlantic Bight resulted from the superposition of high frequency internal waves and tides. However, there are few direct **measurements** of any of the desired quantities; the most often observed **shelf** **internal** wave characteristics are surface signatures **in** **satellite** **rddar** images, waveheight using temperature measurements or acoustic backscatter systems, and velocity fluctuations in the upper part of the water column. **Apel** and Gonzales (1983) have suggested that waveheight might be estimated from **SAR** images by using an appropriate nonlinear

wave theory in an inverse fashion, given a knowledge of the background structure and topography. The saturation idea of Gordon (1978) or simple extension of known internal wave characteristics on the **shelf might** also be used to estimate a rough maximum waveheight for a given depth. In any case, **it may** be anticipated that a knowledge of the effects of background density structure and nonlinearity on the structure and propagational characteristics of internal waves often will be necessary for estimation of the desired quantities from the given information.

There are two possible approaches to the problem of assessing the effects of density structure and nonlinearity. First, **if** the exact background density distribution and nature of possible nonlinearity are known for a particular location, an appropriate nonlinear governing equation **may** be solved numerically (as in Lee and **Beardsley** 1971). This approach has the advantage of obtaining a more nearly exact solution for a given situation and set of assumptions, but **it** is potentially quite cumbersome. Second, a set of models for approximate density structures **may** be constructed and compared analytically for both linear and nonlinear waves, to examine the sensitivity of the general problem to changes in input parameters and assumptions. This approach, while not as exact as the first for a known background structure, has the advantages of generality, ease of solution, and possibly of offering greater physical insight, and **it** is the approach followed here.

An additional reason for following the approximate analytical approach is the possible variability and consequent uncertainty in measurements of the density structure on the continental shelf. For

example, Caldwell (1978) found that the bottom mixed layer on the Oregon continental shelf during August, 1975, varied between about 8-28 m in height over both short (0(hrs)) and long (0(days)) time periods, and varied about the same amount in the cross shelf direction. Trends and meaningful averages were not easily discernable. In the deep ocean, 8-28 m would be a negligible variation in the overall density structure, but on the continental shelf the mixed layer height may represent a large fraction of the depth.

As another example, consider the two density profiles presented in Figure 2.1. They were obtained within 1 hour and within 200 m of the same site in 100 m of water on the Northern California shelf on June 4, 1981, during the high frequency bottom boundary layer experiment of CODE I. Although the surface to bottom density difference is about the same, the apparent interior distribution of density is quite different. Rather than solve internal wave equations numerically for each density profile, the approach used here considers the differences between approximate models for the two profiles. The first profile might be modeled as a 30 m thick surface mixed layer, a 50 m thick interior region with constant stratification with $N_1 \sim 2.5$ cph, and a 20 m thick bottom mixed layer. The second profile might be modeled as a 60 m thick constant density upper layer, a density jump of about 10^{-4} gm/cm^3 , and a 40 m thick constant density lower layer.

In the analysis that follows, the solutions of several models for unidirectional free internal waves in a horizontally homogeneous, inviscid wave guide with rigid top and bottom boundaries are compared. The effects of rotation are ignored, since the present emphasis is on

Figure 2.1: Density profiles obtained with a NBIS CTD system near CODE ■ site C3 in 100m of water, between 21:00 and 22:00 PST on 4 June 1981. Horizontal separation less than 200 m.

high frequency internal waves that are well above the inertial frequency. The models used are simple calculations or extensions of existing solutions; details are presented in Appendix A. Assumptions and simplifications are justified below.

The assumption of a rigid upper boundary is a fairly standard one in internal wave analyses. The ratio of the surface amplitude to the internal amplitude is of the order $\Delta\rho/\rho$, where $\Delta\rho$ is the characteristic change in density over the water column and ρ is the scale density. This ratio is usually of the order 10^{-3} to 10^{-4} , so it is safe to ignore vertical motion of the water surface. Ignoring changes in depth is justifiable if the horizontal length scale of the waves, L_h , is much less than the length scale of the depth variation, $[(\partial D/\partial x)/D]^{-1}$. As an example, consider the midshelf CODE site, which has a fairly steep bottom slope, $\partial D/\partial x \sim 0.005$. For $\omega \sim 0.2N$, where ω is the wave frequency, $L_h \sim 5L_z \sim 10D$ for a mode 1 wave, so $(L_h/D)(\partial D/\partial x) \sim 0.05$, and the bottom may be considered locally flat. Ignoring horizontal variation in density is locally valid if (Mooers 1975)

$$\frac{u}{w} \frac{M^2}{N^2} \ll 1 \quad (2.1)$$

where

$$N^2 = \frac{g}{\rho_0} \frac{\partial \rho}{\partial z} \quad (2.2)$$

is the Brunt-Vaisala frequency and

$$M^2 = - \frac{g}{\rho_0} \frac{\partial \rho}{\partial x} \quad (2.3)$$

is its horizontal equivalent, u is the horizontal particle velocity and w is the vertical velocity. The ratio $u/w \sim L_h/D$, and again using the CODE site

(which has a fairly strong horizontal density gradient) as an example, $M^2/N^2 \sim 0.01$, so $uM^2/wN^2 \sim 0.1$, and horizontal density variation may be ignored at **lowest** order. Finally, assuming that the waveguide is **inviscid** is valid as long as the wave boundary layers are only a small fraction of the depth and the dissipation time scale is much longer than one wave period. This assumption will be made now and checked later for consistency.

The **linear** governing equation for the vertical velocity of a unidirectional internal wave in a codirectional background current $U(z)$, with the wave assumed to be of the sinusoidal form $w = \text{Re}[W(z)\exp(i(kx - \omega t))]$, where $k = 2\pi/L_h$ is the horizontal wavenumber, is

$$\frac{d^2 W}{dz^2} + \left[\frac{N^2 k^2 + k(\omega - kU(z)) \frac{d^2 U}{dz^2} - k^2 (\omega - kU(z))^2}{(\omega - kU(z))^2} \right] W = 0 \quad (2.4)$$

The originally stated objective of the analysis was analytical comparison of approximate solutions, which is not in general possible if $U(z) \neq$ constant. Therefore, $U(z) = U$ is assumed to be the component of a constant barotropic current which is codirectional with the internal wave, which is taken as a reasonable approximation for the present purpose. Then writing the intrinsic frequency (travelling with the current) as $\omega_r = \omega - kU$, the governing equation reduces to

$$\frac{d^2 W}{dz^2} + \left[\frac{N^2 - \omega_r^2}{\omega_r^2} \right] k^2 W = 0 \quad (2.5)$$

which is solved for several different density structure models in Appendix A. For the comparisons **below**, it is assumed that $U = 0$, so that $\omega_r = \omega$, with no loss of generality as long as it is remembered that the general frequency dependence of the results is on ω_r . To

examine the influence of the vertical density structure, solutions are sought for a linear internal wave in a waveguide of depth D , depth averaged Brunt-Vaisala frequency N , defined by

$$N = \left[\frac{1}{D} \int_0^D N^2(z) dz \right]^{0.5}, \quad (2.6)$$

pycnocline thickness H_i , and pycnocline center height z_c . The primary model solved to approximate such a structure has surface and bottom mixed layers and a stratified interior with constant N_i (representing the pycnocline). Its limits, which have standard, simple solutions, are a completely stratified water column with $H_i/D = 1$ and $z_c/D = 0.5$, and two constant density layers with an interfacial density jump, for which $H_i = 0$.

Observations of internal waves on the continental shelf have indicated that most of the high frequency energy occurs at or below some fraction of the local maximum value of N_i . The "knee" in observed energy density spectra usually has its center frequency in the vicinity of $0.5N_i$ (Winant and Olson 1976, Winant and Bratkovich 1981). Halpern (1971) observed that the dominant frequency of the Massachusetts Bay internal wave trains was at about $0.3-0.4 N_i$. Therefore, although initial consideration of the model results that follow will include frequency dependence, most of the analysis will concentrate on the behavior of relatively long, low frequency waves, approximated by the long wave limit of the internal wave problem without rotation. The range of validity of most of the results is thus formally limited to $f \ll \omega \ll N$. While the approximation $\omega \ll N$ is obviously stretched to include

waves of frequency $\omega \sim 0.5N$, the simplification that results and the fact that the energy falls off very rapidly as $\omega \ll N$ are taken as sufficient justification for the present approximate analysis.

To this same approximation, the appropriate governing equation for nonlinear internal waves on the continental shelf should be of the weakly nonlinear, long wave type. This may be demonstrated by calculating an approximate Ursell number (as in Lee and Beardsley 1974), $U = (a/D)/(D/L_h)^2$. Using characteristic values for Massachusetts Bay (Haury, et al. 1979, Trask and Briscoe 1983), $a \sim 20$ m, $D \sim 80$ m, and $L_h \sim 300$ m, so $a/D \sim 0.25$, $L_h/D \sim 4$, and $U \sim 4$. Thus the waves are both nonlinear and relatively long, and should take the form of cnoidal waves for intermediate values of U , approaching solitary waves for large values of U . In Appendix A, a specific form of the solitary wave solution of Benjamin (1966) is calculated, based upon an assumed three layer density structure (surface and bottom mixed layers and a stratified interior). The solitary wave model is taken as representing the strongest appropriate nonlinearity for the present analysis.

In the following section, the mode 1 transfer function between interior amplitude, a , and bottom velocity, u_{bj} , the group propagation speed, c_g , and the ratio of bottom HKE to total depth averaged energy are examined as functions of frequency, density structure, and nonlinearity. The general linear dependence of each (nondimensional) parameter on nondimensional pycnocline thickness H_i/D and frequency ω/N is examined first, by comparing solutions for three vertically symmetrical density structures. These symmetrical structures and schematic diagrams of the mode 1 and mode 2 solutions are shown in Figure 2.2 for

clarity. Dependence on H_i/D and pycnocline height z_c/D is then examined in more detail for first mode waves at the long wave limit. Linear dependencies at the long wave limit are written as being separable. The effect of nonlinearity is written as a multiplicative correction to the the linear dependencies, and in general is a function of all of a/D , H_i/D , and z_c/D . Nonlinearity is only considered in its most extreme form, which is the solitary wave solution.

Formally, the nondimensional mode 1 long wave transfer function is written

$$\frac{aN}{u_{bi}} = \prod_{i=1}^3 e_i = e_1 \left(\frac{H_i}{D} \right) e_2 \left(\frac{z_c}{D} \right) e_3 \left(\frac{2a}{D}, \frac{H_i}{D}, \frac{z_c}{D} \right) \quad (2.7)$$

and the forms of e_i are sought. The nondimensional mode 1 long wave speed is written

$$\frac{c}{ND} = \prod_{i=1}^3 g_i = g_1 \left(\frac{H_i}{D} \right) g_2 \left(\frac{z_c}{D} \right) g_3 \left(\frac{2a}{D}, \frac{H_i}{D}, \frac{z_c}{D} \right) \quad (2.8)$$

where c is the long wave speed ($c_g \approx c$ at the long wave limit), and the forms of g_i are sought. The nondimensional mode 1 bottom HKE is written

$$\frac{0.5 \rho_0 u_{bi}^2}{(E_{tot}/D)} = \prod_{i=1}^3 f_i = f_1 \left(\frac{H_i}{D} \right) f_2 \left(\frac{z_c}{D} \right) f_3 \left(\frac{2a}{D}, \frac{H_i}{D}, \frac{z_c}{D} \right) \quad (2.9)$$

where E_{tot} is the depth integrated total internal wave energy, and the forms of f_i are sought. The results of this analysis enable the calculation of the magnitude of the potential bottom velocity, u_{bi} , the long wave speed, c , and the relation of u_{bi} to E_{tot} , given reasonable estimates of N , D , H_i , z_c , and a .

Figure 2.2: Symmetrical density structures assumed for initial theoretical examination of dependence on pynocline thickness and frequency, and characteristic solutions. Density variation on left, mode 1 solution in center, mode 2 solution (where applicable) on right.

2.4 Results of Modeling:

Internal amplitude to bottom velocity transfer function . In order to estimate bottom velocities from more commonly measured internal wave quantities within the water column, the kinematical relationships describing the vertical structure of the internal wave solution are required. The relationship between maximum internal amplitude, a , and horizontal bottom velocity, u_{bj} , is explored here; it is referred to hereafter simply as the transfer function. The dependence of the transfer function on vertical density structure and nonlinearity is examined for first mode waves only because of the observed dominance of first mode internal waves on the continental shelf and because, as will be shown later, the existence of a bottom mixed layer will tend to further diminish the bottom influence of higher modes.

Several well known aspects of the phase and vertical structure of the mode 1 transfer function are illustrated in Figure 2.2. Vertical deflections are in phase throughout the water column; although not illustrated in Figure 2.2, they are maximum somewhere between the pycnocline and the center of the water column. Horizontal velocity fluctuations with the positive horizontal axis in the direction of wave propagation are in phase with vertical deflections below the deflection maximum, and 180° out of phase above it. Horizontal velocities have local maxima at the surface and bottom for $H_1/D = 1$ and a maximum at the pycnocline for $H_1/D > 0$.

The results of comparing the magnitudes of the linear transfer functions for the three vertically symmetrical density structures of Figure 2.2 are plotted in Figure 2.3a as $a\omega/u_{bj}$ vs. ω/N . At the long

Figure 2.3: Dependence of linear mode 1 transfer function, aN/u_{bi} , on density structure: (a) shows long wave convergence of asymptotic behavior, lack of dependence on H_i/D ; (b) shows dependence of long wave asymptote on z_c/D .

wave/low frequency limit, the three models are asymptotic to the line $aN/u_{bi} = 1$, though they diverge at higher frequency; for $H_i/D = 1$, $a\omega/u_{bi} \gg \infty$ as $\omega/N \ll 1$, while for $H_i/D = 0$, $a\omega/u_{bi} \gg \infty$ as $\omega/N \gg \infty$. In other words, the pycnocline thickness H_i is not important to the mode 1 long wave transfer function, or $e_1 = 1$ in eq 2.7.

Though independent of H_i/D , the long wave asymptote of aN/u_{bi} does depend on the nondimensional pycnocline center height, z_c/D . The form of this dependence may be derived analytically by considering the long wave limit of the two layer internal wave model of Appendix B. It is given by

$$e_2 \left(\frac{z_c}{D} \right) = \left[\frac{z_c/D}{1 - z_c/D} \right]^{0.5} \quad (2.10)$$

and is plotted in Figure 2.3b. Finally, allowing for the most extreme possible nonlinearity by solving the solitary wave problem has a fairly simple effect on the transfer function, which may be derived analytically for the long wave two layer problem. It is

$$e_3 \left(\frac{2a}{D}, \frac{H_i}{D} \frac{z_c}{D} \right) = \frac{c_{lin}}{c_{sol}} = \frac{1}{g_3} \quad (2.11)$$

where c_{lin} is the linear wave speed and c_{sol} is the solitary wave speed. Thus the total long wave mode 1 transfer function between maximum interior amplitude and bottom velocity is given by

$$\frac{aN}{u_{bi}} = e_1 e_2 e_3 = \left[\frac{z_c/D}{1 - z_c/D} \right]^{0.5} \frac{c_{lin}}{c_{sol}} \quad (2.12)$$

with zero phase difference between interior vertical deflections and bottom velocity.

Propagation speed - For dispersive waves, it is possible to calculate two forms of a propagation speed (e.g., Lighthill 1978, Chapter 3). These are the phase speed, $c_p = \omega/k$, which is the speed of propagation of a given wave crest, and the group speed, $c_g = \partial\omega/\partial k$, which is the speed of propagation of a wave group and the speed of energy propagation. In general, $c_g \leq c_p$. For the present purpose, interest focusses on the horizontal flux of internal wave energy and its reduction by the work of bottom friction. The horizontal energy flux may be written

$$E_{f,n} = c_{g,n} E_{tot,n} \quad (2.13)$$

for unidirectional waves of mode n , where $c_{g,n}$ is the group speed in mode n , $E_{tot,n}$ is the depth integrated energy in mode n and $E_{f,n}$ is the horizontal energy flux in mode n ; the total energy flux is the sum of $E_{f,n}$ over all the modes. Thus, for the initial analysis including the frequency dependence of the propagation speed, group velocity is calculated. In the subsequent specialization to the long wave limit, the distinction between c_g and c_p disappears as the system becomes nondispersive.

In Figure 2.4a, the results of comparing nondimensional linear group velocities, c_g/ND , for the three vertically symmetrical density structures of Figure 2.2 are plotted vs. ω/N . The continuously stratified water column ($H_1/D = 1$) has the slowest wave speed, the fastest fall off with increasing frequency, and the highest ratios of higher mode speeds to mode 1 speed. As the pycnocline thickness decreases, the mode 1 speed increases, while the higher mode wave speeds remain similar to the continuously stratified case until they are forced

Figure 2.4: Dependence of linear group velocity, c_g/ND , on density. (a) shows dependence on both H_i/D and ω/N for first three modes, density structures of Figure 2.2; (b) shows long wave dependence on H_i/D for first three modes; (c) shows long wave dependence of mode 1 only on z_c/D .

out of existence as $H_i/D \rightarrow 0$.

The long wave dependence of wave speed on H_i/D is plotted in Figure 2.4b. The upper line in the figure is the function $g_1(H_i/D)$ of eq 2.8; it is well described by

$$g_1\left(\frac{H_i}{D}\right) = 0.5 - 0.182\left(\frac{H_i}{D}\right) \quad (2.14)$$

Again, the decreasing speed of the higher modes relative to the first mode as $H_i/D \rightarrow 0$ is apparent. The dependence of linear long wave speed on the vertical asymmetry of the density structure is illustrated in Figure 2.4c, where the function $g_2(z_c/D)$ is plotted, normalized by the linear wave speed for $z_c/D = 0.5$ and $H_i/D = 0$. The form of g_2 is given by the long wave asymptotic limit of the two layer solution of Appendix A, which is

$$g_2\left(\frac{z_c}{D}\right) = 2\left[\frac{z_c}{D}\left(1 - \frac{z_c}{D}\right)\right]^{0.5} \quad (2.15)$$

Wave speed clearly decreases as the structure becomes more asymmetrical.

The increase in mode 1 long wave speed due to extreme nonlinearity, calculated for the solitary wave solution of Appendix A, may be written to within about 2 percent as

$$g_3\left(\frac{2a}{D}, \frac{H_i}{D}, \frac{z_c}{D}\right) = [1 - A \times \frac{2a}{D}]^{0.5} \quad (2.16)$$

where

$$A = 2.5 \tan\left[\pi\left(\frac{z_c}{D} - 0.5\right)\right] \left[1 - \left(\frac{H_i}{D}\right)^2\right] \quad (2.17)$$

and $2a$ represents the unidirectional waveheight for the solitary wave; a is positive for $z_c/D < 0.5$, negative for $z_c/D > 0.5$. The value of

g_3 is the maximum possible increment in wave speed as long as a weakly nonlinear long wave formulation is appropriate. The geometrical factor A is larger for greater asymmetry and a narrower pycnocline, and $A \rightarrow 0$ as $z_c/D \rightarrow 0.5$ or $H_i/D \rightarrow 1$. That is, the nonlinear correction becomes negligible as the pycnocline becomes very thick and/or as it approaches the center of the water column.

The total mode 1 long wave speed, representing both phase and group velocity, may thus be written

$$\frac{c}{ND} = [1.0 - 0.363 \left(\frac{H_i}{D}\right)] \left[\frac{z_c}{D} \left(1 - \frac{z_c}{D}\right)\right]^{0.5} [1 - A \times \frac{2a}{D}]^{0.5} \quad (2.18)$$

Ratio of bottom HKE to E_{tot} - To consider the effect of the bottom boundary layer on the internal wave field through energy dissipation, it is necessary to relate the bottom velocity fluctuations, u_{bi} , to the total depth integrated wave energy in the water column, E_{tot} . If it is assumed that the energy dissipation in the boundary layer may be expressed in terms of u_{bi} (e.g., LeBlond 1966) and that the group velocity may be expressed in terms of the geometry, the energy balance then may be expressed in terms of the geometry and a single dynamical variable; a formal development is deferred to Chapter 6. In addition, the prior claim that bottom mixed layers tend to insulate the bottom from higher mode energy must be substantiated. To these ends, the ratio of bottom HKE to E_{tot}/D , referred to hereafter as the energy ratio; is examined below.

In Figure 2.5a, the energy ratio is plotted vs. ω/N for the three vertically symmetrical density structures of Figure 2.2. At the long wave limit, bottom HKE is highest for a given E_{tot} for $H_i/D = 1$, and

Figure 2.5: Dependence of linear energy ratio, $0.5 \rho_0 u_{bi}^2 / (E_{tot} / D)$, on density structure: (a) shows dependence on both ω/N and H_i/D for first three modes, density structures of Figure 2.2; (b) shows long wave dependence of first three modes on H_i/D .

this relationship is independent of mode number. The introduction of boundary mixed layers begins to insulate the bottom from interior energy, affecting the higher modes more than the first mode. When $H_i/D = 0$, only mode 1 waves can exist, and the energy ratio is half of its $H_i/D = 1$ value. The **nondimensional** frequency dependence is again more pronounced for greater values of H_i/D .

Figure 2.5b shows the general long wave dependence of the energy ratio on H_i/D . It is clear that for small H_i/D , the bottom is relatively well insulated from higher mode energy in the interior. For example, the analysis of Winant and Bratkovich (1981) shows that the depth integrated energy in each succeeding higher mode on the Southern California shelf is reduced by about a factor of two. Therefore, referring to Figure 2.5b and taking $H_i/D = 0.5$, the bottom HKE due to mode 1 waves might be expected to be a factor of 4-5 greater than the bottom HKE due to mode 2 waves, and about 20 times greater than that due to mode 3 waves. This mode 1 dominance of the energy may be significantly reduced or even reversed in the interior of the water column. The curve for the long wave mode 1 energy ratio in Figure 2.5b is approximated to within 5 percent by the line

$$f_1 \left(\frac{H_i}{D} \right) = 0.96 + 1.08 \left(\frac{H_i}{D} \right) \quad (2.19)$$

In deriving the form of the long wave energy ratio dependence on pycnocline asymmetry, $f_2(z_c/D)$, use is made of the fact that E_{tot} is independent of asymmetry for equivalent internal wave amplitude, a . Then the energy ratio is simply dependent on the square of the transfer function, or

$$f_2(\bar{D}) = \frac{1}{e_2^2} = \left[\frac{z_c/D}{1} \right] \quad (2.20)$$

Similarly, if equipartition of depth integrated energy between kinetic and potential is assumed for nonlinear (solitary) waves, the energy ratio is again simply dependent on the square of the transfer function, or

$$f_3\left(\frac{2a}{D}, \frac{H_i}{D}, \frac{z_c}{D}\right) = \frac{1}{e_3^2} = g_3^2 = \left(\frac{c_{sol}}{c_{lin}}\right)^2 \quad (2.21)$$

Therefore, the total ratio of bottom HKE to depth averaged total energy may be written approximately as

$$\frac{0.5 \rho_o u_{bi}^2}{E_{tot}/D} = [0.96 + 1.08 \left(\frac{H_i}{D}\right)] \left[\frac{1 - z_c/D}{z_c/D}\right] \left(\frac{c_{sol}}{c_{lin}}\right)^2 \quad (2.22)$$

2.5 Example calculations:

To illustrate the use of the relations e_i , f_i , and g_i , three different density profiles have been chosen as examples and analysed in light of the results of the last section to determine bottom velocity, long wave speed, and the ratio of bottom HKE to depth averaged total energy, for a 20 m internal wave. The chosen density profiles are the two previously described approximations to the CODE I density profiles of Figure 2.1, and a profile characteristic of both the Southern California shelf during the summer as described by Winant and Bratkovich (1981), and Massachusetts Bay during the summer as reported by Halpern (1971) and Haury, et al. (1979). The first profile has a 30 m thick surface mixed layer, a 50 m thick pycnocline with $N_i = 2.5$ cph, and a 20 m thick bottom mixed layer. The second profile has a 60 m thick upper mixed

layer, a density jump of $1\text{E-}04 \text{ gm/cm}^2$, and a 40 m thick bottom mixed layer. The third approximate profile is defined as a 5 m thick surface mixed layer, a 20 m thick pycnocline with $N_i = 20 \text{ cph}$, and a 55 m thick lower mixed layer. The geometries of each of the examples and results of the analysis are summarized in Table 2.1, and are discussed below. The wave is assumed to be a solitary wave, so that for each density structure it has a unique characteristic "wavelength", L_s , and characteristic "period", T_s , defined here as $T_s = L_s/c_{s01} = 4l/c_{s01}$, where l is a length scale defined in equation A.26. Thus defined, L_s and T_s are the fundamental wavelength and period of the closest 2nd order Stokes wave form.

The first of the CODE II density profiles represents a typical example of the density structure that tended to prevail during the entire CODE II experiment; i.e., surface and bottom mixed layers and a fairly broad interior region with weak stratification (Huyer 1983, Fleischbein, et al. 1982). Geometrically similar density structure is present in the winter months at the CODE site, but the stratification is much stronger (Fleischbein, et al. 1981). The second of the CODE II density profiles represents an extremum of the possible short term variation in density structure, possibly caused by the passage of an internal wave. It is nevertheless a density profile which, taken alone, might be thought representative of the density structure at that site. Comparison of internal wave parameters calculated for the two CODE II profiles gives a measure of the uncertainty resulting from short term density structure variability. The third example density profile is more typical of summer density structures over continental shelves not

PROFILE NAME	STRUCTURE			WAVE SPEED			KINEMATICS					ENERGY				
	H _i	z _c	D	N _f	N	g ₁	g ₂	c ₁ in	A	g ₃ (20)	$\frac{a N_{f1} n}{u_{bf} \text{ sol}}$	u _{bf} (20)	L _s (20)	T _s (20)	f ₁ f ₂	f ₁ f ₂ f ₃
CODE 1 No. 1	50m	45m	100m	2.5cph	3.1x10 ⁻³ sec ⁻¹	0.41	0.99	0.13 ms ⁻¹	.30	1.03	0.9/0.9	.07ms ⁻¹	.1100m	130 min	1.83	1.94
CODE 1 NO. 2	0m	40m	100m		3.1 x10 ⁻³ sec ⁻¹	0.50	0.98	0.15 ms ⁻¹	.81	1.08	0.8/0.8	.08ms ⁻¹	700m	62 min	1.50	1.75
MASS BAY S. CAL.	20m	65m	80m	20cph	1.7x10 ⁻² sec ⁻¹	0.45	0.78	0.49 ms ⁻¹	3.51	1.37	2.1/1.5	.23ms ⁻¹	240m	6 min	0.28	0.53

Table 2 - Geometries and Results of Density Structure Example
Calculations; All Variables as Defined in Text; 20 refers to 2 m
solitary wave

affected by upwelling. Comparison of internal wave parameters calculated for this third profile to wave parameters calculated for the CODE profiles shows the importance of basic differences in density structure.

The two CODE ■ approximate density profiles are most strongly characterized by the fact that they have the same values of N and D , and a pycnocline center height very close to the center of the water column, $z_c/D \sim 0.5$. They differ mostly in the pycnocline thickness, H_i/D . There are few differences between first mode internal wave characteristics calculated for these two profiles. The influence of H_i/D is evidenced only in the small difference in propagation speed, in the ratio of bottom KE to E_{tot} , which is slightly greater for the larger value of H_i/D , and in the characteristic wavelength/period of a 20 m solitary wave, which is longer for the larger value of H_i/D . Due to the proximity of the pycnocline to the center of the water column, nonlinearity is not very important for either of the profiles, except insofar as it allows for a single, asymmetrical, uniquely determined wave form. It will be shown in the next chapter that the predicted magnitude and period of the bottom velocity fluctuations induced by a 20 m internal wave (7-8 cm/sec at 1-2 hrs) are in fact typical of observations during CODE ■.

The third example profile is markedly different from the CODE ■ profiles, having a much higher value of N and pronounced vertical asymmetry of the density structure with the pycnocline much closer to the surface. The depth is slightly less and the pycnocline thickness is intermediate between the two CODE ■ profiles. The effects of the near surface pycnocline are to: (1) decrease the linear wave speed by about

20 percent, about as much as the effect of the variation in H_i/D between the COE profiles; (2) greatly increase the effect of non-linearity as shown by the values of A and $g_3 = c_{sol}/c_{lin}$; (3) increase the long wave asymptotic value of the nondimensional transfer function, aN/u_{bi} ; (4) decrease the characteristic wavelength/period of the solitary wave; and (5) greatly decrease the amount of bottom HKE for a given value of E_{tot} . The higher value of N directly increases the value of u_{bi}/a for the given structure and directly increases wave speed, leading to an even shorter characteristic period for the solitary wave. The increases in wave speed and u_{bi}/a due to N eclipse the decreases caused by the large value of z_c/D . The characteristic wave period for a 20 m wave, 6 min, and wavelength of 240 m compare favorably to observed wave periods and wavelengths for the same waveheights in Massachusetts Bay (Halpern 1971, Haury, et al. 1979). While the long wave assumption is clearly stretched for waves of such short wavelength/period, it is once again believed that the resultant simplification of the calculations and reasonable agreement with data justify the approximation.

In summary, for density structures like the COE examples with $z_c/D \sim 0.5$, small N , and a moderately thick pycnocline, nonlinearity will produce only small increases in wave speed that are probably indistinguishable from the effects of uncertainty in the density structure. Bottom velocity fluctuations should be dominated by mode 1 internal waves, as long as the bottom mixed layer is an appreciable fraction of the depth. The mode 1 ratio of u_{bi}/a should very nearly equal N , since $z_c/D \sim 0.5$ implies that $aN/u_{bi} \sim 1$. The mode 1 ratio

of bottom HKE to E_{tot} will be fairly high, indicating that energy dissipation by bottom friction may be a relatively important sink for internal wave energy. Wavelengths and periods for large nonlinear waves likely will be longer than those observed on the East Coast and Southern California shelves.

For density structures like the third example profile, characterized by $0.7 < z_c/D < 0.9$, small H_1/D , and high values of N , nonlinearity is likely of first order importance in determining wave speeds and the intensity of bottom velocity fluctuations. Surface velocity fluctuations induced by internal waves will be much greater than bottom velocity fluctuations, but the latter may still be large due to the combined effects of high N and nonlinearity. Wavelengths and periods are shorter than for the CODE II type density structure, and the ratio of bottom HKE to E_{tot} is much smaller than for the CODE II type structure, indicating possibly less relative importance for energy dissipation by bottom friction.

2.6 Discussion and Conclusions:

The most important results of this chapter are:

(1) The high frequency internal wave field on the continental shelf is expected to be energetically dominated by the onshore propagation of mode 1 internal waves. The bottom will be insulated from higher mode internal wave energy if a bottom mixed layer exists and occupies an appreciable fraction of the water column (reasonably typical; e.g., Caldwell 1978 and Pingree and Griffiths 1977), and cross-shelf bottom velocity fluctuations associated with mode 1 internal waves likely will be the most important forcing for the boundary layer in the high frequency

internal wave band.

(2) Long crested trains of topographically generated, high frequency, mode 1 internal waves are commonly observed propagating shoreward on the continental shelf. These wave trains often may be associated with the low, broad peak observed slightly below the local value of N in continental shelf current and temperature spectra, but there is no direct evidence that such is the general case. Trains of large amplitude internal waves can induce large, organized fluctuations in bottom velocity, which have been identified as the strongest forcing for the bottom boundary layer on the Mid-Atlantic Bight during the summer when superimposed on tidal currents (Butman, et al. 1979).

(3) Internal waves on the continental shelf are quite variable in both space and time. Wave trains generated by tidal interaction with topography tend to be localized and intermittent; at a given location, internal waves may be present between 0 percent and about 30-40 percent of the time (Halpern 1971, Butman, et al 1979). On longer time scales, the energy in the internal wave band depends strongly on seasonal stratification.

(4) Using the set of approximate formulae developed in this chapter, it is possible to estimate bottom velocity, wave speed, and the relationship between total, depth integrated internal wave energy and bottom HKE, given a reasonable approximation to the density structure and an estimate of wave height. For example, the maximum likely internal wave induced bottom velocity might be estimated for a particular situation by assuming a maximum wave height (some fraction of the water depth based on experience) and using eq. 2.12 for the mode 1 transfer function between

wave height and bottom velocity.

(5) The results of the theoretical analysis suggest the following practical ordering of effects for mode 1 internal waves - (a) The long wave linear ratio of bottom velocity, u_{bj} , to internal wave amplitude, a , is determined solely by the depth averaged Brunt-Vaisala frequency, N , and the relative height of the pycnocline, z_c/D . For $|z_c/D - 0.5| > 0$, nonlinear effects may act to increase u_{bj} for a given value of a . For $z_c/D \sim 0.5$, $u_{bj}/a \sim N$. (b) The long wave speed, c , is determined at first order by the density difference across the water column, ND . Pycnocline thickness, H_1/D , and z_c/D are of secondary importance. Nonlinear increases in wave speed will tend to counteract linear decreases for $|z_c/D - 0.5| > 0$, so that wave speed will seldom be less than the linear continuously stratified value of $(1/\pi)ND$. Because nonlinearity is not important for $z_c/D \sim 0.5$, wave speed will seldom exceed the linear two layer symmetrical value of $0.5ND$ by more than a few percent. (c) The ratio of bottom HKE to depth integrated total energy is determined at first order by D and z_c/D , and secondarily by H_1/D and the augmenting effect of nonlinearity. The amount of bottom HKE for a given horizontal energy flux ($E_f = cE_{tot}$) hence depends most strongly on D , then about equally on N and z_c/D , and finally on H_1/D and the degree of nonlinearity. This relationship is particularly important for determining the effectiveness of internal wave energy dissipation in the bottom boundary layer.

(6) There are two ways in which nonlinear internal waves may be important to the boundary layer interaction problem. The first, which has been considered in this chapter, is the effect that nonlinearity has

on c and u_{bi}/a . The second is the fact that nonlinear wave solutions introduce time **asymmetry** into the bottom velocity fluctuations, with narrow peaks and broad troughs, and in the extreme allow for transient waves. These effects will be discussed in Section 4.1, but their explicit consideration is in general beyond the scope of this work.

(7) **As** long as the density profiles of Figure 2.1 are reasonably typical of the density structure during **CODE I** (Huyer 1983), it may be anticipated for the **CODE I** internal wave field specifically that $u_{bi}/a \sim N$, that the nonlinear increment in wave speed will be small, and may be indistinguishable from the effects of uncertainty in density structure, and that large amplitude waves will have characteristic periods on the order of 1-2 hrs.

CHAPTER 3

INTERNAL WAVES DURING CODE ■

3.1 Introduction:

To consider the interaction of internal waves and the bottom boundary layer, it is necessary to know (1) the nature and intensity of fluctuations in near bottom velocity induced by internal waves, (2) the relationship between bottom velocity fluctuations and internal wave amplitude and energy in the interior, (3) any additional forcing components or constraints for the bottom boundary layer, and (4) an estimate of the bottom boundary condition - the physical roughness. Chapter 2 has provided a mechanism for estimating some of the internal wave quantities of interest, given an estimate of the density structure and some idea of internal wave amplitude. However, detailed observations of internal waves on the continental shelf from the bottom boundary layer point of view are uncommon, so that there is little in the way of a direct observational basis for the boundary layer-interaction problem.

Analysis of data from the CODE ■ experiment (see Chapter 1) may help to clarify and possibly resolve some of the above questions. There are several possibilities. First, the continental shelf internal wave field during CODE ■ can be examined from the boundary layer interaction point of view; i.e., with primary emphasis on the nature and intensity of fluctuations in bottom velocity in the high frequency internal wave band and on their relation to interior amplitude. Second, the predictive ability of the approximate methods and formulae proposed in

Chapter 2 can be tested for CODE I conditions. Third, results of the internal wave data analysis can be combined with detailed boundary layer measurements and analysis that were carried out as part of the high frequency boundary layer experiment of CODE I (Grant, et al. 1983, Grant, et al. 1984), in order to more closely examine the boundary layer part of the general interaction problem.

The purpose of the present chapter is to examine the internal wave field during CODE I, addressing the first two of the possibilities above and attempting to provide an observational basis for the third, which will be addressed in later chapters. After a brief discussion of the selection of data and the procedures used in preparing it for analysis, the average auto and cross spectral behaviors of the CODE I internal wave field are examined and found to be consistent with expectation from previous studies of internal waves on the continental shelf. An analysis of the short term nature of bottom velocity fluctuations in the high frequency internal wave band is carried out. These fluctuations are found to be dominated by the onshore propagation of large, intermittent mode 1 internal waves, similar to the topographically generated wave packets commonly observed near coastlines (described in Chapter 2), with a few different characteristics. The predictions for CODE I internal waves from Chapter 2 are tested against the data and found to be good. Finally, the results are summarized and discussed.

3.2 Selection and Preparation of Data:

A map of the layout of the CODE I experiment was presented in Figure 1.1. Along the central line, the shelf break is at a depth of

about 150 m at a distance of about 25 km from the coast. The average isobath direction is 317° T. The bottom slope is uniform at about 0.005 from the shelf break to a depth of about 60 m, when it increases sharply to about 0.02. During the **spring/summer** months (April to August) of CODE I, winds were generally strong and upwelling favorable. Huyer (1983) reports that during this time, isotherms, isohalines, and isopycnals sloped persistently upward toward the coast, with the coldest, saltiest and most dense water inshore, and that this large scale pattern was a near steady state. Mean mid-depth currents were primarily longshore towards the south at about 10 **cm/sec**, but the longshore and onshore variance greatly exceeded their **respective** means. Cross-shelf and longshelf phase lags in low frequency response were **common** (CODE I: Moored Array and Large Scale Data Report, 1983).

The present analysis concentrates on site **C3**, in 90 m of water about 7 km offshore. A detailed plan of site C3 and a schematic of the vertical distribution of instruments on moorings C3A and C3S are presented in Figure 3.1. The moorings are separated by about 2 km., and fluctuations at the high frequency end of the internal wave band tend to be statistically incoherent between them, though lower frequency motions are **well** correlated. As a consequence, all of the internal wave band data analyzed here are from **mooring C3S**, while lower frequency analyses take data from both C3A and **C3S**. In addition, two time series from mooring **C4S** in 133 m are briefly utilized. The time period chosen for analysis is ten weeks long, from April 12, 1981 to June 21, 1981, and takes advantage of the period of best data return and most stable large scale density structure.

Figure 3.1 = (a) Detailed plan of site C3 during CODE high frequency bottom boundary layer experiment. (b) Schematic of vertical distribution of VMCM's on moorings C3A and C3S; depth below surface (C3A), height above bottom (C3S), prefix W refers to C. Winant and D. refers to R. Davis, both of Scripps Institute of Oceanography (see also Table B.1).

The data used in this analysis are from Vector Measuring Current Meters (VMCM's) deployed by C. Winant and R. Davis at Scripps Institute of Oceanography. They were originally designed to accurately measure average currents in the presence of large high frequency fluctuations. According to the CODE I: Moored Array and Large Scale Data Report (1983), their threshold speed is about 1 cm/sec, and they are linear above 5 cm/sec. The CODE I total current environment is almost always well above the point of questionable accuracy, though individual frequency components are less energetic. The current meter data are all internally vector averaged and are sampled every four minutes. The horizontal velocity components are rotated into nominal on-offshore (u , $47^\circ T$) and longshore (v , $317^\circ T$) components. The response time of the VMCM thermistors is about 5 min due to the thermal mass of the current meter (CODE I Data Report above).

Detailed descriptions of calculations and error estimates based on temperature measurements at C3 are presented in Appendix B. Internal wave band isotherm height fluctuations (assumed equal to isopycnal height fluctuations) at 51 m above the bottom are estimated from the temperature record of current meter W1 and a derived background temperature gradient field. Neighboring temperature records on C3S are either contaminated by high frequency noise or in a mixed layer too often to be statistically useful for internal wave band calculations. The derived average error bounds on the height fluctuations are ± 1 m, without accounting for straining of the density field due to other internal waves. The error may be greater when the background temperature gradient is very small. Subinertial (20 hr halfpower point

lowpass filtered) time series for the vertical distribution of temperature gradient are estimated, with an average error of about 60 percent for individual temperature gradient estimates which is substantially improved by depth averaging. Finally, a subinertial time varying mixed layer/constant interior stratification model is fit to the data; average mixed layer height error bars are ± 5 m, and the accuracy of the depth averaged interior temperature gradient is estimated as 15 percent.

The ten week average of the surface to bottom depth averaged temperature gradient is thus $0.009 \pm 0.0015^\circ\text{C}/\text{m}$, and the ten week average of the mixed layer model fit to the data has a surface mixed layer 16 ± 5 m thick, an interior temperature gradient of $.0144 \pm .002^\circ\text{C}/\text{m}$, and a bottom mixed layer 20 ± 5 m thick. A temperature gradient to density gradient conversion factor of $0.0016^\circ\text{C}/\text{m}/\text{cph}^2$ has been calculated as an average value from CTD profiles during the high frequency bottom boundary layer experiment of COLE *et al.* Using this value, the surface to bottom depth-averaged value of N_i averaged over ten weeks, is $2.4 \pm .2$ cph, and the average interior value of N_i for the mixed layer model fit is $3.0 \pm .2$ cph.

In Chapter 2, it was shown that the existence of buffering mixed layers ensures that mode 1 internal waves will be most important to the bottom. For example, using the average mixed layer structure above, $H_i/D = 0.6$. Referring to Figure 2.5b and assuming that the energy in each succeeding higher mode is reduced by about a factor of two (e.g., Winant and Bratkovich 1981, as discussed in Section 2.4), the mode 1 bottom HKE should be a factor of 3-4 higher than the mode 2 bottom HKE.

Use of the isotherm height (η) time series from just above the center of the water column (**W61** at 51 m height) and the horizontal velocity component (u and v) time series from the bottom of C3S (**D15** at 7 m height) emphasizes the first mode by taking measurements from regions of expected large amplitude and constant $n-u$ phase. Velocity measured in the interior is not used because mode 1 horizontal velocity fluctuations near the center of the water column may be small and may change phase relative to mode 1 horizontal velocity fluctuations near the bottom if the pycnocline center height moves past the fixed velocity sensor. Higher mode energy may be present randomly in all three time series; for the η time series, shorter vertical wavelengths associated with higher modes imply large changes in the magnitude and phase of higher mode η fluctuations at 51 m as the background field changes, and for the u and v time series, 7 m is sometimes not near the bottom of the mixed layer, so is not as insulated from higher mode energy in the interior as is the bottom. Higher mode energy appears as noise in the present context.

3.3 Average Statistical Behavior:

In Chapter 2, several previous studies of the internal wave environment on the continental shelf (**Winant** and Olson 1976, Gordon 1973, **Winant** and Bratkovich 1981) were compared and similarities in observed average statistical behavior were summarized. Recapitulating, the high frequency internal wave field over the continental shelf is expected to be energetically dominated by first mode waves propagating towards shore (25-45 percent of the total energy). Near surface and near bottom velocity fluctuations induced by these waves are thus strongest

in the cross-shelf direction. Energy spectra often show a clear semi-diurnal tidal peak, are "red" at slightly higher frequencies, and show a second broad, flat peak, or "knee", slightly below the local value of N . In this section, 10 week averages of the auto- and cross-spectra of isotherm fluctuation energy from 51 m height ($\eta(51)$ energy) and HKE from 7 m height ($u(7)$ and $v(7)$ energy) during CODE II are presented and shown to agree with expectation from the summary of Chapter 2.

Ten week averaged energy density spectra of the three time series of interest are presented in Figure 3.2; $\eta(51)$ energy is shown in Figure 3.2a and $u(7)$ and $v(7)$ energy in Figure 3.2b. Each time series has been highpass filtered at 0.004 cph and tapered with a ten percent cosine bell; the spectra cover the entire ten week time period and have 24 degrees of freedom. The straight line drawn in each plot represents an ω^{-2} slope.

The average energy spectra are consistent with expectation. It is clear from Figure 3.2b that the bottom HKE is strongly polarized, with $u(7)$ energy about three times higher than $v(7)$ energy over the internal wave band and $v(7)$ dominating at lower frequencies. There are clear diurnal and semidiurnal tidal peaks, slightly higher in $v(7)$. A recognizable six hour peak appears in the $u(7)$ energy. All three spectra are red at slightly higher than tidal frequency and both $\eta(51)$ energy and $u(7)$ energy show a clear leveling, or "knee" in the spectrum between one and two cph, slightly below a reasonable estimate of the long term average Brunt-Vaisala frequency (2-3cph). The knee is more pronounced in the η spectrum.

Figure 3.2: Average energy density spectra for (a) derived internal wave band isotherm height time series at 51 m height (η energy) and (b) on-offshore bottom velocity (7 m height) (u energy, solid line) and longshore bottom velocity (7 m height) (v energy, dashed line) at site C3S over 10 week period analyzed. Each spectrum has 24 degrees of freedom, with resolution of 0.0073 cph. The inertial (f) and average Brunt-Vaisala (N) frequencies are indicated. The straight line has an ω^{-2} slope.

Ten week averaged cross-spectra are presented in Figure 3.3; the $\eta(51) - u(7)$ cross-spectrum is shown in Figure 3.3a and the $\eta(51) - v(7)$ cross-spectrum is shown in Figure 3.3b. All raw time series were high pass filtered with a half power cutoff period of 20 hrs. Both cross-spectra have 96 degrees of freedom. The $\eta(51) - u(7)$ cross spectrum, with zero phase difference and coherence (squared) in the range 0.3 - 0.5 over the frequency range 0.1 - 3.0 cph, is clearly dominated by the onshore propagation of first mode internal waves. That is, mode 1 internal waves have zero phase difference between interior amplitude and bottom velocity in the direction of wave propagation. That zero phase so clearly and consistently dominates the $\eta(51)-u(7)$ cross-spectrum is therefore consistent with the onshore propagation of mode 1 waves, but by itself does not allow separate determination of energy in the first mode or of energy in the onshore direction. These last are beyond the scope of the present work.

For the $\eta(51)-v(7)$ cross-spectrum, the coherence (squared) is often not significantly different from zero and never higher than 0.2, while the phase varies between -90 degrees and -180 degrees over the frequency range 0.1-3.0 cph. The $\eta(51)-v(7)$ cross-spectrum is much more difficult to interpret than the $\eta(51)-u(7)$ cross-spectrum. There is no clear, consistently dominant phase relationship over the internal wave band, but it is interesting to note the following: (a) for first mode waves propagating directly onshore, the $\eta(51) - v(7)$ phase relationship is -90 degrees if the influence of rotation is included; (b) for first mode waves propagating to the south of onshore, the $\eta(51) - v(7)$ phase relationship is - 180 degrees, neglecting rotation.

Figure 3.3: Ten week averaged cross-spectra: (a) $\eta(51) - u(7)$ and (b) $\eta(51) - v(7)$, with 95 percent different from 0 confidence limits. All raw time series were high pass filtered with a half power cutoff period of 20 hrs. Both cross-spectra have 96 degrees of freedom.

The present energy levels may be compared to deep ocean energy levels by using expressions for average HKE and η energy from Desaubies (1976), for $\omega \gg f$ and $\omega \ll N$. These expressions are derived from the results of Garrett and Munk (1975), and may be written

$$\frac{\langle \eta^2 \rangle}{2} \sim \frac{2}{\pi} r \frac{f}{N(z)} \frac{1}{\omega^2} \quad \text{and} \quad \frac{\langle u^2 \rangle + \langle v^2 \rangle}{2} \sim \frac{2rN(z)f}{\pi \omega^2} \quad (3.1)$$

where r is a nearly universal constant equal to about $300 \text{ m}^2\text{-cph}$. Using the expressions in eq 3.1 with $N(z) = \text{constant} = 3 \text{ cph}$, at $\omega = 0.5 \text{ cph}$, $S_{\eta\eta} = 4.7 \times 10^4 \text{ m}^2/\text{hz}$ and $S_{uu} = 1.3 \times 10^4 \text{ cm}^2/\text{sec}^2/\text{hz}$, where $S_{\eta\eta}$ is the amplitude spectrum and S_{uu} is the HKE spectrum. The data, averaged between 0.49 and 0.51 cph, give $S_{\eta\eta} = 4.2 \times 10^4 \text{ m}^2/\text{hz}$ and $S_{uu} = 0.9 \times 10^4 \text{ cm}^2/\text{sec}^2/\text{hz}$ (for the 7 m HKE), indicating that the energy estimates obtained are reasonable.

In summary, the relative behavior of the average auto- and cross-spectra examined are consistent with previously reported energy spectra and with the expectation that bottom velocity fluctuations will be dominated by the onshore propagation of mode 1 internal waves, and estimated average energy levels are reasonable.

3.4 Variability of the High Frequency Internal Wave Field:

The primary objective of this section of the CODE internal wave data analysis is to establish the nature of the short term behavior of internal waves during CODE I, and in particular the nature of bottom velocity fluctuations induced by high frequency internal waves. Howell (1983) has demonstrated that large internal events whose kinematics are consistent with the onshore propagation of mode one internal waves occur

in a short segment of the CODE I data at a single mooring in 133 m of water. The purpose here is to show that these internal wave events occurred throughout the CODE I experiment, and in fact dominated the behavior of the near bottom velocity fluctuations in the high frequency internal wave band. In other words, the CODE I high frequency internal waves are similar to the packets of waves commonly observed on the East Coast shelf during the summer, though the background density structures are quite different and the CODE I waves are of much lower frequency and less well defined than the East Coast waves.

both direct examination of the time series and a combination of statistical techniques for examining potentially nonstationary, broad-banded processes are used to attempt to put together a convincing, if qualitative, argument. Because of the observed anisotropy in average bottom HKE levels, and because of the present emphasis on the near bottom velocities, the analysis will concentrate on the the on-offshore velocity record from 7 m above the bottom at site C3 ($u(7)$) and on its relation to the isotherm displacement record from 51 m at site C3 ($\eta(51)$). Behavior of the internal wave field on the continental shelf has been shown to be quite frequency dependent (e.g., Hayes and Halpern 1976). Thus, to examine the time and frequency dependence of energy in the internal wave band, the CODE I internal wave spectrum is split into four superinertial frequency bands. These are numbered from high frequency to low, as band 1 (0.5-2.0 cph), band 2 (0.2-0.5 cph), band 3 (0.1-0.2 cph) and band 4 (0.067-0.1 cph). The time variability of energy in each frequency band is examined by considering successive segments of appropriately filtered data records.

There are two possible obstacles to the **straightforward** statistical analysis of data in the internal wave band. Briscoe (1977) has pointed out that the ω^{-2} slope of the internal wave spectrum implies that data records cannot be considered time sequences of strictly independent points; that is, internal wave fluctuations are autocorrelated to some extent. Therefore, standard statistical tests must be modified and/or carefully tested to determine their applicability for internal wave data analysis. In addition, the existence of underlying trends in the data (nonstationarity) can be shown to invalidate standard statistical procedures (e.g., Bendat and Piersol 1971). Techniques for surmounting these obstacles are described below.

Problems arising from the autocorrelation structure of the internal wave data are dealt with here by using a technique similar to that employed by Briscoe (1977). An artificial time series (hereafter referred to as the "test" time series) is constructed, with a spectral shape similar to the measured 10 week $u(7)$ energy spectrum of Figure 3.2; this spectrum is designated $S_{tt,i}$, $i=1,4097$. The phase is generated as a sequence of random numbers uniformly distributed between $-\pi$ and π , ϕ_i , $i=2,4096$, with $\phi_1 = \phi_{4097} = 0$. The test time series is then constructed as the inverse fourier transform of the sequence $[S_{tt,i}]^{0.5}(\cos(\phi_i) + j \sin(\phi_i))$, where $j = [-1]^{0.5}$. The test time series is subsequently processed in the same manner as the actual data, and conclusions based on the data analysis are all made after comparison to the results of the test analysis, thus accounting for the effect of the autocorrelation structure on the chosen statistical procedures.

Before describing the determination of appropriate stationary record lengths, the sense of the word "stationary" as it is used here should be clarified. If the internal waves during CODE I are indeed similar to the organized wave trains common to many continental shelves, they should occur intermittently. That is, the high frequency waves should occur in bursts separated by periods of several hours to a day or two (Halpern 1971, Butman, et al. 1979). If the high frequency internal waves are more representative of a steady state random process, it is unlikely that they will be intermittent in this sense. "Stationarity" is defined for the present purpose in a very specific, weak sense as time invariance of the true mean and variance of the process over time scales longer than any expected intermittency. In the absence of an ensemble of measurements taken from roughly the same height above the bottom in roughly the same location, it must be assumed that the single records available are representative of the processes of interest. The phrase "testing for stationarity" therefore must be interpreted as assuming weak stationarity and attempting to disprove that assumption based on the behavior of a single sample record.

The energy in each of the four frequency bands is tested for stationarity by constructing a sequence of the rms values of the band-pass filtered record, with each rms value calculated for a record segment chosen to be several times the length of the lowest passed period. The segment lengths are the same as the sampling interval of the energy time series to be described below, and are listed in Table C.1 of Appendix C. The sequences of rms values are then tested for stationarity using the runs test (Bendat and Piersol 1971; pp. 234-237).

For example, to test for the stationarity of the energy in frequency band 1 over a period of one week, a sequence of rms values for the filtered data record over successive 6 hr intervals is constructed. The sequences are divided into one week blocks of 28 points each and the number of runs about the week-long median of the rms values is calculated for each block. If this number falls outside the 95 percent confidence interval for the sample size, $10 < R < 19$ for $N = 28$, the hypothesis of stationarity is rejected at the 95 percent confidence level for that block, and if a majority of one week blocks are determined to be nonstationary, the data is said to be nonstationary over a one week period. In this way, assuming local stationarity and working towards longer time periods, the maximum stationary record length for each filtered data record may be determined.

Runs tests on band-pass filtered statistics time series show that energy in the lower frequency bands (3 and 4) may be considered stationary over the entire ten week time period. For band 2, the $\eta(51)$ energy is essentially stationary over the entire time period, but $u(7)$ energy must be considered nonstationary for periods longer than five weeks. Band 1 $\eta(51)$ energy is nonstationary for periods longer than 2.5 weeks, and $u(7)$ energy is nonstationary for periods longer than 1.5 weeks. The test energy is stationary over its entire three week period, for all frequency bands. In other words, the processes which dominate the two highest internal wave frequency bands are nonstationary, while lower frequency processes seem to be stationary. The primary value of testing for stationarity is to determine frequency dependent stationary sample lengths over which other statistical measures may be considered

locally valid (Bendat and Piersol (1971), Chapter 10).

The first procedure used for the analysis of short term variability is an examination of the probability structure of the data. Histograms for stationary segments of the random phase test time series and the $u(7)$ time series are presented in Figure 3.4. Figure 3.4a compares histograms for three week stationary segments of the low-frequency dominated $u(7)$ record, which have been **bandpass** filtered between 0.05 and 1.0 cph, to the **amplitude** histogram of the "test" time series treated in the same way. Figure 3.4b compares histograms for week long stationary segments of the band 1 filtered $u(7)$ record to histograms of week long segments of the band 1 filtered test record. The standardized, normalized histograms are computed by sorting the following statistic into ten bins:

$$y = \frac{1}{N} \left(\frac{x - \bar{x}}{\sigma_x} \right) \quad (3.2)$$

where $\bar{x}$ is the mean value of the process, σ_x is the standard deviation, and N is the total number of data points in the time period of interest. The histograms are tested for normality using the chi-square goodness of fit test with seven (10-3) degrees of freedom (Bendat and Piersol, 1971; pp. 119-122),

$$\chi^2 = \sum_{i=1}^{10} \frac{(f_i - F_i)^2}{F_i} \quad (3.3)$$

where f_i is the measured frequency of occurrence of values of y within bin "i" and F_i is the expected frequency of occurrence from the normal distribution. For the hypothesis of normality of the population to be rejected at the 95 percent confidence level using the chi-square test

Figure 3.4: Amplitude histograms and statistics for the "test" time series (left side) and the u record (right side), calculated as per eq. 3.2 and pp. 119-122 of Bendat and Piersol (1971). (a) shows histograms of three week records band pass filtered from 0.05-1.0 cph; (b) shows histograms of 1 week records band pass filtered from 0.5 to 2.0 cph. The light line is the theoretical normal distribution and the heavy lines are the data. All statistics refer to the data and are averaged values.

with seven degrees of freedom, the value of χ^2 must be greater than 14.

A sample record of a **bandpass** filtered white noise process which is much longer than the longest passed period should have a normal (gaussian) amplitude pdf. Gaussian processes have zero skewness and a kurtosis of three. Borrowing from turbulence literature (Townsend 1976, pp.126-128), the pdf of a strongly nonlinear (asymmetrical) process is skewed towards the peak side of the process mean and an intermittent process has higher kurtosis than a gaussian process. In the present instance **amplitude** histograms are calculated, but these differ from **pdf's** only by a constant. In addition, the process of interest is not "white", but has some autocorrelation structure, as discussed above. However, the average values of χ^2 presented in Figure 3.3 show that the test time series as treated here is normal, so that the auto correlation structure does not affect the histogram calculations. The u(7) records are distinctly non-normal, showing both high kurtosis (peakiness) and high negative skewness; they are skewed in the direction consistent with nonlinear waves of depression. The u(7) records thus seem to be uniformly intermittent and indicate a predominance of negative peaks. Kurtosis is slightly higher and skewness slightly lower for the highest frequency fluctuations, indicating perhaps greater intermittency and slightly more symmetry.

Although high kurtosis and negative skewness of fluctuations in the internal wave frequency band during **CODE 1** are supportive of a scenario of intermittent trains of internal waves propagating past the measurement site, their support for such a scenario is not exclusive.

That is, as Frankignoul (1974) puts it,

"The observed [high frequency] energy bursts can arise from the passage of an energetic wave train or reflect the higher energy level of an ensemble of waves or of the whole internal wave field. Information on the corresponding variability in the wavenumber space could help resolve this uncertainty. "

In the present case, information on the corresponding variability in horizontal wavenumber space in the predominant cross-shelf direction of propagation would be most helpful, but is unfortunately not available. The cross-shelf spacing of moorings during CODE I and the changes in depth between moorings are apparently too great to allow the extraction of horizontal wavenumber information for internal wave frequencies. However, evidence for the onshore propagation of identifiable waveforms is available, and is presented here both on its own merit and as qualitative support for intermittency being due to the propagation of discrete groups of internal waves past the measurement site.

In Figure 3.5, temperature records from 58 m height at C4S (133 m) and 51 m height at C3S (90 m) are compared over a six day period beginning on 22 April 1981. In the plot the C3S record lags the C4S record by 9 hrs. Wave events identified in the C4S record as A thru L are described kinematically in Howell (1983); events clearly recognizable at a later time at C3 are C, D, G', J, K, L, and M. There appears to be much additional wave activity at C3S, but wave forms are not obviously identifiable as being the same as those observed at C4; considering the unknown nature and likely variability of the waveguide between the sites (the moorings are 8 km apart, not directly cross-shelf, and the water depth decreases by 30 percent from C4 to C3) and uncertainty in exact

Figure 3.5 Comparison **between** temperature records from 58 m height at site C4 (upper trace) and 51 m height at site C3 (lower trace), for week with large events reported in Howell (1983) beginning 22 April 1981. Lower trace lags upper trace by 9 hrs, is 8 km inshore. Event identification scheme same as in Howell (1983). Events connected by solid arrows clearly correspond.

propagation direction as the waves pass both moorings, this lack of correspondence is not too surprising. The background temperature gradient was 0(0.01 deg C), so that these records clearly represent large disturbances, and hence may be best represented by some soliton-like solution to a K-dV type equation (as discussed in the previous chapter). Djordjevic and Redekopp (1978) have indicated that a large soliton may break down into a train of smaller waves as it propagates into shallower water, so that wave energy passing a mooring near the shelf break may have changed form significantly by the time it reaches mid-shelf. In any case, Figure 3.5 shows that large internal events do propagate shoreward, with at least a tendency to preserve form and relative spacing.

To examine the time and frequency variability of the bottom HKE during CODE I and its relation to vertical motions in the interior, time series of band-averaged auto- and cross-spectral quantities (referred to as energy and cross-spectral time series) have been produced for each of frequency bands 1-4. Sample lengths and spacing between samples have been determined in each case by a balance between considerations of stability, frequency resolution, and a desire to make each sample record as small as possible in order to maximize the time resolution. For details of the statistical analysis procedures and references, see Appendix C. The results of these calculations are summarized for the $\eta(51)$, $u(7)$, $v(7)$, and test energies and the η - u and u - v cross-spectra in Table 3.1 for reference. These results confirm previous conclusions: (1) the high frequency bottom HKE is dominated by the $u(7)$ energy; (2) the high frequency waves are intermittent - the high variance of the

		BAND 1 0.5 - 20 CPH				BAND 2 0.2 - 0.5 CPH				BAND 3 0.1 - 0.2 CPH				BAND 4 0.067 - 0.1 CPH			
Energy	l x 10 ⁴	Ave	Std Dev	95 Sign	Ave	Std Dev	95 Sign	Ave	Std Dev	95 Sign	Ave	Std Dev	95 Sign	Ave	Std Dev	95 Sign	
Test	Energy	0.144	0.035		1.21	0.378		4.08	1.58								
u	Energy	0.231	0.213		1.49	1.15		4.70	2.84					13.1	7.02		
v	Energy	0.091	0.070		0.544	0.351		2.31	1.24					15.1	6.92		
n	Energy	2.19	2.30		12.7	8.67		45.0	27.4					54.3	36.8		
n-u	Coherence	0.43	0.23	0.15	0.43	0.19	0.15	0.45	0.19	0.28	0.35	0.14	0.28				
n-u	Phase	7.9°	28'	47'	0.8'	27'	44'	-2.4'	35'	50°	-15'	49'	54'				
u-v	Coherence	0.32	0.19	0.15	0.33	0.18	0.15	0.44	0.18	0.28	0.31	0.24	0.28				
u-v	Phase	-49'	71'	50'	-96'	47'	58'	-91'	28'	51°	-50°	72°	52'				

Table 3.1: Statistics of derived energy and cross-spectral time series. Upper half - average stationary period values of mean energy and standard deviation of energy by frequency band, for "test", η -u, and v energy time series. Lower half - average stationary period η -u and u-v coherence and phase, with mean value, standard deviation, and 95 percent difference from zero confidence limits, by frequency band.

$u(7)$, $v(7)$, and $\eta(51)$ energy relative to the test energy is essentially the same as high relative kurtosis; (3) the relationship between $\eta(51)$ and $u(7)$ is dominated by onshore propagating mode 1 waves - fluctuations in $\eta(51)$ and $u(7)$ are in phase and the average coherence (squared) is about 0.4.

The energy and cross-spectral time series are examined for **inter-** and **intra-band correlations** and **relationships** by producing scatter plots of (the log of) energy vs. either appropriately resampled energy from another band or **coherence/phase** from the same band. Linear regressions and correlation coefficients are calculated where appropriate and checked for significance at the 95 percent confidence level as in **Bendat and Piersol (1971)**, pp. 126-133. The correlation coefficient, r_{xy} , is significantly different from zero at the 95 percent confidence level **if**

$$w = \frac{(N - 3)^{0.5}}{2} \ln \left[\frac{1 + r_{xy}}{1 - r_{xy}} \right] \geq 1.96 \quad (3.4)$$

for $r_{xy} > 0$, where N is the number of pairings (x,y) in the sample

Linear regressions between (the log of) $u(7)$ energy in different frequency bands over time periods which are stationary for the lower frequency reveal that measured HKE is significantly correlated at the 95 percent confidence level within bands 1-3. The average band 1 - band 2 value of r_{uu} is 0.74 with $w_{ave} = 4.4$ (average of 3 three-week periods). For band 2 - band 3 $r_{uu} = 0.77$ with $w = 5.8$, and for band 1 - band 3 $r_{uu} = 0.69$ with $w = 4.8$. There is no significant correlation between frequencies for the "test" energy time series. Neither is there any significant correlation between measured band 4 $u(7)$ energy and higher frequency $u(7)$ energy. A stationary random phase wave field by

definition should have no significant correlation between energy at different frequencies. If correlations are calculated over nonstationary record lengths, however, correlation may result simply because energy in all bands increases or decreases together. If the tide is strongly nonlinear, correlations between the tidal and supertidal frequency bands (band 4 and 3, respectively) would be expected, and might even extend to band 2. On the other hand, if the high frequency internal wave field is dominated by strongly nonlinear waves or if the observed intermittency involves energy in all of the high frequency bands, high correlations might be expected between energy levels in the highest frequency bands (bands 1-3). Thus stationary period interband correlations at higher frequencies favor a scenario of strongly nonlinear internal waves and/or a scenario of multifrequency intermittent wave groups. The semidiurnal tide at site C3 is not directly associated with higher frequency energy at CODE ■ site C3.

Scatter plots between $u(7)$ energy and η - u coherence and phase for band 1 are presented in Figure 3.6. They again illustrate the dominance at high frequency of onshore propagating mode 1 internal waves. High η - u coherence, zero η - u phase, and high $u(7)$ energy level are all clearly related. The apparent non-zero value of the dominant phase in these plots is a high frequency artifact of slightly different time bases for the $\eta(51)$ and $u(7)$ records. The relationships between $\eta(51)$ energy, η - u coherence, and u - u phase are similar, but not as clear. This may be because some of the $\eta(51)$ energy results from higher modes that are not very apparent in the $u(7)$ record, or it may be because there are in fact occasional waves with velocity fluctuations stronger

Figure 3.6: Scatter plots of energy and cross-spectral values from the band I energy and cross-spectral time series, for the full 10 week period. (a) ($\log$ of) u energy vs. η - u coherence; (b) η - u phase vs. η - u coherence; (c) η - u phase vs. ($\log$ of) u energy.

in the longshore direction. Neither of these possibilities is truly at odds with the contention that most of the stronger fluctuations in bottom velocity in the high frequency internal wave band during CODE II are due to the onshore propagation of mode 1 internal waves.

The analysis to this point has demonstrated that a number of measures of the short term variability of the internal wave field during CODE II are consistent with a scenario of domination of bottom HKE in the high frequency internal wave band by onshore propagating mode 1 internal waves, which are intermittent and of large amplitude relative to steady state internal waves with the same average energy level. To complete the picture, two examples of internal wave "events" during CODE II are presented below. The first example is the wave "group" designated J, K, L, and M in Figure 3.5, as it propagates past site C3. The second example includes a group of internal waves observed during the high frequency bottom boundary layer experiment of CODE II, some of which have been identified in Grant, et al. (1984) as contributing to the variability of boundary layer measurements during a portion of that experiment; these will be referred to again in Chapter 5.

In Figure 3.7, the large events J, K, L and M of Figure 3.5 are shown in more detail as they propagate past site C3 in 90 m of water. The upper trace shows the $\eta(51)$ record, and the lower plot shows the raw $u(7)$ record and raw $v(7)$ record. There are several obvious points to be emphasized: (1) The waves are very large. The largest wave height in the $\eta(51)$ record is 42 m, or about half of the water depth. The corresponding fluctuation in $u(7)$ is 22 cm/s peak to peak. (2) The $\eta(51)$ record and $u(7)$ record are very clearly in phase, with only slight

FIGURE 3.7: Internal wave events J, K, L, and M of Figure 3.5, at site C3, April 26-27, 1981. (a) shows isotherm displacement record $\eta(51)$, high pass filtered at 20 hrs, (b) shows raw $u(7)$ (solid line) and $v(7)$ (dashed line) records.

differences in the relative amplitudes of successive peaks. The $v(7)$ record shows little evidence of the internal waves, but does show a rather strong ($\sim 15 \text{ cm/s}$) longshore current, possibly tidal. (3) The largest fluctuations are of about 2-3 hr peak width and are asymmetrical in shape, resembling waves of depression. Smaller, more symmetrical, fluctuations of about 0.5 - 1.0 hr period are superimposed, in such a way that the total time trace would be difficult to describe by any classical linear or non-linear wave form. The result is that the waves are best described simply as identifiable larger, lower frequency waves of depression accompanied by smaller, higher frequency waves, without further specifying the form.

Figure 3.8 shows the $\eta(51)$, $u(7)$ and $v(7)$ records from the period of fairly intensive boundary layer data analysis reported by Grant, et al. (1983) and Grant, et al. (1984). The $\eta(51)$ record shows a large (36 m) rise in the isotherms followed by a train of 14 high frequency (0.5 to 1.0 hr) internal waves of 5-20 m wave height. Examination of the $u(7)$ and $v(7)$ records reveals that bottom velocity fluctuations are present and stronger in u , and that the $\eta(51)$ and $u(7)$ records are in phase, but that for the most part the velocity fluctuations are much weaker than those of Figure 3.7. A possible exception are the two waves about two thirds of the way through the records, of about 1 hr period. The effect of these waves on boundary layer measurements is considered in more detail in Chapter 5. The longshore current, $v(7)$, varies between 5 and 10 cm/s, and is uniformly to the north.

Considering the results of the theoretical analysis of Chapter 2, two reasons for the discrepancies between the $\eta(51)$ - $u(7)$ magnitude

FIGURE 3.8: Internal waves during a portion of the high frequency bottom boundary layer experiment of CODE I, at site C3, June 2-3, 1981. (a) shows isotherm displacement record, $\eta(51)$, high pass filtered at 20 hrs, (b) shows raw $u(7)$ (solid line) and $v(7)$ (dashed line) records.

ratios in Figures 3.7. and 3.8 are apparent, both related to the differences in stratification between the two periods. During the wave events of Figure 3.7, the depth averaged value of N was about 2.6 cph (Fleischbein, et al. 1982), while during the period of Figure 3.8, $N \sim 1.8$ cph (see Table 2.1). Assuming that $z_c/D \sim 0.5$ (a good assumption for CODE 1), the long wave transfer function would predict that $u_{bt}/a \sim N$; therefore, for the same wave height, bottom velocity fluctuations in Figure 3.8 should only be about 70 percent as large as those in Figure 3.7. In addition, waves of periods longer than about 45 min ($\omega/N \sim 0.5$) are well approximated by the long wave relationship $u_{bt}/a \sim N$ for Figure 3.7; this criterion encompasses many of the large waves in that figure. Smaller period (shorter) waves should be more confined to the interior, from Figure 2.3a. For Figure 3.8, waves must be longer than about 65 min for the same long wave criterion to hold; in fact, many waves are shorter in this case, hence do not induce very strong fluctuations in bottom velocity.

High frequency $u(7)$ energy levels during the period shown in Figure 3.7 are above the 95th percentile for high frequency $u(7)$ energy over the 10 week period analyzed. These bottom velocity fluctuations are thus among the largest to occur in the high frequency internal wave frequency band during CODE 1. Band 1 $u(7)$ energy for the period including the two waves two thirds of the way through Figure 3.8 is at about the 45th percentile. These waves are probably of below average to average intensity for the CODE 1 period, recalling that internal waves occur intermittently and that the lowest energy levels during CODE 1 hence correspond to the absence of wave activity.

Summary: Energetic **bottom** velocity fluctuations in the high frequency internal wave band during CODE I are usually associated with the intermittent onshore propagation of large amplitude mode 1 internal waves. These waves often occur in irregular groupings of a large, lower frequency wave of depression accompanied by **smaller**, higher frequency oscillations, leading to correlations between the energy throughout the **high** frequency internal wave band. The high frequency energy at site C3 is not correlated with the tide at C3, but this does not deny the possibility of tidal generation of the waves at another location. Maximum wave heights appear to be on the order of half of the water depth (~45 m at C3) and maximum peak to peak bottom velocity fluctuations are on the order of 20 **cm/s**. More typical bottom velocities are on the order of 5-10 **cm/s** peak to peak. Typical wave periods are about 1-4 hrs. Further generalization about wave form or frequency of occurrence is hard to justify.

3.5 Comparison of CODE I Data to Theoretical Predictions:

In Chapter 2, the results of an approximate theoretical analysis for the sensitivity of internal waves to background density structure and assumptions about linearity were used to predict several specific aspects of internal waves during CODE I. For typical CODE I density structure, it was predicted that (1) u_{bi}/a should approximately equal N , (2) the nonlinear increment in wave speed should be small and might be indistinguishable from the effects of uncertainty in the density structure, and (3) large **amplitude** waves should have characteristic periods on the order of 1-2 hours. In this section, the predictions of

Chapter, 2 are tested against data from CODE II.

The last of the predictions for typical wave periods has already been addressed. In Figures 3.7 and 3.5 the large amplitude waves have periods on the order of 1-3 hours. An examination of Figure 3.5 reveals that wave event periods are sometimes longer, but as the CODE II wave events do not necessarily correspond to solitary waves of permanent form, exact agreement is not to be expected. The predicted wave periods are of the correct order of magnitude for CODE II.

In Figure 3.9a, a ten week time series of surface to bottom depth averaged temperature gradient is plotted. It is calculated as described in Appendix B, from the low pass filtered (20 hr halfpower point) derived background temperature gradient field during CODE II. The temperature gradient to density gradient conversion factor of $0.0016 \text{ }^{\circ}\text{C}/\text{m}/\text{cph}^2$ calculated from CTD profiles during the high frequency bottom boundary layer experiment of CODE I is assumed as a reasonable way of accounting for the influence of salinity during CODE II. Dividing the conversion factor by the time varying temperature gradient and taking the square root, an estimate of $1/N$ is obtained. This is plotted as the solid line in Figure 3.9b, and represents a theoretical estimate of a/u_{bi} according to the second of the above predictions from Chapter 2.

To calculate an estimate of a/u_{bi} from the high frequency internal wave data during CODE II, the band 2 $\eta(51)$ energy time series is divided by the total band 2 HKE ($u(7) + v(7)$) time series. The square root of each value is taken as a reasonable estimate of a/u_{bi} , and is plotted as the dotted line with symbols in Figure 3.9b. The band 2 energy is used as best satisfying the nonrotating long wave assumption for which

Figure 3.9: Test of predicted long wave transfer function for CODE 1, $a/u_{bi} \sim N$ (a) shows the 10 week subinertial time series of depth averaged temperature gradient, (b) shows an estimate of $1/N$ from (a) (heavy solid line) and an estimate of a/u_{bi} from the band 2 internal wave energy time series during CODE 2 (dot-dash line with triangles).

the relationship $u_{bi}/a \sim N$ is valid (i.e., $f \ll \omega \ll N$), while still representing high frequency internal wave energy. The generally reasonable agreement demonstrates that the relationship $u_{bi}/a \sim N$ is a fairly good estimator for bottom velocity fluctuations during CODE II. The occasional lapses may result from changes in the temperature gradient to density gradient conversion factor due to changes in the importance of salinity, or they may result from the presence of strong fronts (see further discussion in Appendix B).

In Figure 3.5, temperature records from site C3 (90 m) and C4 (133 m) were compared, showing that at least some of the waveforms passing C4 were identifiable and had the same relative spacing as they passed C3. The best fit (by eye) of the two temperature records was achieved by lagging the C3 record by 9 hrs. At 8 km separation, this gives an average wave speed of about 25 cm/s.

Fleischbein, et al. (1982) present two cross shelf transects of density profiles at the CODE II central line during the period shown in Figure 3.5. The first is from April 25, 1981 (day 13 in Figure 3.4) and includes three profiles at or between sites C3 and C4. The second is two days later, with three profiles in approximately the same locations. Using the techniques proposed in Chapter 2 to approximate the density structure of each profile and estimate long wave speeds, the cross shelf variation in theoretical wave speed, $c(x)$, has been estimated for each transect by linear interpolation between the sites of the density profiles. The travel time, T , for an internal wave to reach C3 from C4 can then be estimated as

$$T = \int_0^{8000 \text{ m}} \frac{dx}{c(x)} \quad (3.5)$$

with $c(x)$ expressed in m/s. Four calculations have been carried out for each transect. These are: (1) minimum linear $c(x)$ based on the surface to bottom density difference, $c = (1/\pi)ND$; (2) linear $c(x)$ estimated from the reported density profiles (referred to as "measured" in Table 3.2); (3) maximum linear $c(x)$ based on the surface to bottom density difference, $c = 0.5ND$; and (4) nonlinear $c(x)$ for a 30 m internal solitary wave, estimated from the reported density profiles (again referred to as "measured"). The calculated travel times and average wave speeds ($c = 8000m/T$) are listed in Table 3.2.

Of the theoretical travel times, the 30 m solitary wave estimates are closest to the estimate from CODE ■ data in Figure 3.5. The theoretical solitary wave speeds are about 10 percent faster than the equivalent theoretical linear wave speeds. However, the estimate of travel time from Figure 3.5 is only approximate, as are the estimates of wave speed from the density profiles. In addition, the estimated theoretical wave speeds assume no codirectional current; the difference of 2-3 cm/s between "measured" linear and solitary wave speeds might easily be overwhelmed by advection of the waves by an onshore component of the background current. Thus, theoretical estimates of solitary wave speed are slightly greater than estimates of linear wave speed, and are in better agreement with a rough estimate of travel time from the CODE ■ data, but the differences are too small and the uncertainties too large to make any definite conclusions about which is better. Given the uncertainties involved, the estimates of travel time from the "measured" density structure are remarkably good. Note that the fastest solitary wave speed is still less than the maximum possible linear wave speed

TABLE 3.2

	April 25, 1981		April 27, 1981	
	<u>T (hrs)</u>	<u>c (cm/s)</u>	<u>T (hrs)</u>	<u>c (cm/s)</u>
<u>Linear</u>				
ni ni mum	13.5	17	12.6	18
"measured"	10.1	22	10.5	21
maximum	8.6	26	8.0	28
<u>Solitary (30m)</u>				
"measured"	9.2	25	9.7	23

TABLE 3.2: Calculated travel times and average wave speeds for various approximations to observed cross-shelf transects of density profiles on April 25, 1981 and April 27, 1981.

based on the surface to bottom density difference.

Conclusion: Predictions for CODE I internal waves using the approximate theoretical analysis of Chapter 2 have been shown to be good. Thus, for the CODE I type density structure, $u_{b1}/a \sim N$, large wave periods are on the order of hours, and the nonlinear increment in wave speed is finite but small. The general ability to predict wave speed using the results of Chapter 2 seems to be quite good.

3.6 Discussion:

The principal results of this chapter are:

- (1) The average behavior of the internal wave field at a single central shelf mooring during CODE I is consistent with expectation from previous studies, especially with regard to the relationship between near bottom velocity and interior wave amplitude. The wave field is energetically dominated by the onshore propagation of mode 1 internal waves, with a low, broad peak, or "knee", in the $\eta(51)$ and $u(7)$ energy spectra slightly below the average value of N .
- (2) The short term behavior of the high frequency energy is dominated by the onshore propagation of large amplitude, intermittent mode 1 internal wave events. The waves often occur in irregular groups best described as a large, lower frequency wave of depression accompanied by smaller, higher frequency oscillations. Maximum wave heights are on the order of half of the water depth, maximum bottom velocities are on the order of 20 cm/s peak to peak, typical bottom velocities are 5-10 cm/s peak to peak, and typical wave periods are on the order of hours. There is no obvious short term regularity to the occurrence or the form of these

waves.

(3) Predictions for CODE I internal waves based on the results of the approximate theoretical analysis of Chapter 2 compare favorably to data from CODE I. These predictions include wave speed, wave period for large amplitude nonlinear waves, and the relationship between bottom velocity and interior amplitude, given a knowledge of wave height and the density structure.

The internal waves observed during CODE I are similar to the onshore propagating, **intermittent** packets of high frequency waves commonly observed travelling on the ~~summer~~ pycnocline near coastlines, which were discussed in more detail in Chapter 2. There are some notable differences. The waves during CODE I had periods on the order of hours, and were not in general very organized or regularly occurring. Bottom velocity fluctuations, while occasionally large, were not often the primary forcing for the boundary layer. Packets of high frequency internal waves on the continental shelf as exemplified by the **Massachusetts** Bay observations (Halpern 1971, Haury, et al. 1979, Trask and Briscoe 1983) and Mid-Atlantic Bight observations (**Butman**, et al. 1979, Fu and **Holt** 1982) have periods on the order of minutes, are more organized **into** regular packets of waves, and are more clearly nonlinear waves of depression with broad troughs and narrow peaks. **Butman**, et al. (1979) observed that near bottom speed peaks associated with such waves, superimposed on tidal velocities, were the strongest forcing for the bottom boundary layer during the summer on the Mid-Atlantic Bight. The differences between the internal waves during CODE I and these higher frequency wave packets are at least partly due to the differences in

density structure, as discussed in Chapter 2. They may also be due to the nature of the generation mechanism, which may determine the existence and intensity of these wave trains.

The most **commonly** proposed mechanism for the generation of solitary wave **packets** on the **shelf** or in shallow seas is tidal flow over a sill or the shelf edge (**e.g.**, review articles by Huthnance 1981 and Levine 1983). Although there was no observed tidal regularity to the midshelf wave field at CODE, the variable nature of the **shelf** wave guide prevents ruling out tidal generation. Furthermore, a **SEASAT** SAR image of the CODE site (Fu and Holt 1982) shows nearshore surface striations generally taken to be indicative of topographically **ly** generated internal waves (**e.g.**, **Apel** and Gonzales 1983, Trask and Briscoe 1983). The physics of topographic generation are controlled by the geometry of the topographic feature and the densi**ometric** Froude number, $U/(ND)$, where U is the magnitude of the barotropic current, N is the depth averaged Brunt-Vaisala frequency, and D is the water depth (**e.g.**, Haury et al., 1979; Maxworthy 1979 and Farmer and **Smith** 1980). If the Froude number is higher than some critical **value(s)**, internal lee waves or hydraulic jumps are possi**ble**, which may subsequently evolve into the observed shoreward propagating events.

If **the** internal wave generation mechanism is tidal and depends on the existence of the right combination of geometry and densiometric Froude number, then topographically generated waves may be **localized** or seasonal (Huthnance 1981). In particular, Fu and Hol**t** observed that the section of the west coast shelf between 35 and **40°N** lat., which includes the CODE site, is generally an area of low soliton activity. This is

probably the result of a lack of favorable conditions for generation, **if** **it** is not due simply to lack of observations. Clearly, wave generation **is** favored by the lower **shelf** break stratification established during the spring transition of the CODE site to an upwelling dominated region. Lower **shelf** break stratification is maintained throughout the spring/**summer** months (Huyer 1983), **i**ncreasing the densimetric Froude number at the shelf break. Huyer (1983) has pointed out that upwelling favorable winds in the CODE region, between Pt. **Reyes** and Pt. Arena, are the **strongest** of the entire California Current system. **Therefore**, the CODE region may be particularly favorable to topographic wave generation during the upwelling season, while other regions and other seasons may not be as favorable. Interestingly, Hayes and Halpern (1976) reported a **large**, unexplained increase in high frequency internal wave energy during a strong upwelling event off the Oregon Coast, which disappeared after the upwelling event.

The CODE **■** data is of value for the boundary layer interaction problem for several reasons. First, observations of near bottom velocity fluctuations induced by internal waves during CODE **■** do not exist in a vacuum, but are simultaneous with accurate boundary layer **measurements** (Grant, et al. 1983, Grant, et al. 1984) and measurements of longer period currents and density (**Allen**, et **al.** 1982). During the boundary layer experiments, data on surface wave bottom velocities was **also collected**. Subsequent analysis has enabled the calculation of physical bottom roughness and the demonstration of the importance of wave-current interaction in the boundary layer (Grant, et **al** 1983, Grant, et **al.**,1984). Therefore, the internal wave boundary layer

interaction problem can be considered in one sense as an addition to the already extensive **CODE** I boundary layer analysis. Second, simultaneous long time series of near bottom velocity fluctuations and interior vertical displacements in the internal wave band, and **information** about the background **density** structure, have enabled comparison of the predictions of Chapter 2 to an extensive data set. The favorable results of the comparison support the methodology and conclusions of Chapter 2. Third, the analysis has shown that large, onshore propagating high frequency internal wave events can exist in a region of low stratification like the **CODE** site and that they can induce strong, intermittent fluctuations in bottom velocity at frequencies intermediate between surface waves and tides.

The existence of large **amplitude**, onshore propagating internal waves has several **implications** for boundary layer processes. Large bottom velocity relative to other current components, even if sporadic, takes such waves **out** of the realm of low energy background noise. A model incorporating an additional intermediate frequency boundary layer forcing into the surface wave-current boundary layer interaction problem is developed in Chapter 4, and possible dynamical effects are examined. Possibilities for contamination of boundary layer velocity profile measurements by internal waves are treated in Chapter 5. Finally, the fact that wave propagation appears to be onshore without reflection raises the question of the (apparent) dissipation mechanism. Bottom friction is just one of the possibilities for a reduction in energy as the shore is approached, but its study is clearly appropriate to the present study. The problem of quantifying the effect of bottom friction

on internal wave dissipation on the continental shelf is addressed in Chapter 6.

CHAPTER 4

A THREE COMPONENT BOUNDARY LAYER MODEL

4.1 Introduction:

Consideration of the interaction of internal waves and the bottom boundary layer on the continental **shelf** has concentrated thus far on the internal wave part of the problem. Intermittent, large amplitude high frequency internal waves have been **implicated** as a possible forcing for the bottom boundary layer on the continental **shelf**. Though intermittent and possibly localized, the bottom velocity fluctuations induced by these waves may be of comparable **amplitude** to forcing at both lower ("steady" current) and higher (surface wave) frequencies. Thus, when internal waves are present they may be an important component of the general boundary layer problem on the **shelf**.

The bottom boundary layer on the continental shelf is a region of complex interaction between the various forcings **imposed** by flow conditions away from the boundary, the constraints due to finite water depth and **temperature/salinity** stratification, and the nature of a potentially bioturbated, moveable bed. Under sediment transporting conditions, interactions in the boundary layer become particularly complex. The problem of particular interest in this chapter is how the boundary layer responds to the imposition of an additional forcing at intermediate frequency between steady currents and high **frequency** surface waves. A well developed theory exists for the nonlinear **interaction** of a surface wave and a current in the bottom boundary layer, and recent measurements support its importance (Grant et al., 1984,

Wiberg and Smith 1984). However, no formal model has been developed to extend this theory to more than two components, though a model allowing partial interaction has predicted that a third, intermediate frequency may particularly affect conventional boundary layer measurements (Grant 1982).

The internal wave analysis of Chapters 2 and 3 indicates that high frequency internal waves on the continental shelf may be both nonlinear in form (narrow peaks, broad troughs) and at least partly transient, occurring in discrete packets of several waves. The complications introduced into the multiple frequency forcing problem by including nonlinear, transient forcing are beyond the scope of this work. Therefore, the model that is developed here must be considered a lowest order, qualitative look at the effects of including internal waves in the general boundary layer problem. The simplified approach used is nevertheless both valid and useful. In the next section, discussion of the anticipated effects of weakly nonlinear, transient forcing indicates that these effects are likely of either higher order importance or limited extent.

The boundary layer model developed here is an extension of the wave-current treatment of Grant and Madsen (1979) to include a third, intermediate frequency component. The flow is assumed to consist of two sinusoidal oscillatory components with different frequencies and arbitrary orientation in combination with a steady current. The steady current is subject to rotation, while the oscillatory components are assumed to be of high enough frequency to ignore rotation at first order. All current components are assumed uncorrelated, so that the

governing equations may be separated. The amplitudes and frequencies of the oscillatory components are taken to be constant and the velocities just **outside** the boundary layer are specified. All boundary layer interaction is assumed to be representable as enhancement of a scalar, time invariant, vertically varying eddy viscosity. The boundary layer is assumed to be fully rough turbulent and the physical bottom roughness is assumed to be fixed and known.

Both characteristics of individual solutions and the generalized dynamical effect of the third component as reflected in the stress felt by the steady current are examined. Predicted effects are an extension of existing wave-current solutions. Near boundary turbulent mixing is enhanced. A third, intermediate "log layer" is introduced into the steady current profile, with different apparent slope and intercept. The **dynamical** effect of adding a third component is predicted to be significant over a wide range of conditions, including conditions typical of event occurrences in the CODE data.

4.2 Formulation of the Boundary Layer Model:

The Reynolds averaged horizontal momentum equation for the x component of a near boundary, turbulent, nonrotating flow can be written

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{a}{\partial z} \frac{\tau_{zx}}{\rho} + O\left(\frac{a}{L} \cdot \frac{U}{T}\right) \quad (4.1)$$

(e.g., Trowbridge 1983) where u is the x component of the velocity, w is the vertical (z) component, p is the pressure, t is time, and $\tau_{zx} = -\rho u'w'$ is the Reynolds stress; a is the boundary layer length scale, L is the x length scale of the flow, U is the scale horizontal velocity, and T is

the time scale of the flow. The boundary layer approximation may be written $s/L \ll 1$, which leads to the further approximation that $\partial p / \partial z \sim 0$, so that $\partial p / \partial x$ is given by its value outside of the boundary layer. If the forcing for the boundary layer flow is a wave, the scales L and T are related through the dispersion relationship for the wave motion; $L/T = c$ (the phase velocity of the wave). Then the terms on the left hand side (LHS) of eq. 4.1 scale as:

$$\frac{\partial u}{\partial t} \sim O\left(\frac{U}{T}\right); u \frac{\partial u}{\partial x} \sim O\left(\frac{U^2}{L} = \frac{U}{c} \frac{U}{T}\right) \quad (4.2)$$

and

$$w \frac{\partial u}{\partial z} \sim O\left(w \frac{U}{\delta}\right)$$

Within the boundary layer, $w/U \sim O(\delta/L)$, so the last scaling in eq. 4.2 becomes

$$w \frac{\partial u}{\partial z} \sim O\left(\frac{w}{U} \frac{U^2}{\delta}\right) \sim O\left(\frac{U^2}{L} = \frac{U}{c} \frac{U}{T}\right) \quad (4.3)$$

Assuming a sinusoidal forcing for the boundary layer, $u(z \rightarrow \infty) = a \exp(i\omega t)$, is equivalent to assuming a linear wave solution in the present context, for which $U/c \ll 1$. The second and third terms on the LHS of eq. 4.1 (the advection terms) are smaller than the first term by the factor U/c , hence may be neglected for the boundary layer under a linear wave. Then eq. 4.1 reduces to

$$i\omega u(z) = i\omega u(\infty) + \frac{\partial}{\partial z} \frac{\tau_{zx}}{\rho} \quad (4.4)$$

where $u(\infty) = u(z \rightarrow \infty)$ and the pressure gradient has been set equal to

its value at $z \rightarrow \infty$, $-(1/\rho) \partial p / \partial x = au(\infty) / \partial t$. This is the standard form for the linearized oscillatory boundary layer equation.

The Reynolds stress term, τ_{zx} , may be related formally to the vertical gradient of the horizontal velocity through an eddy viscosity,

$$\frac{\tau_{zx}}{\rho} = K(z, t) \frac{\partial u}{\partial z} \quad (4.5)$$

where $K(z, t)$ is generally a function of both distance above the boundary and time. For the present formulation, the eddy viscosity is assumed time invariant. The use of such an eddy viscosity in high frequency oscillatory flows (i.e., surface waves) has been well documented. Kajiura (1964), Grant (1977) and Brevik (1981) have all demonstrated its success in predicting the magnitude of the velocity in a high frequency oscillatory boundary layer. The prediction of phase within the boundary layer relative to the free stream has been shown to depend most strongly on the form of the vertical variation of the eddy viscosity (Long 1981, Brevik 1981, Trowbridge 1983). Either linear variation near the boundary matched to a constant value above some scale height or linear variation near the boundary in combination with an exponential decay result in good estimates of the phase. Smith (1977), Grant (1977) and Grant and Madsen (1979) have presented physical arguments for time invariance of the eddy viscosity, and Trowbridge (1983) has demonstrated formally that time variation is unimportant at first order for sinusoidal forcing. Finally, Grant et al. (1984) have pointed out that uncertainty in the physical bottom roughness will generally overshadow any measurable predicted differences resulting from different chosen forms of the eddy viscosity, as long as the vertical

variation near the boundary is linear.

A possible problem with extending the use of a time invariant eddy viscosity to low frequencies is the necessity of assuming a time invariant scale height. In his formal consideration of time varying eddy viscosity, cited above, Trowbridge (1983) allowed the magnitude of the eddy viscosity to vary with time but kept the scale height of the eddy viscosity constant. The scale height may be expected to depend on the history of the flow in the outer part of the boundary layer or on the time scale of the largest eddies. It seems physically reasonable that this large eddy time scale may be of the same order as a surface wave period (seconds), so that the scale height should be roughly constant over the surface wave period. However, physical intuition balks at the thought of large eddies with time scales on the order of hours, which must be the case if a constant eddy viscosity scale height is to make sense for internal waves.

In fact, the period of the wave may be shown to be a reasonable time scale for the large, diffusive eddies in a wave driven boundary layer by reference to the turbulence spectrum and accompanying discussion presented by Hinze (1975, pp. 227-231). For the large eddies responsible for the diffusion of turbulent energy, the time scale may be written

$$\tau_d \propto \left(\frac{K(h)}{\epsilon} \right)^{1/2} \quad (4.6)$$

where $K(h)$ is the (maximum) value of the eddy viscosity at the scale height, h , and ϵ is the dissipation of turbulent energy, $\epsilon \propto u_*^3/h$, where u_* is the shear velocity, $u_* = [\tau_b/\rho]^{1/2}$. In standard

oscillatory rough turbulent boundary layer formulations, $K(h)$ a h and h a u_* / ω , where ω is the radian frequency of the oscillation; in general, u_* may be a function of time. Using eq. 4.6 and the expressions for $K(h)$, h , and ϵ ,

$$t_d \propto \left(\frac{u_*^2 / \omega}{u_*^2 \omega} \right)^{1/2} = \frac{1}{\omega} \quad (4.7)$$

so the time scale of the large diffusing eddies is proportional to the wave period and the assumption of a constant scale height should be a good one regardless of the wave period.

The discussion to this point has been limited to purely sinusoidal oscillations under linear waves. However, the analysis of Chapters 2 and 3 indicated that large amplitude internal waves on the continental shelf may be nonlinear and intermittent, often governed by some form of the $K-\epsilon$ equation. Allowing for nonlinear wave forcing would introduce several complications into the problem. Nonlinear waves are asymmetrical in time (narrow peaks, broad troughs), asymmetrical in direction (unidirectional in the extreme case of a solitary wave), may be transient (for a solitary wave or for the evolution of an initial disturbance), and may induce particle velocities which begin to approach the wave phase speed. Thus, the advective terms on the LHS of eq. 4.1 are larger, while the approximation of a time invariant eddy viscosity may not be as good as for a purely sinusoidal forcing.

Trowbridge (1983) has considered the solution for a rough turbulent boundary layer under a 2nd order Stokes wave with time varying eddy viscosity, $u_* = u_*(t)$ but $h \neq h(t)$. He found that, in fact, the

lowest order features of the boundary layer solution are not much affected by either the nonlinearity or the time varying eddy viscosity. These features are the boundary layer scale height and the characteristic value of the shear velocity. Second order quantities were affected - steady streaming, the details of the stress history at the boundary, and the details of the velocity profile. His solution was for steady state, continuous waves, so that the issue of transience was not addressed.

Solitary waves represent an extreme of transience, and solutions for the evolution of soliton wave packets (e.g., Lee and Beardsley 1974, Djordjevic and Redekopp 1978) also show at least initial transience. Observations on the continental shelf show intermittent packets of waves, with varying amplitude within the packet. In an eddy viscosity formulation, transience will introduce time variation into the scale height, h , which scale height depends on the history of the flow in the outer part of the boundary layer. For a transient wave, there is no history of the flow until at least $0.5 - 1$ wave "period" into the wave. Thus a steady state wave boundary layer solution is at best a crude approximation to the boundary layer under the leading wave of a wave packet propagating into a quiescent region, but should not be bad thereafter. Variable waves within a packet are also a form of transience, but Grant, et al. (1984) and Wiberg and Smith (1984) have found that predictions based on wave-current interaction are good when local surface wave characteristics within a slowly varying wave group are used. Since the present model is an extension of the same wave-current interaction concepts, the use of local internal wave

characteristics to predict the lowest order features of the boundary layer is reasonable as long as the differences between successive waves are not extreme.

Finally, existing models which include time variation in the eddy viscosity formulation (Trowbridge 1983, Lavelle and Mofjeld 1983) and forcing by a nonlinear wave (Trowbridge 1983) do so for a wave alone; none of the complications of time varying eddy viscosity and nonlinear forcing have been considered in the context of the wave-current interaction problem. The full solution of the transient, nonlinear wave-current boundary layer interaction problem would require fully time dependent, numerical treatment, which is simply beyond the scope of this work. For the present purpose, linear wave forcing is assumed and the eddy viscosity is taken as time invariant. The results are expected to reasonably predict basic features such as (1) the introduction of intermediate length and time scales into the surface wave-current boundary layer problem, (2) the generation of additional shear stress at the boundary and consequent enhancement of total mixing, and (3) the fundamental shape and direction of the boundary layer velocity profile as a function of time. Details of the velocity profile and details of the time history of shear stress at the boundary are not expected to be predicted well, and second order effects like steady streaming are not predicted at all. Predicted behavior is expected to be at best qualitatively correct for the leading edge of a transient wave packet.

Before proceeding with the mathematical formulation of the boundary layer model, three final assumptions should be discussed. First, the assumption of rough turbulent flow is quite reasonable. With a moveable

bed, the bottom roughness usually will be dominated by either ripples or large **bedload** transport in storms (Grant and **Madsen** 1982, Grant and Glenn 1983). Fine sediment beds are usually highly bioturbated (Cacchione et al., 1983). For the typical CODE value used in subsequent calculations, Grant et al. (1984) estimate that $k_b \sim 6$ cm, which for corresponding u_* values is fully rough turbulent. Second, the flow is taken to be neutrally stratified, which should be a reasonable approximation with regard to **temperature/salinity** stratification for the most important, near bed part of the boundary layer so long as there is a bottom mixed layer. Bottom mixed layers are common on the continental shelf (**Caldwell** 1978, Pingree and Griffiths 1977) and are quite large during CODE I (average ~ 20 m). Third, a fixed boundary roughness and no suspended sediment stratification are assumed for simplicity. Techniques exist to allow a **moveable** bed to readjust under an imposed forcing (e.g., Grant and **Madsen** 1982) and to account for suspended sediment stratification (e.g., Glenn 1983), but these are considered an unnecessary complication for the present model. Suspended sediment stratification has been found to be unlikely under typical **spring/summer** conditions at the CODE site, and **Butman**, et al. (1979) report a reasonably **stable** bed and little suspended sediment during the summer on the Mid-Atlantic Bight.

For uncorrelated forcing components, the governing equations may be separated, so that the form of eq 4.4 is appropriate for each oscillatory component. Writing each oscillatory component as $u_n = U_n(z)\exp(i\omega_n t)$, where ω_n is the appropriate radian frequency, and defining a velocity deficit as $u_{dn} = U_n(z) - U_n(\infty)$, eq 4.4 becomes

$$i \omega_n u_{dn}(z) = \frac{\partial}{\partial z} \left(K(z) \frac{\partial u_{dn}(z)}{\partial z} \right) \quad (4.8)$$

A similar analysis can be carried out for the steady component in a rotating frame of reference. The imaginary plane becomes the horizontal velocity component plane rather than the time phase plane. The phase angle for the steady component will represent turning through the boundary layer instead of time phase lag. The governing equation is identical, except that the Coriolis parameter, f , replaces ω_n .

For all components, the boundary conditions for fully rough turbulent flow are

$$\begin{aligned} u_{dn} &= -U_n(\infty) \text{ at } z = z_0 \\ u_{dn} &= 0 \text{ as } z \rightarrow \infty \end{aligned} \quad (4.9)$$

where $U_n(\infty)$ is the inviscid bottom velocity for the oscillatory components (u_b for the surface wave and u_{bi} for the internal wave) and the geostrophic velocity for the steady current (U_c).

Various forms for the vertical variation of the eddy viscosity have been used successfully in the past. The most important characteristic for the vertical variation is the linear increase with height at the boundary, corresponding to a constant stress layer. That is $K(z)$ a

$u_* z$ at the boundary where u_* is the shear velocity, $= [\tau_b / \rho_0]^{1/2}$. In the present instance the linear exponential form is chosen. It was used successfully by Businger and Arya (1974) for the steady planetary boundary layer and by Long (1981) for unsteady flows. This general form is

$$K(z) = \kappa u_* z e^{-z/h} \quad (4.10)$$

where κ is von Karman's constant ~ 0.4 and h is the decay scale of the mixing. This form has the advantages of linear behavior near the boundary, continuity, and physically realistic decay above the scale height, h . It has been used recently by Wiberg and Smith (1984) and (in a hybrid model) by Glenn (1983) and Grant and Glenn (1983) for a wave and a current. Its disadvantage is that it requires numerical solution; however, this turns out to be straightforward.

For the three component model, $K(z)$ is defined as

$$K(z) = \kappa z (u_{*cw} - u_{*ci}) e^{-z/h_{cw}} + (u_{*ci} - u_{*c}) e^{-z/h_{ci}} + u_{*c} e^{-z/h_c} \quad (4.11)$$

The three components are meant to be a steady Current, a surface Wave, and an Internal wave, thus the notation. This formulation can be reduced to any component alone or any combination of two components simply by dropping the unwanted subscripts. Note that $K(z_0) \sim$

$$\kappa u_{*cw} z_0.$$

The u_{*} 's are defined as follows:

$$u_{*cw}^2 = K(z_0) \left(\left| \frac{\partial u_w}{\partial z} + \frac{\partial u_i}{\partial z} + \frac{\partial u_c}{\partial z} \right|_{z_0} \right)_{\max} \quad (4.12)$$

$$u_{*ci}^2 = K(z_0) \left(\left| \frac{\partial u_i}{\partial z} + \frac{\partial u_c}{\partial z} \right|_{z_0} \right)_{\max} \quad (4.13)$$

$$u_{*c}^2 = K(z_0) \left| \frac{\partial u_c}{\partial z} \right|_{z_0} \quad (4.14)$$

where u_w is the surface wave velocity, u_i is the internal wave

velocity and u_c is the steady current velocity. That is, the scale value of η in each subregion of the boundary layer is defined by the maximum shear stress appropriate to that region, so u_{*cwi} is within the surface wave boundary layer and is proportional to the maximum combined shear including all components, etc.

The scale heights h_n play the same role as matching heights or sub-boundary layer heights in other formulations. They effectively patch the inner solution for each layer to its outer solution. The physical reasoning of Businger and Arya (1974) is extended to derive an expression for h . Near the boundary,

$$\frac{\partial u}{\partial t} > 0 \text{ and } \frac{\partial \tau}{\partial z} \sim \frac{\partial p}{\partial x} \quad (4.15)$$

integrating up,

$$\tau(z) - \tau_b \sim \frac{\partial p}{\partial x} \cdot z \quad (4.16)$$

If h is the height at which the stress approximately goes to zero, then

$$-h \frac{\partial p}{\partial x} \sim \tau_b \quad (4.17)$$

Now,

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{\partial u(\infty)}{\partial t} = i\omega u(\infty) e^{i\omega t} \quad (4.18)$$

and

$$\frac{\tau_b}{\rho} = u_*^2 e^{i(\omega t + \phi_0)} \quad (4.19)$$

where ϕ_0 is the phase lead of the boundary stress, so

$$h \cdot i\omega u(\infty) e^{i\omega t} \sim u_*^2 e^{i(\omega t + \phi_0)}$$

$$h \cdot i \omega u(\infty) e^{-i\phi_0} \sim u_*^2 \quad - 118 -$$

Taking the real part,

$$h = \frac{u_*^2}{u(\infty) \omega \sin \phi_0} \quad (4.20)$$

Writing

$$C = \frac{u(\infty) \sin \phi_0}{u_*} \quad (4.21)$$

the more familiar form of $h = u_*/C\omega$ is obtained. The value of C is not specified but is allowed to float as part of the solution.

For each component appropriate values of $u(\infty)$, ω , ϕ_0 , and u_* are used. Thus

$$h_{cwi} = \frac{u_{*cwi}^2}{u_b \omega_w \sin \phi_w} \quad (4.22)$$

where u_b is the inviscid amplitude of the surface wave bottom velocity, ω_w is the surface wave frequency, and ϕ_w is the bottom phase shift of the surface wave stress. Similarly,

$$h_{ci} = \frac{u_{*ci}^2}{u_{bi} \omega_i \sin \phi_i} \quad (4.23)$$

$$h_c = \frac{u_{*c}^2}{u_c f \sin \phi_c} \quad (4.24)$$

with corresponding variable definitions. The individual values of shear stress are defined by

$$\vec{\tau}_w = \rho_0 K(z_0) \left. \frac{\partial \vec{u}_w}{\partial z} \right|_{z_0} \quad (4.25)$$

$$\vec{\tau}_i = \rho_0 K(z_0) \left. \frac{\partial \vec{u}_i}{\partial z} \right|_{z_0} \quad (4.26)$$

$$\vec{\tau}_c = \rho_0 K(z_0) \left. \frac{\partial \vec{u}_c}{\partial z} \right|_{z_0} \quad (4.27)$$

The problem is thus completely formulated. There are three governing equations of the form 4.8, three sets of boundary conditions of the form 4.9, and an eddy viscosity defined by 4.11, 4.12-4.14, and 4.22-4.24.

The solution procedure is fairly standard: Guess initial values for the $u_\star$'s and ρ 's, calculate the form of $K(z)$ from the $u_\star$'s, ρ 's and input parameters, solve each of the three governing equations, calculate new $u_\star$'s using equations 4.12-4.14, new ρ 's directly from the solutions, and iterate until the desired accuracy is achieved. This takes from about 4 iterations for 1 percent accuracy to about 10 for 0.1 percent accuracy.

The numerical solution of each of the governing equations is done as in Glenn (1983), using an even grid of the logarithmically stretched variable $X = \ln(z/z_0)$, and solving a finite difference form of each governing equation with the double sweep algorithm of Abbott (1979), pg. 171. The grid spacing must be small enough to resolve the smallest boundary layer, and the domain must be large enough to extend beyond the largest boundary layer. Each solution of an equation requires about $4n$

operations, where n is the number of grid points. For example, a solution with 100 grid points takes about 400 operations per component equation, about 1200 operations per iteration, and about $O(10^4)$ operations to achieve 0.1 percent accuracy.

4.3 The Dynamical Effect of the Third Component - Method of Analysis:

Perhaps the most straightforward indicator of the dynamical behavior of a boundary layer is the magnitude of the shear stress at the bottom. It responds directly to a change in any input parameter, and the amount of response reflects the relative importance of the change. In available two component models, the greatest stress enhancement is that felt by the steady current in the presence of the wave. Therefore, the steady **shear** stress is used as an indicator of dynamical effect. The analysis proceeds from a **single** component model through a two component model to the full three component model.

A single component boundary layer depends upon six variables; formally $f(\tau_b, \rho, \mu, k_b, U(\infty), \omega) = 0$, where μ is the kinematic viscosity and k_b is Nikuradse equivalent bottom roughness ($z_0 = k_b/30$ for a fully rough turbulent flow). The assumption that $\rho =$ constant and that there is no dependence on μ is made, leaving a dependence on four variables which may be written $f'(u_*, k_b, U(\infty), \omega) = 0$. Dimensional analysis reduces this dependency to two non-dimensional variables, only one of which is independent, so

$$\frac{u_*}{U(\infty)} = g\left(\frac{U(\infty)}{z_0 \omega}\right) \quad (4.28)$$

For an Ekman layer, $U(\infty)/(z_0\omega) = U(\infty)/(z_0f)$ is the boundary Rossby number. For an oscillatory boundary layer, **it** is proportional to the inverse of the relative roughness, $k_b/A_b = k_b\omega/U_b = 30 (z_0\omega/U_b)$.

Proceeding in the same manner,

$$\frac{u_{*c}}{U_c} = h \left(\frac{U_c}{z_0 f}, \frac{u_b}{U_c}, \frac{\omega_w}{f}, \theta_{cw} \right) \quad (4.29)$$

for a wave-current boundary layer, where θ_{cw} is the angle between the wave and current directions. Note that an analogous expression could be written for the interaction of a surface wave and internal wave or an internal wave and a current. Finally,

$$\frac{u_{*c}}{U_c} = H \left(\frac{U_c}{z_0 f}, \frac{u_b}{U_c}, \frac{u_{bi}}{u_c}, \frac{\omega_w}{f}, \frac{\omega_i}{f}, \theta_{cw}, \theta_{ci} \right) \quad (4.30)$$

for the interaction of all three components, where θ_{ci} is the angle between the internal wave and the current. Clearly, even a non-dimensional formulation becomes unwieldy. Note that specifying any quantity at a height other than z approaching infinity introduces an additional dependence on the specification height, z .

The analysis procedure is as follows:

1. Establish the single component case as a base state, $(u_{*c}/U_c)_1$.
2. For the two component case, write

$$\left(\frac{u_{*c}}{U_c} \right)_2 = E \left(\frac{u_{*c}}{U_c} \right)_1 \quad (4.31)$$

so E is a multiplicative enhancement factor.

3. Fix θ_{cw} (which has the least effect) and either ω_w/f or u_b/u_c . Contour values of E in the space of the remaining two variables ($u_c/z_0 f$ and either u_b/u_c or ω_w/f).
4. For three components, write

$$\left(\frac{u_{*c}}{u_c}\right)_3 = F \left(\frac{u_{*c}}{u_c}\right)_2 = F \cdot E \left(\frac{u_{*c}}{u_c}\right)_1 \quad (4.32)$$

So F is the additional enhancement due to the third current component.

5. Fix θ_{ci} and pick either ω_i/f or u_{bi}/u_c (in the case presented subsequently ω_i/f is fixed). Use one of the E contours as a base state, and calculate values of F for discrete values of either u_{bi}/u_c or ω_i/f (the former in the present instance).

In short, the stress enhancement due to the addition of successive current components to a steady boundary layer is sought. In the examples that follow, $z_0 = 0.2$ cm, $f = 0.0515$ cph (the CODE values) and $\theta_{cw} = 90^\circ$, $\theta_{ci} = 110^\circ$. For two components, three examples taken in succession $\omega_w = 15$ sec, $\omega_i = 1$ hr, and $u_b/u_c = 1.0$. For three components, one example is presented with $\omega_w = 15$ sec and $\omega_i = 1$ hr; u_{bi}/u_c is varied to find values of F.

4.4 Results:

Summary - characteristics of individual solutions: For a single high frequency oscillatory boundary layer, the model agrees well with

the best available data, that of Jonsson and Carlsen (1976), test 1 (as reinterpreted by Grant 1977). This agreement for the linear exponential form of the eddy viscosity was first reported by Long (1981), whose formulation is the same as the present one for a single component except for the definition of the scale height, h . For a wave and a current, the present model predicts the same effects as existing models. The enhanced eddy viscosity in the wave boundary layer acts on the current so as to produce a nested log layer with a smaller slope, while the slope of the log profile above the wave boundary layer is higher and there is a larger apparent zero intercept than in the absence of the wave. With a continuous eddy viscosity there is no apparent "kink" in the velocity profile, as there is in discontinuous models. Under the present assumptions, the effects of adding a third, intermediate frequency component are a simple extension of the wave-current case. An intermediate subregion is introduced, additionally retarding the steady current and increasing the apparent roughness seen by the current in the outermost part of the boundary layer. Further nonlinear interaction is evidenced by higher individual shear stresses than otherwise, and larger values of h . Scale heights change more than in other models; in addition to an increase in the value of u_* , there is a decrease in the value of C in eq. 4.21.

Figure 4.1 shows the comparison between the model and the Jonsson test 1 data. The same physical parameters are used as in Grant (1977); it is possible to play with the model-data fit by varying roughness and displacement height, but this would only obscure the point. The dashed line on the left is the nondimensional eddy viscosity, the solid line is

Figure 4.1: Model solution for a single oscillatory component, for comparison to the data of Jonsson and Carlsen (1976) (JC76), test 1. The magnitude of the velocity profile is represented by the solid line for the model and the pluses (+) for the JC76 data, and is nondimensionalized by u_b . The phase of the velocity profile is represented by the dot-dash line for the model and the crosses (x) for the JC76 data; units are degrees. The eddy viscosity profile is the dashed line; it is nondimensionalized by the value of the eddy viscosity for the lowest frequency component alone, without enhancement, so that its boundary value in this case is 1.0. Input parameters are $u_b = 213.4$ cm/sec, $z_0 = 0.031$ cm, $T_w = 8.39$ sec, and a displacement thickness of 0.1 cm as determined by Grant (1977).

the nondimensional velocity profile, and the dot-dash line is the phase. Agreement is generally good; the phase agreement is better than with a simple linear eddy viscosity (Grant 1977). Long (1981) presented essentially the same comparison except that he fixed the value of C at 6.0. The comparison is included here because the present parameterization for the scale height, h , was originally developed for an Ekman layer. This demonstrates that it works equally well for a high frequency oscillatory boundary layer.

In individual profile plots, the eddy viscosity is nondimensionalized by the value that it would have at the boundary if the lowest frequency were present alone, emphasizing the near boundary enhancement due to higher frequency components. The velocity profiles are nondimensionalized by their respective velocities outside the boundary layer.

Figures 4.2a-4.2f show examples of each component separately, the two oscillatory components interacting separately with the steady current, and all three components interacting together. The components are a surface wave of 15 sec period and 10 cm/sec amplitude, an internal wave of 1 hr period and 10 cm/sec amplitude, and a 10 cm/sec current. For these examples all three components are taken as colinear and $z_0 = 0.2$ cm. Several things are apparent from the surface wave (a), internal wave (b), and current (c) boundary layers alone. The eddy viscosity is larger near the boundary for higher frequencies, but is more confined to the wall region. The logarithmically linear part of the velocity profile extends beyond the linear portion of the eddy viscosity profile. The scale height, h (the height of the maximum eddy

Figure 4.2: Model runs (a) for a surface wave alone, with $T_w = 15$ sec and $u_b = 10$ cm/sec; (b) for an internal wave alone, with $T_i = 1$ hr and $u_{bi} = 10$ cm/sec; (c) for a steady current alone, with $u_c = 10$ cm/sec; (d) for the surface wave and current colinear; (e) for the internal wave and current colinear; and (f) for surface wave, internal wave, and current colinear. In every case, $z_0 = 0.2$ cm. All axes, line definitions, and nondimensionalizations are as in Figure 4.1, with the highest boundary layer profile corresponding to the lowest frequency component in each plot.

viscosity), is clearly related to the boundary layer height. Examination of the surface wave-current (d), internal wave-current (e) and both waves and current (f) boundary layer profiles reveals the following. The surface wave enhances the total eddy viscosity more than the internal wave does near the boundary, but the greater extent of the internal wave enhancement affects the current shear velocity more (plots d and e). All shear velocities are increased by interaction, but the internal wave and current u_* 's are increased by successively larger percentages than the surface wave u_* . The different logarithmic regions of the current velocity profile (the uppermost profile in each plot) are clearly distinguishable. Finally, boundary layer interaction acts to decrease the phase lag for the oscillatory components and increase the turning angle for the steady current, though this effect is minor.

Summary - dynamical effects: The present model predicts dynamical effects that are very similar to those predicted by existing models for one component and for wave-current interaction. The drag coefficient for one current component is about 90 percent of that predicted with a linearly increasing eddy viscosity, which is as might be expected given the nature of the respective eddy viscosity formulations. For the same physical bottom roughness, the present model for two components predicts a drag coefficient that is significantly enhanced but only about 80 percent of that predicted by the Grant and Madsen formulation; again, this might be expected. Under the present assumptions, low frequencies enhance the steady stress more than high frequencies for the two component case. The introduction of an internal wave to the surface

wave-current problem leads to additional enhancement of the same order as that due to the surface wave alone (C_d additionally increases between about 20 percent and about 190 percent for typical CODE internal wave events).

Figure 4.3 shows the dynamical behavior of any one of the single component solutions of the present model. The independent variable may be considered a boundary Rossby number for Ekman layers, with f substituted for ω , or an inverse relative roughness for oscillating boundary layers (A_b/z_0). The plot of u_* / U_{inf} shows behavior that is expected; values from the present model are about 95 percent as large as those from a linearly varying eddy viscosity model, plotted for comparison. Values of the phase difference across the boundary layer, ϕ , decrease uniformly from about 30° at $Rb = 10^3$ to 12° at $Rb = 10^7$. Referring to eq 4.21, this decrease in ϕ might be taken to imply that the constant C grows without bound as Rb increases. However, the decrease in ϕ is matched by the decrease in u_* / U_{inf} , so that C approaches 6 as Rb becomes very large; this was reported in Long (1981). C decreases for Rb less than 10^5 ; in the neighborhood of inverse relative roughnesses characteristic of surface waves ($O(100)$), C is about 3.5. Finally, the horizontal line at $u_* / U_{inf} = 0.045$ is equivalent to a constant geostrophic drag coefficient of .002.

Figures 4.4a-4.4c illustrate the dynamical effect of single wave-current interaction on the steady current as reflected in the value of the enhancement factor E , (4.31). Contour plots of E are presented over a wide range of values of current boundary Rossby number vs. the ratio of u_b / U_c for a 15 sec surface wave in (a), for a 1 hr internal

Single Component Solution

Figure 4.3: Dynamical behavior of single component oscillatory or rotating boundary layer solutions. Horizontal axis is an inverse relative roughness for oscillatory solutions, a boundary Reynolds number for rotating solutions. Solid curve is $u^*/U(\infty)$, as in eq 4.28; u^* is from solution of linear eddy viscosity model; dot-dash curve is phase lead (rotation angle for rotating solution) relative to the free stream; dashed line is the value of C in eq 4.21. Straight horizontal line shows location of constant drag coefficient = 0.002.

Figure 4.4: Enhancement factor, E (4.31), for two component boundary layer solutions. Contour plots of enhancement (a) for a 15 sec surface wave and a steady current at right angles, (b) for a 1 hr interval wave and a current at right angles, and (c) for a wave of arbitrary frequency and amplitude equal to the current speed at right angles. Horizontal axes are a boundary Rossby number, vertical axes for (a) and (b) are wave amplitude relative to current speed, vertical axis for (c) is wave period relative to inertial period. Contour interval is 0.1 for (a) and (b), and 0.05 for (c). Contours above and to the right of labelled contour in (c) are both 1.25.

wave in (b), and vs. wave period for a constant speed ratio of 1 in (c). Two general results are apparent. Enhancement clearly increases for increasing relative wave speed and for increasing wave period. The increase with relative wave speed is greater for longer period waves. In other words, under the present assumptions an internal wave alone would affect a steady current more than would a surface wave alone, especially for strong waves relative to the current. Comparison of the present model to the Grant-Madsen model runs listed in Grant et al. (1984) shows that present predicted enhancement is about 90 percent of the Grant and Madsen predicted values. It should be pointed out that, for typical CODE values, U_{inf} is about 1.5 times $u(1\text{ m})$ and that enhancement of the drag coefficient, C_D , is equal to the present enhancement squared.

At this point, an example of the use of Figure 4.4a seems appropriate. Taking typical CODE values of $z_0 = 0.2\text{ cm}$, $u_c = 10\text{ cm/sec}$ and $f = 0.0515\text{ cph}$, $R_0 = 5.6 \times 10^5$ is calculated. Referring to Figure 4.3, a base value of $u_*/U_{inf} = 0.047$ is obtained. Then, taking $u_b = 5\text{ cm/sec}$ for a 15 sec surface wave, calculating $u_b/U_c = 0.5$ and referring to Figure 4.4a, E is found to be $= 1.11$. Therefore $u_{*c}/U_c = 1.11 \times 0.047 = 0.052$, and $u_{*c} = 0.52\text{ cm/sec}$.

Finally, Figure 4.5 shows the effect of adding a 1 hr internal wave to the 15 sec surface wave-current problem. The base contour plot is the same as Figure 4.4a. Plotted above it are surfaces of the value of the second enhancement factor F , (4.32), for constant ratios of the internal wave speed to the value of the current speed, u_{bi}/U_c . To continue the above example, suppose a 1 hr internal wave with $u_{bi} =$

Figure 4.5: Additional enhancement factor, F (4.32), for three component boundary layer solutions. Surfaces of additional enhancement resulting from the addition of a 1 hr internal wave to the surface wave-current problem of Figure 4.4a; the base contour plot is the same as Figure 4.4a. Each surface represents values of F for a constant ratio of internal wave bottom velocity to current speed, u_{bi}/u_c .

3 cm/sec is added. Then the value of F on or just below the first surface and directly above the point of interest in the base contour plane is about 1.1, so $u_{*c} = 1.1 \times 0.52 = 0.57$ cm/sec. The (weaker) internal wave has caused as much additional enhancement of the steady shear velocity as the enhancement due to the surface wave alone.

4.5 Discussion:

There are several minor differences between existing time invariant eddy viscosity formulations and the present formulation. The most obvious is the addition of a third frequency component. The formulation of the eddy viscosity makes this straightforward; either the addition of more frequencies and/or the relaxation of the neutral stratification assumption would be equally straightforward. The use of oscillatory boundary layer scale heights that have no arbitrary constant of proportionality (fixed value of C , eq 4.21) is also new. The importance of the scale or matching height has been documented in Trowbridge (1983) or Brevik (1981), for example, but available data are insufficient to test predicted differences between boundary layer wave-current interaction models with different eddy viscosity formulations (Grant et al. 1984; Wiberg and Smith 1984).

It must be emphasized that the present model is a simple extension of existing concepts of wave-current interaction. As such, the general agreement with other models for one and two components is to be expected; slight differences result from different assumed forms for the eddy viscosity profile and different definitions of the scale height (or matching height or sub-boundary layer height in other models).

Therefore, the predicted significant possible dynamical effect of adding a third, lower frequency oscillatory component is reasonable, given the assumptions, and the dependence on the relative strength of the components is to be expected.

Any weaknesses of the present model and predictions based upon it will result from the violation of basic assumptions. Possible problems may be broadly classified into two interrelated areas, both of which were discussed in section 4.2. First, the analysis of Chapters 2 and 3 indicated that large amplitude internal waves on the continental shelf may be nonlinear in form and often propagate in discrete groups, hence are at least partly transient. The model results, based on the assumption of sinusoidal, steady state oscillations, are thus limited to predicting the lowest order properties of the internal wave boundary layer and to elucidating the mechanism and consequence of interaction between internal waves, surface waves, and currents in the boundary layer.

For the present purpose, this level of sophistication is sufficient. The quantities that are expected to be predicted reasonably well are the basic boundary layer length and time scales, the fundamental shape and direction of the total boundary layer velocity profile as a function of time, and the likelihood of enhanced turbulent mixing/momentum transfer/boundary shear stress due to boundary layer interaction between forcing components. Details of the velocity profile and stress history may not be predicted well for nonlinear forcing (e. g. , Trowbridge 1983) and second order quantities like wave induced mass transport will not be predicted at all. The model must be applied

with caution to transient situations like the first wave in a discrete wave packet. The full nonlinear, transient problem will probably require fully time dependent, numerical solution.

The second, related set of **assumptions** that may be subject to question involve the parameterization of the eddy viscosity. In the present formulation the eddy viscosity is assumed to be time invariant and it is based upon the maximum scale shear velocities for each sub-boundary layer. These assumptions have been employed successfully for high frequency oscillatory boundary layers (**e.g.**, Grant 1977, Smith 1977, Long 1981) and for surface wave-current interaction models (**e.g.**, Grant 1977, Smith 1977, Grant and **Madsen** 1979, Wiberg and Smith 1984). Time invariance and maximum shear velocity scaling are used in the present formulation for relatively low frequency internal waves because they are simple assumptions and because there is no apparent reason not to use them at the present time. High quality data for low frequency oscillatory boundary layer **flows** and/or comparison to a full numerical solution would provide valuable insight into the questions of time invariance and choice of scale shear velocity.

The present model predicts significant enhancement of the steady shear stress in the presence of internal waves of magnitude comparable to or greater than the magnitudes of the other current components, assuming steady state waves. However, observations on the continental **shelf** indicate that large **amplitude** bottom velocity fluctuations due to internal waves are in fact quite intermittent, occurring only a fraction of the **time** at a given site. The true dynamical influence of internal waves will then be site specific, depending on the amount of time they

are present, the amount of bottom affected, and the relative strengths of the other forcing components for the boundary layer. During the summer on the East Coast continental shelf, surface waves are weak and internal wave bottom velocities are one of the primary forcings for the bottom boundary layer (Butman, et al. 1979), so the dynamical effect of the internal waves may be important. During the CODE II experiment, internal waves were often weaker than either surface wave induced bottom velocities or "steady" currents. The strong events shown in Figure 3.7 represent an extremum of the 10 week period analysed. Therefore, it is unlikely that internal waves were a major dynamical influence during CODE II.

CHAPTER 5

THE INFLUENCE OF INTERNAL WAVES ON BOUNDARY LAYER

VELOCITY PROFILE MEASUREMENTS

5.1 Introduction:

A common method of estimating bottom boundary shear stress and roughness is to assume a constant stress layer, then to determine stress and roughness from the best fit of a semi-logarithmic line to an appropriately averaged, measured boundary layer velocity profile. Shear velocity, u_* , is given by the slope of the line and roughness, z_0 , by the z intercept. Internal waves have been implicated as a source of noise in geophysical boundary layers; that is, unsteadiness in the internal wave frequency band is thought to contribute to deviation from the so called "log" profile and to variability in measured estimates of stress and roughness. Grant (1982) briefly discusses the problem without allowing full interaction of all current components, and Grant et al. (1984) attribute observed degradation of log profiles to the simultaneous presence of internal waves.

The purpose of this chapter is to consider the problem formally, expanding the treatment of Grant (1982) to more fully investigate the theoretical nature of possible errors in estimates of boundary stress and roughness using the velocity profile techniques, due to the presence of an unaccounted for steady state internal wave boundary layer. The three component boundary layer model of Chapter 4 is used in a numerical experiment that mimics the kinematics of a boundary layer velocity

profile experiment. Because the boundary layer model of Chapter 4 assumes that all wave components are sinusoidal, steady state oscillations, because the eddy viscosity is assumed time invariant, and because the model is untested to date, the results of the kinematical numerical experiment must be taken as **qualitative**. The discussion of Section 4.2 indicated that ignoring nonlinearity and time variation of the eddy viscosity may lead to errors in higher order aspects of the boundary layer solution (**e.g.**, Trowbridge 1983). The effects of transience are unknown, but assumed to be of limited extent. Therefore, the present numerical experiment should reproduce the **lowest** order aspects of observed contamination of boundary layer velocity profile measurements by internal waves, but may not predict aspects of the observations that depend on higher order details of the velocity profile and stress history.

Inclusion of physical dimensions and current meter configurations makes general presentation of the results extremely **difficult**. Thus, the numerical analysis concentrates on describing the detailed kinematics of a particular example referred to as the base state, and the sensitivity of the problem to variation of input parameters around their base state values. Subsequently, a short segment of the CODE I high frequency bottom boundary layer experiment data set is presented. The data presented span a period including a short group of internal waves (noted in Section 3.4 and Figure 3.8), and qualitatively illustrate some of the predicted effects. In their analysis of boundary layer data from the CODE I experiment, Grant et al. (1984) exclude the

time period presented here as subject to too much error, possibly due to the contaminating effects of the internal waves.

A key element of the present analysis is comparison of truly unsteady effects to effects predicted by considering the flow to be quasi-steady. Unsteady refers to explicit inclusion of an oscillatory internal wave boundary layer solution. Quasi-steady refers to using the same time series of velocity at a given height, but assuming the flow to be a slowly varying steady current. In both cases, interaction with an assumed constant surface wave field is explicitly included.

There are two distinct ways in which internal waves may affect measured bottom boundary layer velocity profiles, depending on the heights of the current meters and the chosen averaging time. The first is an actual change in the steady current profile slope, which occurs within the internal wave sub-boundary layer. This effect was discussed in Chapter 4, and is equivalent to the smaller slope of the steady current profile inside the wave boundary layer predicted by two component wave current boundary layer models (Grant 1977, Grant and Madsen 1979, Smith 1977). Roughly (since the present model does not have stacked constant stress layers, but predicts a smooth variation in stress), within the internal wave boundary layer,

$$\frac{\partial u_c}{\partial (\ln z)} = \frac{u_{*c}}{\kappa} \left(\frac{u_{*c}}{u_{*ci}} \right) \quad (5.1)$$

so that the stress estimate obtained directly from the slope of the profile and ignoring the presence of the internal wave sub-boundary layer will be too small by $(1 - u_{*c}/u_{*ci}) \times 100$ percent, under the assumptions of the present model (u_{*ci} is always greater than or equal

to u_{*c}). According to the results of the last chapter, the under estimate thus obtained may be substantial; it is directly related to the additional enhancement of the steady shear stress caused by the internal wave. This additional enhancement by internal waves was predicted to be as large as the enhancement due to surface waves alone for internal waves of about the same amplitude as surface waves or slightly smaller. It must be emphasized that this effect formally depends on the assumption of steady state internal waves and on the existence of an internal wave sub-boundary layer where wave-current interaction occurs. The hypothesized dynamical influence of internal waves is reasonable but not verifiable at present.

The second predicted set of effects of internal waves on velocity profiles is strictly kinematical, and results from using an averaging time which is only a fraction of the internal wave period. A second time varying velocity profile is added on to the true steady current profile, increasing apparent variability and possibly introducing curvature into the profile (Grant 1982). Curvature is likely most pronounced if the top of the internal wave log layer falls within the range of the current meters. Specific details may depend on the model used, but the existence of some kinematical influence depends only on the existence of a separate internal wave boundary layer with different scale height than the steady current. These kinematical effects should be more apparent in the data record than the change in slope resulting from the dynamical influence of the waves.

Summary of results: The results of the numerical simulation of a velocity profile experiment indicate that unsteadiness due to internal

waves causes much more variability of roughness and drag coefficient estimated from velocity profile "measurements" than if the flow is considered to be quasi-steady. The fixed phase, nearly inverse relationship of "measured" drag coefficient and averaged speed at 1 m and the greater variability of the "measured" drag coefficient for the unsteady case result in a smaller range of stress estimates than for the quasi-steady case. Predicted degradation of the log profile fit seems to be directly related to the amount of predicted variability. Internal waves may cause underestimation of the stress actually felt by the steady current. All effects are predicted to be greatest when internal wave bottom velocities are comparable to or greater than the current and colinear with it. A segment of the CODE I high frequency boundary layer experiment data containing internal waves shows qualitative agreement with predictions of trends in drag coefficient and roughness variability which cannot be explained by surface wave-current interaction alone.

Observed degradation of log profile fits is greater than that indicated by the results of the numerical experiment, which is not surprising in light of the simplifications and approximations of the boundary layer model.

5.2 Methods and Criteria for the Numerical Experiment:

In Chapter 4, the full three component boundary layer model was shown to be a function of eight nondimensional variables. For the present analysis, dimensional dependence (2 variables), dependence on current meter heights, and dependence on chosen averaging time must be added, so that the general problem becomes much too unwieldy.

A numerical simulation of an example experiment is carried out here, and the nature of internal wave induced contamination of boundary layer velocity profile "measurements" is described based on that example. The sensitivity of the system to variation of the input parameters is then explored. The chosen example will be referred to as the base state. Its geometry, presented in Figure 5.1a, is patterned after the intensive boundary layer data analysis reported in Grant et al. (1984) (see Figure 3.8). For the base state example, wave and current characteristics are only approximate, and the internal wave amplitude has been magnified for the sake of clarity.

The base state: The surface wave is assumed to be constant at 15 sec period and $u_b = 5 \text{ cm/sec}$, directed slightly to the north of onshore. The steady current speed at infinity is taken to be 12 cm/sec, and the direction is 90° to the north of the surface wave direction. The internal wave period is 1 hr, $u_{bi} = 5 \text{ cm/sec}$, and the wave direction is 45° to the south of the surface wave, or slightly to the south of onshore. The physical bottom roughness is taken to be $z_0 = 0.2 \text{ cm}$, in accord with the analysis of Grant et al. 1984, where it was demonstrated to give reasonable agreement between the data and the Grant-Madsen surface wave-current model for cases in which surface waves and current alone were present. Current meter heights are approximately taken from the CODE BASS bottom tripod geometry as 25 cm, 50 cm, 100 cm, and 200 cm above the (nominal) bottom. Finally, averaging times for calculating profiles are taken as either 4 or 5 min for the base state, in order to show as much of the structure of the variability as possible

Figure 5 - The "base state" - (a) assumed geometry, with vectors representing the magnitude and direction (propagation direction for surface wave and internal wave) for each of the current components; (b) phase (dot-dashed line) and magnitude (solid line) of the boundary layer velocity profile solutions, for the steady current (heavy lines) and the internal wave (light lines). The nominal height range of the BASS current meters is also shown in (b), for the present value of z_0 only.

(the averaging time in the **CODE** data presented later is 4.67 min). In the theoretical analysis, physical bottom **roughness** and surface wave parameters are held constant. In the real world, variation of these factors can lead to variation in measured stress and roughness (Grant et al 1984), but such is not the point of this analysis.

Base state solutions of the three component boundary layer model of Chapter 4 are presented in Figure 5.1b, with the range of the BASS current meter heights shown. Solutions are presented only for the steady current and internal wave, but the effect of the surface wave is apparent near the boundary. Note that all of the current meters are within the internal wave sub-boundary layer, and in fact that the internal wave boundary layer profile is nearly logarithmic through the meters. Several of the base state numerical experiment results may be anticipated from this figure:

1. The average shear stress will be underestimated, given the location of the current meters within the predicted internal wave sub-boundary layer.
2. Variability in stress and roughness estimates will result from vector addition of pieces of the time varying, nearly logarithmic internal wave boundary layer profile to the steady current profile. Variability will also result from the slight back and forth tilting experienced by the internal wave boundary layer due to the phase change across the current meters.
3. There will be little degradation of the log profile fit, since there is very little curvature in either profile; adding two

log profiles gives another log profile, except for the phase change.

Calculations are carried out as follows. The boundary layer model is run and the resultant steady current and internal wave profiles are stored. The internal wave period is divided up into subsequent subperiods, each T_{ave} long, and the average speed over the subperiod for the internal wave component at each current meter height is calculated by

$$u_{i,ave}(z) = \frac{1}{T_{ave}} \int_t^{t+T_{ave}} u_i(z) \cos [\omega_i \alpha + \phi_i(z) + \gamma] d\alpha \quad (5.2)$$

where $u_{i,ave}(z)$ is the averaged internal wave speed, $u_i(z)$ is the magnitude of the internal wave boundary layer profile, $\phi_i(z)$ is the phase lead of the profile relative to the free stream, and γ is a specifiable constant initial phase shift. At each height, the "measured" steady current speed, $u_{cm}(z)$, is calculated as the magnitude of the vector sum of the internal wave velocity (calculated average speed and given direction) and the z -dependent steady current velocity. A semi-logarithmic line is fit in a least squares sense to the speed profile; the "measured" shear velocity, u_{*m} , is determined by

$$u_{*m} = \kappa \frac{u_{cm}}{\ln z} \quad (5.3)$$

and the "measured" roughness, z_{om} , by the z intercept of the line.

The "measured" drag coefficient, C_{dm} , relative to 1 m, is defined by

$$C_{dm \ 100} = \left(\frac{u_{*m}}{u_{m \ 100}} \right)^2 = \left[\frac{\kappa}{\ln \left(\frac{100 \text{ cm}}{z} \right)} \right]^2 \quad (5.4)$$

Statistical uncertainty in the value of u_{*m} is a function of the regression coefficient and the number of instruments. The percent error in u_{*m} may be written

$$e = 100 \, t_{(\alpha/2, n-2)} \left[\frac{1}{n-2} \left(\frac{1-R^2}{R^2} \right) \right]^{1/2} \quad (5.5)$$

where e is the ($\pm$) percent error in u_{*m} , $t_{(\alpha/2, n-2)}$ is the Student t probability distribution at the $(1-\alpha) \times 100$ percent confidence level, and n is the number of current meters (e.g., Grant et al., 1984, Gross and Nowell 1984). Neglecting the uncertainty in u_{m100} , percent error in C_{dm} is about $2e$. For example, with four current meters at the 95 percent confidence level $t = 4.303$, so if $R^2 = 0.993$, $e = 25$ percent. Calculations are carried out for each successive subperiod and the results stored as time series of "measured" shear velocity, roughness, drag coefficient, velocity at the current meter heights, and regression coefficient squared. The final result is a simulation of a boundary layer velocity profile experiment in the presence of an internal wave.

Unsteadiness: The influence of internal wave unsteadiness is examined by comparing results of the above unsteady analysis to results obtained by assuming that each successive velocity value at 1 m represents a slowly varying steady current, which interacts only with the assumed surface wave. The chosen indicator of variability is the "measured" drag coefficient, C_{dm} . The variation in z_{om} is larger, but less directly meaningful, and in any case is linked to the variation in C_{dm} by eq 5.4. Results of single wave-current interaction models

(Grant 1977, Grant and Madsen 1979, Smith 1977) indicate that the magnitude of the steady drag coefficient is primarily controlled by surface wave characteristics if surface waves are the dominant near boundary current component within the boundary layer, so that the drag coefficient should not vary much if the surface wave is taken to be constant and the current is treated as quasi-steady.

For the sensitivity analysis, all input parameters except the one being varied are held constant at their base state values. The indicator of variability is the range of C_{dm} over the internal wave cycle. The indicator of possible underestimation of steady shear stress is taken as the percent difference between the average of "measured"

values and the "actual" steady u_* value predicted by the three component boundary layer model, u_{*c} . The amount of degradation of the log profile fit is represented by the minimum value of the regression coefficient squared for each boundary layer experiment. Assuming a slowly varying steady current will obviously give regression coefficients of one.

5.3 Results of the Numerical Experiment:

Results of running simulated boundary layer experiments for the base state with and without including effects of unsteadiness are presented in Figure 5.2. The averaging period for these runs was 4 min. Figure 5.2b represents the velocity vectors at 1 m predicted by the three component boundary layer model and the full unsteady simulation. These are taken as input to the succession of runs considering only a slowly varying steady current and the chosen surface

Figure 5.2: Results of running simulated boundary layer experiments for the base state, comparing unsteady effects to effects derived assuming that the velocity at 100 m is a slowly varying steady current, or "quasi-steady". (a) successive best fit log profiles; (b) corresponding velocity vectors at 100 m; (c) speed at 100 m; (d) "measured" values of drag coefficient, C_D , with error bars for the unsteady experiment values derived from eq 5.5; and (e) values of the regression coefficient (squared). Each successive value represents a four min average.

wave. The speed at 1 m is plotted in 5.2c for clarity. Figure 5.2a shows the successive velocity profiles resulting from both experiments. Inclusion of unsteady effects results in much greater variability in apparent roughness, z_{om} , but less variability in slope, as the independent internal wave log profile is vectorially subtracted from or added to the steady current profile.

The variation in z_{om} anticipates the variation in C_{dm} , plotted in 5.2d with 95 percent confidence limits from eq. 5.5. For the quasi-steady experiment, C_{dm} varies inversely with speed, but the variability is small so that stress will nearly scale as speed squared. For the unsteady experiment the corresponding drag coefficient variability is much greater, and it leads an inverse phase relationship with speed (the minimum in C_{dm} occurs slightly before the maximum in speed). This is due to the phase advance of velocity at the lowest current meter relative to the phase at the highest current meter. Although the quasi-steady drag coefficient is about equal to the average of the unsteady drag coefficient, the quantity of dynamical interest is the "measured" stress whose magnitude is the product of the drag coefficient and the speed (5.2c) squared. The large, nearly inverse variation of unsteady C_{dm} with speed causes the total range of stress variability for the unsteady case to be small relative to the range for the quasi-steady case; for the unsteady experiment, the range of predicted stress is 0.14-0.67 dynes/cm², .17 percent smaller than the range for the quasi-steady runs, which is 0.11-0.74 dynes/cm². The value of C_{dm} corresponding to either the average value of u_{*m} for the quasi-steady experiment or the value of u_{*m} for the unsteady

experiment with the internal wave averaged out is indicated as "average of "measured" C_d " in 5.2d. The value of C_d predicted by the full three component model is denoted "actual C_d ", and is about 1.5 times as large as the "measured" value.

The variable range of 95 percent confidence limits for unsteady C_{dm} is due to variation in the "measured" regression coefficient squared, R^2 , which is plotted in Figure 5.2e for both unsteady and quasi-steady experiments. Clearly, the quasi-steady results give $R^2 = 1.0$, or no error. The variability in R^2 for the unsteady experiment is slight by usual field experiment standards; the minimum value of 0.994 is just at the level of 25 percent accuracy in estimates of u_* quoted as acceptable by Grant et al., 1984. However, even this slight variability leads to fairly substantial possible error in estimates of shear stress, which is particularly notable because of the idealized nature of the present theoretical example. Although the error bars for unsteady C_{dm} reduce the statistical difference between unsteady and quasi-steady drag coefficients, the systematic nature of unsteady C_{dm} variability argues against a purely random source. In other words, the difference between unsteady C_{dm} and quasi-steady C_{dm} is real and the pattern of variation is uniformly repeatable.

Sensitivity of the unsteady analysis to variation of the input parameters around their base state values is summarized in Table 5.1. Parameters varied are the strength of the internal wave, the internal wave period, the angle between the internal wave and the current (by varying the current direction), the strength of the current, and the averaging period and initial phase angle, γ , in eq 5.2. Listed for each

5.1a Vary Strength of Internal Wave

u_{bl} (cm/sec)	% underestimate u_{*c}	Range of "measured" C_D ($\times 10^{-3}$)
1	5	5.1 - 6.3
2	9	4.6 - 7.0
3	12	4.3 - 7.9
4	15	4.0 - 8.9
*5	17	3.7 - 9.9
6	19	3.5 - 10.5
7	21	3.3 - 11.1
8	22	3.1 - 12.2
9	23	3.0 - 13.2
10	24	2.7 - 14.0

5.1b Vary Internal Wave Period

T_{IW} (hr.)	% underestimate u_{*c}	Range of "measured" C_D ($\times 10^{-3}$)
*1	17	3.7 - 9.9
2	17	4.2 - 7.8
3	16	4.4 - 7.1
4	16	4.6 - 6.7
5	16	4.7 - 6.5

5.1c Vary Steady Current Direction

θ_{cl} (angle) between I.W. and current)	% underestimate u_{*c}	Range of "measured" C_D ($\times 10^{-3}$)
180°	19	3.9 - 11.5
*135°	17	3.7 - 9.9
90°	8	4.2 - 6.6
45°	11	4.6 - 7.4
0°	19	3.9 - 11.5

5.1d Vary Strength of Current

u_c (cm/sec)	% underestimate u_{*c}	Excluding Profile Reversals		
		Range of "measured" C_D ($\times 10^{-3}$)	R^2 min	Number of Reversals
2	19	1.1 - 10.7	.683	3
4	27	1.1 - 12.9	.684	1
6	25	2.3 - 21.7	.978	0
8	22	3.1 - 14.3	.983	0
10	19	3.5 - 11.7	.990	0
*12	17	3.7 - 9.9	.994	0

5.1e Vary Averaging Period and Initial Phase

T_{ave} (min)	Initial Phase Shift (γ)	% underestimate u_{*c}	Range of "measured" C_D ($\times 10^{-3}$)
* 5	0°	17	3.7 - 9.9
10	0°	17	3.9 - 9.4
15	0°	17	4.0 - 9.5
15	45°	17	4.0 - 8.1
30	0°	18	4.4 - 7.3
30	45°	18	4.4 - 8.2
30	90°	17	4.9 - 7.3
45	45°	24	5.1 - 6.4
45	0°	20	5.0 - 5.6
45	45°	15	5.1 - 5.3
45	90°	12	5.0 - 5.8
60	0°	18	5.73

Table 5.1: Sensitivity of simulated log profile analysis to variation of input parameters around their base state values. In each series of runs, * refers to the base state. For the base state runs, $T_{ave} = 5$ min for this analysis and $u_{*c} = 0.704$ cm/sec.

case are the average percent underestimate of "true" predicted steady shear velocity,

$$100 \times \left(1 - \frac{u_{*m}}{u_{*c}}\right) \quad (5.6)$$

where u_{*c} is the steady shear velocity predicted from the 3 component boundary layer model, eq. 4.14, and the range of measured drag coefficient over the internal wave period. Degradation of the log profile as reflected in **minimum** R^2 value is only listed for cases where the **strength** of the steady current is varied.

For the cases considered, actual steady shear velocity is always underestimated by between about 10 and 20 percent, which translates to a 20 to 40 percent underestimate of stress. Drag coefficient variability and degradation of the profiles increase for larger internal waves, for shorter internal wave periods, for more nearly **colinear** internal waves and currents, and for shorter averaging times. The increasing importance of **larger** internal waves **might** be expected. Stronger internal waves will lead to greater shifting of successive unsteady experiment profiles, as in Figure 5.2a. Referring to the base state boundary **layer** profiles of Figure 5.1 and the definition of the scale height, h , eq 4.20, the reason for the increasing effect of shorter wave periods becomes apparent. **Shorter** wave periods lead to smaller scale heights, hence bring the **"overshoot"** part of the internal wave boundary layer nearer to the current meters. **Some** curvature is introduced, and the lower slope and greater magnitude of the internal wave profile cause greater variability of measured roughness and drag coefficient. Relative

direction between internal wave and current is important because velocity components are added vectorially; an internal wave colinear with the current obviously modulates speed more than other directions do. Note that this implies that the amplitude of an internal wave is not necessarily well represented by speed records alone. Possible disguising of internal wave amplitude in speed records is important because, although strictly kinematical effects are related primarily to the magnitude of the speed variation, the boundary layer solution and possible dynamical effects depend more on the amplitude of the wave. The effect of averaging time is quite reasonable, if the boundary layer scale height does not change. Longer averages will simply average out more of the internal wave variability.

Large degradation of log profiles occurs for the internal wave bottom velocity comparable to or greater than the current velocity. This is most obvious when varying the strength of the steady current in the present sensitivity analysis. Low minimum R^2 values are always accompanied by a very large range of "measured" drag coefficient and roughness variability. As stated previously, the systematic nature of the underlying variability belies a random source. For very weak current, the velocity profile may even reverse briefly with larger speeds near the bottom near stall conditions, due to the phase difference between top and bottom current meters as the wave reverses direction.

5.4 CODE ■ Boundary Layer Data:

Data from the CODE ■ high frequency bottom boundary layer

experiment are presented in Figure 5.3, for the period including the event identified in Figure 3.8. This particular subperiod was not used in the analysis of Grant et al., 1984. Their analysis concentrated on showing the validity of the surface wave-current interaction hypothesis, and questions concerning the effects of internal waves on velocity profiles and unacceptable large error bars on shear velocity estimates during this subperiod lead to rejection of the data for their purposes.

The on-offshore velocity data plotted in 5.3a are from a Neil Brown Instrument Systems Acoustic Current Meter (NBIS ACM) mounted at the top of the El Camino tripod during the 90 m experiment, at about 2.7 m above the bottom. The data (connected circles) in 5.3b and 5.3c are from a BASS current meter system on a second tripod, Bayshore, located about 400 m away from El Camino along isobath to the north. The CODE site was described in section 3.2 and a map of the 90 m site presented in Figure 3.1a. Velocity data from both instruments have been presented and described in different form in Grant et al., 1983 and Grant et al., 1984. The BASS values used here are 4.67 min averages of the 5 hz sampled data, and the NBIS ACM values are 4.7 min running means of the 1 hz sampled data. None of the time series are high pass filtered. The large trough at the beginning of the on-offshore velocity record of 5.3a is the maximum excursion of the first wave in the small group under consideration, and it clearly corresponds to the peak in the speed time series from the NBIS ACM and the BASS 1 m sensor, presented in 5.3b. The BASS data in 5.3c are corresponding estimates of the drag coefficient, determined from the BASS current profile. The dashed lines in 5.3c are predictions of C_d using the present model, measured speed

Figure 5.3: Data from the CODE ■ high frequency bottom boundary layer experiment starting at 0500 hrs on June 3, 1981. (a) On-offshore velocity data at 2.7 m height on the El Camino tripod (see Figure 3.1), lowpass filtered with a 4.7 min running mean. (b) Corresponding speed data from the same instrument (solid line), and 4.67 min averages of the BASS speed from 1 m on the Bayshore tripod (connected circles). (c) Estimates of steady drag coefficient, C_{d100} , from the BASS velocity profiles (connected circles) and predicted drag coefficient using only the speed at 1 m and the measured surface wave parameters (dashed line; quasi-steady treatment). Error bars for the shear velocity estimates are presented in Table 5.2.

Point No	S.W. Period, T_w (sec)	u_b (cm/sec)	u_{c100} (cm/sec)	u_* (cm/sec)	R^2	Percent error in u_*
1	14.9	4.6	10.7	1.1	0.98	43
2	13.4	4.5	11.5	1.2	0.90	100
3	13.8	4.9	11.4	1.1	0.96	62
4	13.6	4.9	9.7	0.9	0.98	43
5	14.3	3.9	8.3	0.8	1.00	0
6			7.2	0.7	0.98	43
7			5.9	0.6	0.96	62
8			6.5	0.8	0.98	43
9	15.1	6.8	6.3	0.8	0.92	90
10	13.8	4.2	6.3	0.8	0.94	77
11	13.5	4.6	6.3	0.6	0.98	43
12	16.8	4.9	6.3	0.5	0.98	43
13	14.2	6.2	6.9	0.6	0.94	77
14	13.6	4.4	7.7	0.8	1.00	0
15			7.8	0.6	0.98	43

TABLE 5.2: Data used for figure 5.3c, from Bayshore bottom tripod. Surface wave period and value of u_b obtained from pressure records as explained in Grant et al. (1984); u_{c100} is the measured value from the BASS current meter at 1 m; u_* and R^2 values are obtained from log profile fits to BASS measured current profiles, and error estimates are calculated from eq 5.5.

at 1 m, measured surface wave characteristics, and a constant $z_0 = 0.2$ cm, assuming that the speed at each time point is quasi-steady. Table 5.2 summarizes the BASS data used for Figure 5.3c and lists values of u_* estimates, R^2 and percent error in u_* estimates obtained from the BASS data analysis.

The drag coefficient predicted by the quasi-steady surface wave-current model in Figure 5.3c (not explicitly including the internal waves) does not agree well with the drag coefficient estimated from the data [C_D (data)], though the error bars for C_D (data) are often large enough to make the difference statistically insignificant (C_D percent error is about twice the u_* percent error listed in Table 5.2). Grant et al. (1984) found that when internal waves were either not present or very weak, a much greater percentage of the log profile regression fits were acceptable ($R^2 > 0.993$ for less than 50 percent error in C_D) and the surface wave-current model predictions were quite good. Thus, the internal waves significantly degrade the log profile measurements. In addition, the trends in C_D (data) are in qualitative agreement with the predictions of the unsteady simulation of this chapter (e.g., Figure 5.2). Notably, C_D (data) and speed at 1 m have a roughly inverse phase relationship and the range of C_D (data) is much greater than the range of the predictions of the quasi-steady surface wave-current model. The systematic nature of C_D (data) variability argues against purely random error. A boundary layer data set containing a more clearly defined internal wave packet would be needed for quantitative comparison to an unsteady simulation run. Such an explicit comparison does not seem justified here, but the qualitative agreement discussed above supports the

present contention that a separate internal wave boundary layer must be accounted for.

5.5 Discussion:

In the context of the present theoretical model, there are two ways in which internal waves may lead to significant degradation of the log profile fit. The first, which applies to the cases considered above, holds when the current meters are all within the calculated internal wave sub-boundary layer, and degradation only occurs during limited portions of the internal wave cycle. The second applies to cases when the current meters span the top of the internal wave boundary layer, adding curvature to the total boundary layer profile. Both mechanisms depend on the internal wave being an important component of the entire boundary layer; the addition of the internal wave velocity must be significant.

In the first case, degradation results from the vector addition of the internal wave boundary layer profile, whose magnitude is logarithmic, to the steady current profile, as the internal wave reverses direction and passes through zero velocity. The phase difference between top and bottom current meters leads to velocities at different heights vectorially adding or subtracting nonuniformly, and is particularly likely to cause large degradation in the log profile fit if the internal wave velocities are strong and the total velocity profile is near stall. The mechanism is complicated by the fact that experimental profile methods fit to speed at each height, without taking account of direction.

The case of current meters spanning the top part of the internal wave boundary layer, thus introducing curvature to the logarithmic

profile, was evaluated by rerunning the unsteady base state experiment with current meters at 1m, 2m, 4m, and 8m so that the current meters span the internal wave overshoot region in Figure 5.1b. Although the upper current meters are now strictly speaking out of the "constant stress" region of the steady boundary layer, Figure 5.1b shows that the steady current speed profile is still reasonably logarithmically linear. The curvature introduced into the internal wave profile does increase the degradation of the log profile fit; the lowest base state value of R^2 is reduced to 0.989 as opposed to 0.994 for the original base state. However, lower R^2 values are at the expense of much greater variability in drag coefficient and roughness. Some of the increase in variability of drag coefficient and roughness may be statistical, but most of the increase is due to the fact that the internal wave boundary layer profile is more nearly vertical. The 30 percent greater range of C_{dm} relative to the original base state leads to 40 percent further reduction in the range of stress values.

Neither possible mechanism for degradation of the log profile fit predicted by the present theoretical simulation is fully consistent with the amount of degradation estimated from the limited data set. In addition, all cases considered in the present theoretical context improve log profile fits with increasing averaging time, a tendency opposite to that observed from the CODE data analysis and reported by Grant et al. (1984). Theoretical underestimation of the amount of observed degradation of log profiles is not surprising, considering the differences between theoretical assumptions of steady state, single frequency linear waves and time invariance of the eddy viscosity, and

the reality of intermittent, possibly nonlinear internal waves, variable surface waves, and **likely** time variability of turbulent mixing. Effects resulting from the differences between assumptions and reality are discussed **below**.

Assuming that the observed degradation is physical, it can probably be traced to either the introduction of significant curvature into the velocity profile or effective non-stationarity of the bottom roughness which causes the steady current profile to readjust. Curvature must vary in both space and time for its effect to grow worse with increasing averaging rather than better. In other words, if the shape function of the boundary layer profile is time invariant and if the magnitude varies symmetrically over the wave period, then its effect should be averaged out as the averaging time approaches the wave period, which is in fact what occurs in the present simulations.

The present theoretical boundary layer model assumes steady state, sinusoidal waves and a time invariant eddy viscosity. Trowbridge (1983) has shown that allowing for weakly **nonlinear** waves and time varying eddy viscosity introduces higher harmonics into the boundary layer velocity profile, which lead to time asymmetry of the profile shape. In addition, transient forcing should give rise to a transient boundary layer, which grows through the current meter array. The **asymmetries** and additional curvature resulting from time variability and nonlinearity presumably are higher order phenomena, which will increase the degradation of the log profile fit but should not affect the **lowest** order predictions of basic profile slopes and scale heights. Transience is not as easily accounted for, but should be of limited extent.

Grant et al. (1984) point out that the physical bottom roughness does not change significantly over the period of the CODE I high frequency bottom boundary layer experiment. However, it is possible (Grant, personal communication) that changes in the surface wave field within an averaging period may lead to significant variability in apparent bottom roughness, causing some degradation of log profiles as the "steady" boundary layer profile adjusts. This would not be predicted by a model which assumes no variability of the surface wave field, and it is uncertain what the magnitude of the effect would be. In any case, results of surface wave changes over an averaging period are likely confined to the lowest part of the steady boundary layer.

In summary, it must be re-emphasized that the present simulation of a boundary layer velocity profile experiment does reasonably well at predicting trends in variability of measured drag coefficient and bottom roughness that result from using the velocity profile technique in the presence of internal waves. This variability is a direct result of unsteadiness induced by the internal waves and is not predicted if the boundary layer is considered to be quasi-steady; the existence of variability depends only on the existence of a separate oscillatory boundary layer with different scale height than the steady boundary layer. The amount of variability will depend on the exact boundary layer model used, but analysis of the CODE data to date does not allow discrimination between models. The three component boundary layer model used indicates possible underestimation of actual drag felt by the steady current, which depends on the hypothesized dynamical influence of the internal wave. All effects are predicted to be greatest for

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internal wave bottom velocities large relative to and **colinear** with the steady current.

Considering the idealized nature of the present boundary layer model, significant degradation of the log profile due to internal wave unsteadiness is predicted. The amount of degradation predicted is directly related to the range of variability of C_d and z_0 over an internal wave cycle. However, the amount of log profile degradation observed in the **field** in the presence of internal waves is larger than the predictions would indicate; this is not surprising, given the differences between field conditions and model assumptions. This disagreement does not necessarily imply that the **lowest** order quantities predicted by the boundary layer model are not reasonable estimates of real phenomena.

The primary value of the present analysis is as a diagnostic tool for examining results of boundary layer experiments which may be contaminated by internal wave unsteadiness, and in particular for evaluating "instantaneous" estimates of boundary **roughness** and drag coefficient for reliability and relevance to true average conditions.

CHAPTER 6

DISSIPATION OF INTERNAL WAVE ENERGY IN THE CONTINENTAL SHELF BOTTOM BOUNDARY LAYER

6.1 Introduction:

The statistical studies of energy in the internal wave band on the continental **shelf** referenced in Chapter 2 **all** indicate that the dominant vertical structure of temperature and velocity fluctuations is consistent with the onshore propagation of mode 1 internal waves. In addition, observations of packets of high frequency internal waves on the shelf indicate general onshore propagation, seemingly without reflection. Moreover, estimates of cross-shelf differences in energy density and energy flux show shoreward reduction of both quantities over the entire **shelf** (Gordon 1978, Torgrimson and Hickey 1979). A number of mechanisms for decreasing shoreward energy flux in a given frequency band are possible, including shear **instability**, wave breaking (Cacchione 1970, Cacchione and Wunsch 1974), scattering of strongly nonlinear waves into higher frequency energy (**Djordjevic** and Redekopp 1978), scattering into **barotropic** energy (Huthnance 1981), and bottom friction. The problem is of particular interest for internal tides; various authors (**e.g.**, review article by Wunsch 1975) have discussed the possibility that a nontrivial part of the total tidal dissipation may be due to dissipation of the internal tide. Dominance of any one of the above mechanisms for a given situation is hypothetical; **it** is the purpose of this chapter to theoretically quantify, in as **realistic** a way as possible, the amount of

dissipation of internal wave energy occurring in the bottom boundary layer on the continental shelf for a given energy **level** and environment.

There is at least an order of magnitude discrepancy between published internal wave energy dissipation rates estimated from measurements on the continental shelf and published theoretical predictions of deep ocean internal wave energy dissipation rates by bottom friction (Wunsch and Hendry 1972, D'Asaro 1982). Wang and Mooers (1977) and Petrie (1975) have ascribed observed cross-shelf decreases in tidal energy **entirely** to bottom friction. Wang and Mooers (1977) estimate a dissipation rate of 5 **ergs/cm²/sec** at a site where a ray of the internal tide encounters the bottom on the Oregon continental shelf. From the results reported by Petrie (1975), estimates of dissipation rates can be made which range between 0.3 and 6 **ergs/cm²/sec** at the Nova Scotia shelf edge. In contrast, Wunsch and Hendry (1972) and D'Asaro (1982), using **simple** boundary layer models, have predicted virtually negligible bottom frictional internal wave dissipation rates for the continental slope and abyssal plain, respectively; both found dissipation rates of about 0.01-0.02 **ergs/cm²/sec**. LeBlond (1966) concluded that bottom frictional dissipation of internal waves is likely to be dominant in shallow seas, but his theoretical analysis assumed a very high value of a constant eddy viscosity (1000 **cm²/sec**).

The central hypothesis of this chapter is that the full flow dependent nature of the internal wave drag coefficient, as enhanced by nonlinear interaction with surface waves and currents on the continental shelf, will lead to greater dissipation of internal wave energy than if internal waves are considered alone and a constant drag coefficient is

assumed. Note that this implies that the internal wave dissipation rate in the deep ocean bottom boundary layer may also be higher than previous theoretical predictions, if interaction between internal waves and the steady current occurs. To test this hypothesis, output of the three component boundary layer model of Chapter 4 is used in a model considering the fate of a shoreward propagating internal wave subject to shoaling and bottom frictional energy dissipation, with both constant and flow dependent frictional parameterizations. Sensitivity to surface wave and current strength, initial internal wave strength, and bottom slope are examined. For purposes of comparison, equivalent values of a constant eddy viscosity are calculated. Results are qualitatively compared to previously reported estimates of continental shelf internal wave energy dissipation rates.

Summary of results: Consideration of the variable, flow dependent nature of the drag coefficient is found to be very important. Theoretical estimates of bottom frictional dissipation of internal waves are significantly enhanced when surface wave/internal wave/current boundary layer interaction is included, especially for surface waves and currents large relative to the internal wave. Low frequency, long internal waves are less affected per unit bottom or unit time than high frequency, short internal waves. The inner shelf bottom boundary layer is very effective at dissipating internal wave energy, especially if surface waves are allowed to shoal. Local dissipation rate depends strongly on internal wave amplitude, as is to be expected, but efficiency of energy dissipation (defined as percent reduction of incident energy flux) is fairly insensitive to wave amplitude. Broad,

gently sloping shelves are predicted to have slightly lower local bottom frictional dissipation rates than narrow, steep shelves, but should be much more efficient dissipators in an integral sense. Equivalent values of a constant eddy viscosity are found to range between 0.02 and 20 cm²/sec, but depend on both internal wave amplitude and frequency.

The use of a flow dependent drag coefficient which includes the effects of interaction with reasonable surface waves and currents and anticipation of large internal wave amplitude on the shelf bring predicted dissipation rates up to the same order of magnitude as dissipation rates estimated from continental shelf data by Wang and Mooers (1977) and Torgrimson and Hickey (1979). Bottom friction is predicted to account for much of the estimated decrease in energy flux over the wide Northwest African shelf reported by Gordon (1978).

6.2 Methods and Materials:

For mode 1 linear internal waves propagating in the positive x direction subject to dissipation, with any barotropic background current constrained to be at right angles to the direction of wave propagation, the depth integrated energy conservation equation may be written

$$\frac{\partial E_{tot}}{\partial t} + \frac{\partial}{\partial x} (c_g E_{tot}) = -d(E_{tot}; x, t) \quad (6.1)$$

where E_{tot} is the depth integrated internal wave energy density, c_g is the local horizontal group velocity, and d is the local dissipation rate. The present problem is assumed to be time invariant, and the dissipation to be due only to turbulent bottom friction, for which the

dissipation term may be written as the depth integral of the time averaged turbulence production. To the accuracy of the of the boundary layer approximation (i.e., $0[\delta/D]$ where δ is the boundary layer thickness), the time averaged turbulence production may be taken as the time averaged work of the bottom shear stress (e.g., Trowbridge 1983). For typical continental shelf conditions the internal wave boundary layer thickness is on the order of meters and the depth is on the order of tens of meters, so the ratio of the boundary layer thickness to the water depth is $O(10^{-1})$ and the second formulation is of sufficient accuracy. Under these assumptions, the governing equation for E_{tot} reduces to

$$\frac{\partial}{\partial x} (c_g E_{tot}) = -d \quad (6.2)$$

and the dissipation term may be written (Jonsson 1968)

$$d = \frac{1}{2} \tau_b u_{bi}^* = \frac{1}{8} \rho_0 C_d u_{bi}^3 \cos \phi_b \quad (6.3)$$

where u_{bi}^* is the complex conjugate of the local inviscid internal wave bottom velocity, τ_b is the boundary shear stress, and ϕ_b is the phase change across the boundary layer. For the present analysis take $\rho_0 = 1.026 \text{ gm/cm}^3$ and $\phi_b = 15-20^\circ$ (e.g., Figure 4.3), so $\rho_0 \cos \phi_b \sim 1 \text{ gm/cm}^3$.

The drag coefficient, C_d , may be either taken as constant or calculated in a manner similar to that of Chapter 4, where the (square root of the) steady current drag coefficient was calculated as a function of a boundary Rossby number and surface and internal wave characteristics. For the present analysis, it is possible to carry out

analogous calculations for the (square root of the) internal wave drag coefficient as a function of an internal wave inverse relative roughness $[u_{bi}/(\omega_i z_0)]$ and the relative intensities of the surface wave and current to the internal wave, for assumed surface wave and internal wave frequencies and fixed problem geometry. Figure 6.1 presents an example calculation, in the form of contour plots of enhanced u_{*i}/u_{bi} (where u_{*i} is the amplitude of the shear velocity for the internal wave) for a one hour internal wave propagating onshore in the presence of an onshore propagating 15 sec surface wave and a longshore current. The values contoured are the final enhanced values, analogous to $(u_{*c}/U_c)_3$ in eq 4.32. The corresponding local drag coefficient for an internal wave can be calculated as $(u_{*i}/u_{bi})^2$ by interpolation between and extrapolation of the stored matrices that form the basis for Figure 6.1, for arbitrary relative surface wave and current strength. A corresponding set of matrices has been derived for a six hour internal wave with all other parameters unchanged. The two sets of matrices are the basis of the variable drag coefficient dissipation analysis that follows.

One hour and six hour internal waves are chosen as being representative of the high frequency and broad middle frequency parts of the internal wave band, respectively. Reference to Figure 4.5c shows that the dynamical influence of one wave on a steady current only changes slightly over the internal wave frequency band, so it may be anticipated that enhancement of the internal wave drag coefficient will also be fairly insensitive to frequency. Thus, the two chosen wave periods should effectively cover much of the internal wave frequency

Figure 6.1: Contour plots of enhanced u_{*i}/u_{bj} for a 1 hr internal wave propagating onshore in the presence of an onshore propagating surface wave with 15 sec period and a longshore steady current. Each plot represents a surface of constant ratio of current speed to internal wave bottom velocity, u_c/u_{bj} ; the horizontal axis is an inverse roughness, $u_{bj}/(\omega z_o)$, and the vertical axis is the ratio of surface wave magnitude to internal wave magnitude, u_b/u_{bj} . In (a), $u_c/u_{bj} = 0.0$; in (b), $u_c/u_{bj} = 1.0$, and in (c), $u_c/u_{bj} = 3.0$.

band.

The model developed here calculates the theoretical trade-off between internal wave shoaling and energy dissipation by bottom friction, for a shoreward propagating mode 1 internal wave. The calculation is similar to the shallow water surface wave dissipation problem. Equation 6.2 is solved by simple forward finite differencing. The horizontal dimension is divided into equally spaced intervals, each Δx wide, and the energy at succeeding grid points calculated by

$$E_{tot,n} = \frac{C_{g,n-1} E_{tot,n-1} - \Delta x d_{n-1}}{C_{g,n}} \quad (6.4)$$

for $n = 1, 2, \dots$. The model takes as input the constant bottom slope, an assumed constant depth averaged value of N , the constant physical bottom roughness, an initial internal wave bottom velocity, $u_{bi,0}$, an initial surface wave bottom velocity, $u_{b,0}$, and a constant longshore current, u_c . The dissipation rate is calculated at each step by a discretized version of eq 6.3,

$$d_n = \frac{1}{2} C_{d,n} u_{bi,n}^3 \quad (6.5)$$

where an appropriate value of $C_{d,n}$ either is taken as an input constant or is calculated by quadratic interpolation between or extrapolation of values stored as above. Extrapolation is cut off when relative surface wave or current strength greatly exceed the stored range, a time and space saving step that does not much affect the results.

To solve equations 6.4 and 6.5, $E_{tot,n}$ must be expressed in terms of $u_{bi,n}$. The analysis of Chapter 2 showed that both the ratio

of E_{tot} to u_{bi}^2 , and the value of c_g are dependent in general on the assumed density structure and the degree of nonlinearity. For the present purpose, only the linear constant stratification structure model ($H_i/D = 1$, $z_c/D = 0.5$) and the linear symmetrical two layer structure model ($H_i/D = 0$, $z_c/D = 0.5$) are used and the waves are assumed to be long. For the former case, $E_{tot} = 0.25 \rho_0 u_{bi}^2 D$ and $c_g = (1/\pi) ND$. For the latter case, $E_{tot} = 0.5 \rho_0 u_{bi}^2 D$ and $c_g = 0.5 ND$. These two simple models bracket or approximate many cases of practical interest. The input internal wave is marched in towards shore, allowing the surface wave to shoal but keeping the roughness and current strength constant. Allowing the surface wave and internal wave to shoal while keeping bottom roughness, current strength, stratification and bottom slope constant is an approximation, but adding cross-shelf variation in roughness, current, stratification and slope would complicate the analysis far beyond the present purpose. Linear surface and internal wave shoaling are straightforward to calculate, and may have a large effect. For example, the bottom velocity for a 15 sec surface wave (without dissipation) roughly triples from 140 m to 70 m depth, while the bottom velocity for an internal wave without dissipation doubles in the same distance. The calculation is stopped when the internal wave energy is completely dissipated or when the calculated internal wave bottom velocity, u_{bin} , exceeds the local internal wave phase velocity. The latter limit is taken as a rough breaking criterion, or a maximum limit of validity of the present linear formulation.

The model has been tested by comparing it to the analytical

solution of eq 6.2 for a constant drag coefficient and a flat bottom, which is (e.g., Ole Madsen, course notes)

$$u_{bi}(x_0 + \Delta x) = \frac{u_{bi,0}}{1 + K_f u_{bi,0} \Delta x} \quad (6.6)$$

where

$$K_f = \frac{C_d \cos \phi_b}{ND^2} \quad (6.7)$$

at the long wave limit of the two layer structure model. It also has been tested against the solution for linear inviscid shoaling, simply conserving shoreward energy flux. In both cases, the agreement is very good; further, calculated behavior with a variable drag coefficient is quite reasonable.

To fully examine predicted behavior, both cross-shelf characteristics of individual solution runs and integral dissipative characteristics of shelves with different flow environments and geometries are considered. "Integral" refers to treatment of the entire shelf (or some fraction of the entire shelf) inshore of a fixed depth as a bulk dissipator, or energy sink. A base state representative of a possible CODE central shelf situation is defined. The CODE central shelf bottom slope of 0.005 and previously used roughness value of $z_0 = 0.2$ m are assumed. The steady longshore current is taken as $u_c = 10$ cm/sec, initial surface wave bottom velocity is taken as $u_{b,0} = 5$ cm/sec, and initial internal wave bottom velocity is taken as $u_{bi,0} = 4$ cm/sec. The two layer vertical density structure model is used with depth averaged $N = 3$ cph. After presenting a single series of model runs

illustrating the cross shelf behavior of individual solutions, the integral dissipative effects of varying surface wave and current strength, initial internal wave bottom velocity, and bottom slope are examined. Two indicators of integral effect are used: (1) the average dissipation rate per unit bottom between two fixed depths in $\text{ergs/cm}^2/\text{sec}$ measures an average absolute level of energy dissipation rate, and (2) the percent reduction in incident energy flux between the same two depths measures the integral efficiency of energy dissipation. For the present sensitivity analysis, calculations are started at 133 m depth and carried in to 67 m depth, for both one hr and six hr internal waves. Results derived using a flw dependent, enhanced drag coefficient are compared to results derived assuming a constant drag coefficient of 0.002 (about equal to the number used by Wunsch and Hendry [1972], and about twice as large as the number used by D'Asaro [1982]).

For qualitative comparison of model results to previous observations, a common basis of average dissipation rate per unit bottom in $\text{ergs/cm}^2/\text{sec}$ is used. When published dissipation rates are not given in these units, they are converted, and when only energy density or energy flux values are given, dissipation rates are estimated as best as possible from the given data.

6.3 Results:

An example of a series of model runs illustrating theoretical effects of increasing surface wave and current strength on a one hr internal wave as it propagates shoreward is presented in Figure 6.2, Bottom slope, roughness, and initial internal wave bottom velocity are

Figure 6.2: Cross shelf behavior of a series of model runs with increasing surface **wave/current** intensity, with other parameters fixed at their base state values. Assumed shelf geometry is shown in (c), with a vertical exaggeration of 100. The traces of predicted internal wave bottom velocity, u_{bi} , are plotted in (a); the **lowest** corresponds to a surface **wave/current** combination of (10 cm/sec, 20 cm/sec) at the left boundary, and the highest corresponds to inviscid propagation; calculations are stopped when u_{bi} equals the phase velocity, plotted as the dashed line. Calculated traces of corresponding internal wave drag coefficient, C_d , are plotted in (b).

taken as in the defined base state of the previous section. The variable internal wave drag coefficient is calculated as described in the previous section, for cases with progressively larger surface wave/current combinations; successively, $(u_{b,0}, u_c) = (0,0), (2,4), (4,8), \dots$ cm/sec.

There are three points to be emphasized. First, the internal wave drag coefficient is greatly enhanced if its flow dependent nature is considered, and is particularly enhanced for large surface wave/current combinations. Flow dependent C_d with no enhancement is about 2.5-3 times larger than a constant value of $C_d = 0.002$, for the present value of $u_{bi,0}/(\omega_i z_0) = 1.1 \sim 10^5$. When enhanced by a combination of large surface waves and currents, C_d is an order of magnitude larger than 0.002. Second, because of the inverse dependence of C_d on u_{bi} , a dependence which is magnified by surface wave/internal wave/current interaction, a "critical point" phenomenon occurs. As long as internal wave shoaling is dominant and u_{bi} is increasing, both $u_{bi}/(\omega_i z_0)$ and the strength of the internal wave relative to the other current components increase and C_d decreases. Once u_{bi} begins to decrease, both $u_{bi}/(\omega_i z_0)$ and the relative strength of the internal wave begin to decrease and C_d starts to increase, which leads to an even faster decrease in u_{bi} . Thus, the point of reversal in the trend of u_{bi} , or the point at which internal wave shoaling and dissipation of energy are balanced, is a "critical point" in the theoretical model. Third, allowing surface waves to shoal causes C_d to increase even more, especially in shallow water when surface wave velocities become relatively very large. Note that

the cut-off in the rise of C_d near the shore in Figure 6.2b is an artifact due to truncation of the **extrapolation** procedure, explained in the last section.

Theoretical integral dissipative characteristics of different idealized **shelf** environments and geometries are **illustrated** in Figures 6.3 to 6.5. In each figure, **all** parameters except the one being varied are defined as in the base state. Results derived using a flow dependent, enhanced drag coefficient are compared to results derived assuming a constant drag coefficient of 0.002. As explained in the previous section, both average dissipation and change in energy flux are defined in an integral sense over the distance between 133 m depth and 67 m depth, which is an arbitrarily chosen **shelf "halfwidth"**. Thus, most of the model runs are not affected very much by the "critical point" behavior mentioned above, which reduces the range of the results but **makes** the runs more directly comparable. Figure 6.3 shows the effect of varying surface wave and current strength in combination, just as in Figure 6.2. Figure 6.4 shows the effect of varying the **initial** internal wave bottom velocity, u_{b10} . Figure 6.5 shows the effect of **varying** the bottom slope, for fully enhanced internal wave dissipation only. Calculations are presented for both one hr and six hr internal waves, and the location of the base state is indicated in each plot.

Figure 6.3 indicates that greater surface **wave/current** strength leads to more energy dissipation, as might be expected. Both the average dissipation rate per unit bottom and the integral efficiency of energy dissipation as measured by percent reduction in incident energy flux are increased over their constant C_d values by factors of 2 to 8.

a. Average Dissipation Rate

b. Change in Energy Flux

Figure 6.3s Integral dissipative characteristics between 133 m depth and 67 m depth for varying surface wave/current intensity, with all other parameters held constant at their base state values. Average dissipation rate is shown in (a), and percent reduction in incident energy flux in (b). The solid lines represent internal waves with fully enhanced drag; the upper line in each plot is for a 1 hr wave and the lower is for a 6 hr wave. The dashed line represents both internal waves with a constant drag coefficient of 0.002. The arrow indicates the base state in each plot.

a. Average Dissipation Rate

b. Change in Energy Flux

Figure 6.4: Integral dissipative characteristics between 133 m depth and 67 m depth for varying initial internal wave bottom velocity, $u_{bi,0}$, with all other parameters held constant at their base state values. Average dissipation rate in (a) (note log axis) and percent reduction in incident energy flux in (b); lines defined as in Figure 6.3. The arrow indicates the base state in each plot.

Figure 6.5: Integral dissipative characteristics between 133 m depth and 67 m depth for varying bottom slope, **b**, with all other parameters held constant at their base state values. The shelf width for a given slope is shown in (a), the average dissipation rate in (b), and the percent reduction in incident energy flux in (c), for fully enhanced drag only. The arrow indicated the base state in each plot.

The six hr (longer) wave is always affected less than the one hr (shorter) wave. Figure 6.4a shows that constant C_d dissipation has an approximately cubic dependence on $u_{bi,0}$, as expected from eq 6.3, while flow dependent C_d dissipation has a lower slope because of the general inverse relationship between C_d and u_{bi} ; as discussed above, this inverse relationship is largely a function of the strength of the internal wave relative to the other current components. For constant C_d and low values of $u_{bi,0}$, the predicted average dissipation rate is of the same order of magnitude as theoretical predictions reported by Kunsch and Hendry (1972) and D'Asaro (1982) for deep ocean internal wave bottom energy dissipation (0.01-0.02 ergs/cm²/s). For flow dependent, enhanced C_d , low amplitude average dissipation rates are almost an order of magnitude higher (0.1-0.2 ergs/cm²/s) than constant C_d dissipation rates. Figure 6.4b shows that, though average dissipation rate depends strongly on $u_{bi,0}$, the integral efficiency of energy dissipation is not much affected. That is, for increasing $u_{bi,0}$ more total energy is dissipated, but the fraction of incident energy flux dissipated remains about the same. The results of Figure 6.5 show that broad, gently sloping shelves have slightly lower average dissipation rates than narrow, steep shelves, but are much more efficient dissipators in an integral sense; there is simply more bottom. For a shelf with the CODE slope and base state conditions energy flux is only reduced by 20-30 percent by midshelf, while bottom friction might completely dissipate an internal wave by midshelf for a shelf with one fifth of the CODE slope. The lower average dissipation rates of broader shelves are simply due to lower average values of u_{bi} .

The results of the present model may be compared to constant eddy viscosity formulations by equating respective expressions for dissipation rate per unit time in an oscillatory boundary layer, or

$$\frac{1}{2} \rho_o C_d u_{bi}^3 \cos \phi_b = \frac{1}{2} \rho_o u_{bi}^2 \left(\frac{K_m \omega_i}{2} \right)^{1/2} \quad (6.8)$$

where K_m is the constant vertical eddy viscosity. The expression on the RHS is a version of the classical solution for constant eddy viscosity (e.g., LeBlond 1966). Therefore, for a given value of C_d and given internal wave characteristics, equivalent values for K_m are calculated by

$$K_m \sim 1.8 C_d^2 u_{bi}^2 / \omega_i \quad (6.9)$$

assuming that $\cos^2 \phi_b \sim 0.9$. For a one hr internal wave, using $C_d = 0.002$ gives $K_m = 0.02-0.4 \text{ cm}^2/\text{sec}$ for $u_{bi} = 2-10 \text{ cm/sec}$. Using an enhanced drag coefficient for the same wave and same range of speeds and assuming $u_b = 5 \text{ cm/sec}$, $u_c = 10 \text{ cm/sec}$, and $z_o = 0.2 \text{ cm}$, $C_d = 0.023-0.008$ and equivalent values of K_m are $2-6 \text{ cm}^2/\text{sec}$. For a six hour wave and enhanced drag, $C_d = 0.015-0.005$ and $K_m = 6-17 \text{ cm}^2/\text{sec}$, again for $u_{bi} = 2-10 \text{ cm/sec}$. Thus, an approximate range of equivalent K_m values for high frequency internal waves is about $0.02-20 \text{ cm}^2/\text{sec}$, but these values are highly dependent on frequency and amplitude.

The first data set to be referenced for comparison to predictions of the present model is from the Coastal Upwelling Experiment (CUE) of the summer of 1973, off the coast of Oregon. Internal wave analysis at

a single mooring during an upwelling event was reported by Hayes and Halpern (1976). Wang and Mooers (1977) reported on changes in interior mixing and cross-shelf energy flux during the same event; of particular interest is their estimate of the dissipation rate of the internal tide at the bottom boundary in about 50 m water depth. For a situation in which calculated rays of the internal tide were expected to reflect from the bottom slightly offshore of the 50 m isobath, they calculate a dissipation rate per unit bottom of about $5 \text{ ergs/cm}^2/\text{sec}$ and report a definite downward energy flux. Torgrimson and Hickey (1979), in reanalyzing the larger data set, estimate average dissipation rates for the internal tide of about $3.8 \text{ ergs/cm}^2/\text{sec}$ from 200 m to 100 m, about $1.3 \text{ ergs/cm}^2/\text{sec}$ from 100 m to 50 m, and about $0.6 \text{ ergs/cm}^2/\text{sec}$ from 50 m to the coast. The numbers have been converted from W/m^3 using the approximate published geometry. Referring to Figure 6.4a, and assuming that the values that enter into calculation of the theoretical dissipation rates plotted are not unreasonable for the CUE situation, the estimated dissipation rates from CUE are in fact seen to be within the theoretically feasible range for energy dissipation by bottom friction ($0.2\text{--}5 \text{ ergs/cm}^2/\text{sec}$), if the drag coefficient is enhanced by current and surface wave action. In addition, higher reported dissipation rates clearly correspond to situations in which the internal tide bottom velocities are likely to be greater, in agreement with theoretical expectation. Therefore, the hypothesis that a nontrivial portion of the observed dissipation of the internal tide may be due to bottom friction is supported by the present theoretical analysis.

Gordon (1978) presents a detailed study of cross shelf changes in

internal wave energy density and energy flux over the wide shelf off Northwest Africa, and concludes that internal wave energy levels on the shelf are nearly saturated, and that the shoreward flux of energy is almost all dissipated. The dissipation mechanism is not specified. To determine how much of the estimated dissipation might be due to bottom friction, model runs for one hr and six hr internal waves were calculated using the approximate reported shelf geometry, characterized by a slope of about 0.002, an approximate averaged Brunt-Vaisala frequency of 3.2 cph, assuming (for lack of better information) that $z_0 = 0.2$, $u_{b0} = 5 \text{ cm/sec}$, and $u_c = 10 \text{ cm/sec}$, and matching the reported estimates of average first mode energy density and energy flux at 100 m by using an internal wave with amplitude equal to the rms speed in corresponding frequency bands and the constant stratification vertical structure model. Because chosen surface wave, current, and roughness values are arbitrary, theoretically predicted dissipation can only be considered as an order of magnitude estimate, but both the efficiency of bottom frictional dissipation for broad shelves and the importance of using a flow dependent, enhanced drag coefficient are clearly illustrated. The model runs with enhanced C_d predict almost 100 percent of the observed decrease in internal wave energy flux between 100 m and 60 m, while using a constant value of $C_d = 0.002$ leads to prediction of 25-45 percent of the observed decrease over the same range.

6.4 Discussion:

Two results of the present analysis are particularly notable.

First, consideration of the full flow dependent nature of the internal wave drag coefficient on the continental shelf, **allowing** for boundary shear stress enhancement by surface waves and currents, theoretically increases both the amount and the integral efficiency of dissipation of internal wave energy in the bottom boundary layer. In combination with hypothesized large internal wave bottom velocities on the shelf, the enhanced drag is theoretically capable of reproducing dissipation rates of the same order of magnitude as estimates from data. Large bottom velocities ~~may~~ result **from** large, intermittent waves as in CODE, bottom reflection of rays of the internal tide (to the extent that ray theory is valid on the shelf), or shoaling and/or bottom velocity intensification effects (Wunsch 1969, Cacchione and Wunsch 1974, Eriksen **1982**), the last applying more directly to the continental slope. The dynamical importance of shear stress enhancement by nonl**inear** interaction of waves and currents in the bottom boundary layer was predicted by Grant (**1977**), **Smith** (**1977**), and Grant and **Madsen** (**1979**), has been demonstrated locally in field **experiments** by Cacchione and Drake (**1983**), Grant et al. (**1984**) and Wiberg and **Smith** (**1984**), and has been theoretically demonstrated for **continental shelf** wave dynamics by Brink (1982). The results **reported** here for internal wave dissipation provide another example of the dynamical importance of wave-current boundary layer interaction concepts.

The second main result is that it is theoretically possible to anticipate the **relative** importance of the bottom boundary layer as a sink for internal wave energy, for a given continental shelf, by considering several basic characteristics of that shelf and its flow environment. The character**istics** considered in this chapter were shelf

slope/width, characteristic internal wave amplitude, and surface wave and current strength relative to the internal wave. In addition, the mean stratification, the vertical density structure, and characteristic bottom roughness are important. Discussion of these factors follows.

1. Shelf slope/width is most important to the integral efficiency of a given geometry as a sink for internal wave energy. That bottom friction is not predicted to be a very efficient internal wave energy sink for narrow shelves is not surprising. Consider the CODE shelf: a six hr wave in a two layer density structure with depth averaged Brunt-Vaisala frequency of 3 cph has about a 6 km wavelength in 100 m of water (eq. B.13), so only about one wave fits between moorings C4 and C3. If bottom frictional dissipation made more than a fractional difference in a single cycle, basic assumptions and parameterizations of the present model would be questionable - that it does not is thus both expected and reassuring.

2. Characteristic internal wave amplitude is a controlling factor in determining the amount of local dissipation - the functional dependence of dissipation rate varies between u_{bio}^2 and u_{bio}^3 . Internal wave amplitude is expected to be largely a function of conditions at the shelf break and over the slope (e.g., Huthnance 1982), and may vary considerably from one location to another.

3. Relative strength of the surface waves, internal waves, and steady current affects dissipation by increasing the internal wave drag coefficient by up to a factor of five for quite reasonable conditions. It is important to emphasize the word "relative", since all wave-current boundary layer layer interaction theories (e.g., Grant and Madsen 1979)

predict results that depend on the relative strength of the current components, not their absolute strength. This fact was somewhat obscured in the present analysis, since the strengths of all components except the one being varied were held constant.

4. The importance of assumptions about vertical density structure for energy flux calculations was pointed out in Sections 2.4 and 2.5. Differences in vertical density structure between the examples of Section 2.5 (Table 2.1) made up to a factor of 6 difference in depth integrated energy density estimates from bottom velocity, and up to about a factor of 8 difference in energy flux estimates. Density structures with slow wave speeds tend to increase bottom frictional dissipation because energy propagates slower, allowing more time for bottom friction to work.

5. Higher mean (depth averaged) stratification will lead to less bottom frictional dissipation of internal wave energy, since there will be less bottom HKE for a given total energy level (Figure 2.3c), and since propagation speeds are faster, allowing less time for bottom friction to work (Figure 2.3d).

6. Bottom roughness must be estimated from data, especially for situations in which internal waves are not the dominant sediment transporting mechanism and the topography is not well known. Fortunately, exact knowledge of the bottom roughness is not critical for reasonable stress estimates, though its order of magnitude must be known.

The present model is somewhat limited by the nature of the boundary layer model upon which it is based. That is, the model is fine as long as it is used as a diagnostic tool only. Transience may limit the full

development of the internal wave boundary layer and Trowbridge (1983) has indicated that including time variation in the eddy viscosity may reduce estimates of energy dissipation by 20 to 30 percent.

Nevertheless, internal wave energy dissipation in the bottom boundary layer should be enhanced in the presence of surface waves and currents even if the oscillatory nature of the internal waves is ignored and they are considered as a slowly varying steady current. The concept of enhanced dissipation is robust, but its exact quantification depends on modelling assumptions.

A short discussion of other possible mechanisms for the uniformly observed shoreward decrease in internal wave energy flux seems appropriate. First, some of the shoreward decrease in energy flux may be due to radial or refractive spreading of energy. Topographically generated internal waves do not always propagate directly onshore, and are often short crested and somewhat curved. There are many examples of SEASAT SAR images that illustrate these tendencies; some particularly good coastal images are presented in Fu and Holt 1982. Second, large amplitude, lower "frequency" solitary waves break down into trains of smaller, rank-ordered solitons as they enter shallower water (e.g., Djordjevic and Redekopp 1978), so that there may be a transfer of energy to higher frequencies as the shore is approached. The additional possibility of transfer of energy to nearshore barotropic motion has been postulated (Huthnance 1981). Third, shear instability and gravitational overturning may generate significant turbulence in the interior of the water column, and at least some of that turbulent energy must eventually be dissipated, taken here to mean the loss of total

macroscopic energy. That internal waves break in part by gravitational overturning or collapse as the shore is approached has been demonstrated in the **laboratory** by Cacchione (1970), and Winant (1974) reports large internal bores of tidal origin propagating shoreward on the Southern California shelf. Finally, turbulent **mixing** of a stratified water column, however produced, must transfer some of the kinetic energy of the mixing mechanism to increased potential energy of the water column by reducing the stratification. This process may be important to shelf dynamics that depend on the stratification. Note that, of the four mechanisms for shoreward decrease of internal wave energy in a **particular** frequency band mentioned besides boundary **layer** turbulence, only the third is actually dissipative.

The amount of dissipation of internal wave energy in the bottom boundary layer may be important in determining where and how much of the water **column** over the shelf is mixed by the **incident** internal wave energy flux. Some of the boundary generated turbulence may initially go into the increased potential energy of a bottom mixed **layer**, **relative** to a stratified state, but once a bottom mixed layer is established stratification should not be very important to the internal wave boundary layer. Aside from possible generation of steady currents by second order processes, **all** internal wave energy not dissipated at the bottom (and possibly the surface) boundary **must** eventually go into turbulent mixing of the interior of the water column. Some of this energy is dissipated, but some is transferred into increased potential energy, as above.

A simple thought experiment illustrates some of the nature of the trade-off between boundary **layer** dissipation and interior mixing. It

may be shown that the excess potential energy (hereafter EPE) of a water column of depth D and constant stratification characterized by Brunt-Vaisala frequency N_2 over the potential energy of a water column of the same depth but stratification N_1 , where $N_2 < N_1$, is

$$EPE = \frac{\rho_0 D^3}{12} (N_1^2 - N_2^2) \quad (6.10)$$

If a wedge with constant bottom slope b , outside depth D_0 , an inside depth $= 0$ (i.e., the shoreline), and initial stratification N_1 is mixed so as to reduce the stratification to N_2 , then the total EPE of the wedge in its second state over the wedge in its initial state is

$$EPE_{tot} = \frac{\rho_0 D^4}{24} \frac{\bar{N} \Delta N}{b} \quad (6.11)$$

where $\bar{N} = (N_1 + N_2)/2$ and $\Delta N = N_1 - N_2$. If this EPE is all supplied by an incident internal wave energy flux E_f , then it will take $\Delta t = EPE_{tot}/E_f$ (time units) for this mixing to occur. For example, if $N_1 = 3$ cph, $N_2 = 2.5$ cph, $D_0 = 60$ m, $b = 0.02$, $E_f = 1$ joule/m/sec (similar to the CODE inner shelf), and all of E_f somehow goes into homogeneously mixing the wedge, $\Delta t = 32$ hrs. Focussing on the effect of bottom slope in eq 6.11, it is clear that lower slope will increase EPE_{tot} for the wedge. For a given internal wave energy flux at the shelf break, lower slope also will increase the efficiency of bottom dissipation, decreasing the amount of energy available for inner shelf mixing. In other words, for this hypothetical case lower slope both reduces the amount of energy flux available for mixing inshore of D_0 and increases the total energy necessary to mix up the inner

shelf. The nature of the **shelf** internal wave guide may determine not only the amount of internal wave energy dissipation in the bottom boundary layer, but **also** the nature and extent of **mixing** of the interior of the water column by the incident internal wave energy **flux**.

CHAPTER 7

SUMMARY AND CONCLUSIONS

Geophysical turbulent boundary layers are regions of complicated, inherently nonlinear flow, affected by forcing at different length and time scales, by density stratification, and by the response and uniformity of the boundary. Because boundary layer dynamics are nonlinear, the addition of any single complicating factor may affect processes at many different scales. For this reason, the bottom boundary layer on the continental shelf is a region of potentially complex, variable flow. Continental shelf bottom boundary layer processes are generally of first order importance to the overall dynamics of the shelf flow field, and may control patterns and amounts of sediment transport. Thus, the ability to reasonably predict the magnitude and variability of boundary stress and roughness on the shelf is both important and potentially very difficult. Major advances in understanding and predicting bottom boundary layer behavior have been made in recent years (e.g., for a comprehensive review see Nowell 1983), but the resultant models are only as good as the assumptions upon which they are based.

The present study has examined one possible complication to measuring and predicting characteristics of continental shelf bottom boundary layers that, until now, has not been considered formally. The theoretical analyses presented here have suggested that nonlinear boundary layer interactions involving internal waves may be significant, increasing the boundary shear stress felt by lower frequency ("steady")

currents and increasing the variability and decreasing the **reliability** of boundary layer measurements of drag coefficient and **roughness** using the common "log profile" technique, when internal waves are present. Levels of internal wave energy dissipation by bottom friction are also **increased** by **interaction**. "Steady" stress enhancement and contamination of boundary layer measurements are predicted to be increasingly important as internal waves become **large relative** to other current components. Intermittent, shoreward propagating packets of **large amplitude** high frequency internal waves often are observed on the continental **shelf** when the water column is stratified, and these waves may induce large fluctuations in bottom velocity (**e.g., Butman et al., 1979**). Intermittent, shoreward propagating mode 1 internal wave events have been shown here to dominate near bottom velocity fluctuations in the high frequency internal wave band during CODE I. Therefore, though it is not clear how to incorporate the intermittency of **large** internal waves into a **long** time or broad spatial average, it is clear that boundary layer interaction with **large amplitude** internal waves may be **locally** important during those times that internal waves are **present**, especially if **local**, short period measurements of boundary layer processes are to be extrapolated to represent **longer term** average behavior.

The general problem of interaction of internal waves and the bottom boundary layer on the continental shelf has been treated by viewing it from a number **of** different perspectives. First, Chapter 2 reviewed previous observations of characteristics of internal waves on the continental **shelf** and presented a method for **estimating** quantities of interest for the boundary layer interaction problem from a knowledge of

the approximate density structure and an estimate of wave height. Intermittent, large amplitude, shoreward propagating mode 1 internal wave trains were reported to be a common occurrence on the continental shelf, and cross-shelf fluctuations in bottom velocity induced by the passage of onshore propagating mode 1 internal waves were predicted to be the most important forcing for the bottom boundary layer in the high frequency internal wave band. Chapter 3 presented an analysis of the internal wave field during the CODE II experiment, concentrating on high frequency internal wave induced fluctuations in near bottom velocity. The wave field during CODE II was dominated by large amplitude, onshore propagating mode 1 events similar to observations in other locations, but having longer periods and occurring with less regularity. Predictions for the behavior of the CODE II waves from the theoretical analysis of Chapter 2 were tested against the CODE II data and found to be good, lending credence to the approximate methods developed in Chapter 2.

Chapter 4 developed a simple boundary layer model for the interaction of surface waves, internal waves, and a steady current in a neutrally stratified, fully rough turbulent boundary layer, and showed how the internal wave might lead to additional enhancement of the boundary shear stress felt by the steady current (referred to as dynamical effect). Wave frequencies, relative current component strength, geometry, and roughness representative of possible event occurrences during the CODE II experiment were used for example calculations. Although the assumption that all wave components are sinusoidal, steady state oscillations is a great simplification, especially for the internal waves, it is believed that the lowest order

features of the general boundary layer solution are predicted reasonably well. These features are the basic boundary layer length and time scales, the fundamental shape and direction of the total boundary layer velocity profile as a function of time, and the likelihood of enhanced turbulent mixing/momentum transfer/ boundary shear stress due to boundary layer interaction between forcing components. Details of the velocity profile and stress history, and second order quantities like wave induced mass transport, may not be predicted well for nonlinear forcing (e.g., Trowbridge 1983). The model must be applied with caution to transient situations like the first wave in a discrete wave packet. The full nonlinear, transient problem will probably require fully time dependent, numerical solution.

Chapter 5 considered the contamination of velocity profile boundary layer measurements in the presence of an internal wave boundary layer (referred to as kinematical effects), and predicted large variability in estimates of drag coefficient and roughness and some degradation of "log profile" fit. Predicted trends of drag coefficient variability were qualitatively consistent with observed trends in a short segment of the CODE I high frequency boundary layer experiment data known to contain internal waves, lending credence to one of the fundamental hypotheses of the boundary layer model of Chapter 4 - that the explicit addition of an internal wave to the boundary layer problem will introduce an additional length and time scale. Finally, Chapter 6 considered the problem of dissipation of internal wave energy in the bottom boundary layer on the continental shelf, and found that dissipation might be significantly enhanced by the simultaneous presence of surface waves and a steady

current interacting with large amplitude internal waves. The analysis showed that it is possible in general to anticipate the likely importance of a given continental shelf as a sink for internal wave energy by considering several basic characteristics of that shelf and its flow environment. It was pointed out in the discussion of Chapter 6 that the relative importance of bottom frictional dissipation of internal wave energy may in part determine the nature and extent of mixing of the interior of the water column by breaking internal waves.

The three component boundary layer model of Chapter 4 predicts significant enhancement of the steady shear stress in the presence of internal waves of magnitude comparable to or greater than the magnitudes of the other current components, assuming steady state waves. However, observations on the continental shelf indicate that large amplitude bottom velocity fluctuations due to internal waves are in fact quite intermittent, occurring only a fraction of the time at a given site. The true dynamical influence of internal waves will then be site specific, depending on the amount of time they are present, the amount of bottom affected, and the relative strengths of the other forcing components for the boundary layer. During the summer on the East Coast continental shelf, surface waves are weak and internal wave bottom velocities are one of the primary forcings for the bottom boundary layer (Butman, et al. 1979), so the dynamical effect of the internal waves may be important. During the CODE I experiment, internal waves were often weaker than either surface wave induced bottom velocities or "steady" currents. The strong events shown in Figure 3.7 represent an extremum of the 10 week period analysed. Therefore, it is unlikely that internal

waves were a major dynamical influence during CODE I. In any case, the models developed here are as valid for situations like that reported by Butman et al. (1979) as they are for the CODE I situation.

One obvious question to come out of the present study is whether internal wave characteristics for a given shelf location and season may be anticipated or predicted, especially with regard to their possible influence in the bottom boundary layer. The answer to the question of prediction must be "usually not" at the present time. If the statistical characteristics of the wave field are desired, or even average behavior of groups of topographically generated waves, there are some instances in which the behavior of the internal wave field is predictable simply because the waves are well documented and clearly due to some regular generating mechanism like the semidiurnal tide. This may be the case for several East Coast shelf locations; the wave packets reported by Apel et al. (1975) and Butman et al. (1979) have an average separation that is clearly tidal, and Trask and Briscoe (1983) report that wave packets generated by tidal flow over Stellwagen Bank in Massachusetts Bay are actually fairly predictable. However, in general the details of the generation mechanism and the subsequent propagational characteristics of internal wave packets are not understood well enough to say that at a given location and time, a certain number of waves of given amplitudes will be present, which is the prediction of importance to boundary layer processes. It is possible to anticipate the maximum possible influence of internal waves for a particular location and density structure using the results of Chapter 2. That is, if it is known that large amplitude internal waves are likely to be present at

some time, and the vertical density structure is reasonably stable and known, it is possible to make a **simple** estimate of the maximum internal wave bottom velocity by assuming a maximum wave height and using the formulae developed in Section 2.4.

Lower frequency fluctuations (tidal and inertial) have been **s**lighted in the present study in favor of higher frequencies, mainly because it is **uncertain** whether the present theoretical boundary layer problem formulation is extendable to very **low** frequencies. The solution for internal waves ignores rotation, treating the boundary layer as purely oscillatory; while this is a valid **simplification** for the high internal wave frequencies, it is clearly not valid for tidal and inertial oscillations. Inclusion of both oscillation and rotation is possible within the present framework, but the additional complication is of questionable worth if the oscillatory boundary layer scaling used here is not valid at low frequencies. The assumptions that a **low** frequency boundary layer **will** behave as an oscillatory boundary layer and that constant oscillatory boundary layer scaling may be used at low frequencies are untested, though there is no apparent reason to doubt their validity for predicting **lowest order** quantities. Boundary layer solutions for **high** frequency surface waves based upon a constant scale height have been found to be reasonably accurate for predicting the first order velocity profile and boundary shear stress (Kajiura 1964, Grant 1977, Long 1981, Brevik 1981, Trowbridge 1983), and predicted effects of boundary layer interaction between a steady current and **high** frequency waves assuming constant scaling have been verified in the **field** (Cacchione and Drake 1983, Grant et al. 1984, Wberg and Smith

1984). At this point, there is simply no high quality data available for low frequency oscillatory boundary layer flows that are not constrained by topography or depth.

There are several interesting possibilities for further research that are suggested by the present results. First is a more detailed **theoretical** analysis of the oscillatory boundary layer problem, **allowing** for **nonlinear**, transient forcing and/or a fully time varying eddy viscosity, which may be accomplished by use of higher order turbulent closure models or numerical time dependent eddy viscosity **formulations**, and an investigation of possible theoretical frequency dependence of wave-current interaction. In fact, this problem is **already** being **tackled**; a time dependent wave-current model using 2nd order turbulence closure techniques is being developed by R. L. Soulsby, and W. Grant is presently working on developing a time dependent eddy viscosity scale height **formulation** (W. Grant, personal communication). Second, **it** may be interesting to investigate the nature of the internal wave field at the CODE site during the winter months. **As** mentioned in the discussion of Chapter 3, the presence of large nonlinear events at the CODE site may depend upon the existence of favorable **generation** conditions at the **shelf** edge. Favorable generation conditions may depend upon the **low** stratification during the summer, hence the **relatively** strong stratification during the winter months may preclude the presence of large amplitude intermittent internal events. Third, **it** may be possible to do a fairly simple experiment to better determine the bottom boundary layer response to the passage of a **well** defined train of **internal** waves, simply by choosing a site better suited to such an observation than the

CODE site. For example, the internal wave packets in Massachusetts Bay reported most recently by Trask and Briscoe (1983) are reasonably predictable and often quite well defined. Finally, **it** should be possible to use the present model to predict the **likelihood** of sediment motion under **combined** internal waves and tidal currents on the mid-Atlantic bight during the summer months, as suggested by **Butman et al. (1979)**, and to incorporate those predictions into a rough consideration of the patterns and amounts of resultant sediment transport.

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APPENDIX A

INTERNAL WAVE MODEL CALCULATIONS

In this Appendix the details of the model calculations that provide the basis for the theoretical analysis of Chapter 2 are presented. The models are either known solutions or simple extensions of known solutions; they are presented for clarity and reference.

For the **linear** solutions, assume unidirectional waves propagating in a horizontally homogeneous, inviscid, nonrotating wave guide with rigid top and bottom boundaries. The form of the wave is

$$\eta = a \phi(z) \exp i(kx - \omega t) \quad (\text{A.1})$$

where η is the local streamline deflection, a is the maximum wave amplitude, $\phi(z)$ is the vertical structure function, k is the horizontal wave number, and ω is the radian frequency. Note that, for all **linear** solutions, a variable retains its significance once defined.

Defining w as the vertical velocity and N as the local Brunt-Vaisala frequency ($N^2 = -g/\rho_0 \cdot (\partial\rho/\partial z)$), the **linear** governing equation may be written (eq. 2.5):

$$\frac{d^2 w}{dz^2} + \frac{(N^2 - \omega^2)}{\omega^2} k^2 w = 0 \quad (\text{A.2})$$

Solutions for the various density structures differ only in the appropriate values of N and in the boundary conditions applied. A WKB scaling for the vertical structure is not attempted, since the lowest internal mode is of most interest, and all of the density structure changes are contained within half of a vertical wavelength.

For each **linear** case, the following expressions are presented:

- a.) The dispersion **relation**
- b.) The kinematical **relation** between maximum first mode amplitude and bottom velocity amplitude
- c.) The vertically integrated energy density
- d.) Group **velocity**, $c_g = \partial\omega/\partial k$

The distinguishing boundary conditions are also presented.

As Case 1, consider a constant, stable density gradient from top to bottom, so N is constant throughout the water column to the accuracy of the Boussinesq approximation. Then the only two boundary conditions are $w = 0$ at $z = 0$ and $z = D$ (see Figure A.1 for definition of variables for linear solutions). The solutions are:

Dispersion relation -

$$\frac{m}{k} = \frac{n\pi}{kD} = \left[\frac{N^2 - \omega^2}{\omega^2} \right]^{0.5} \quad (A.3)$$

where m is the vertical wave number and n is the (integer) mode number

Kinematical relationship

$$a = \frac{u_{bi}}{\omega} \cdot \frac{k}{m} = \frac{u_{bi}}{\omega} \left[\frac{\omega^2}{N^2 - \omega^2} \right]^{0.5} \quad (A.4)$$

where u_{bi} is the amplitude of the bottom velocity.

Vertically integrated energy density

$$E_{tot} = \frac{\rho_0}{4} u_{bi}^2 D \left[\frac{N^2}{N^2 - \omega^2} \right] \quad (A.5)$$

Group velocity

$$c_g = \frac{ND}{n\pi} \left(1 - \frac{\omega^2}{N^2} \right)^{3/2} \quad (A.6)$$

Figure A.1: Definition of geometry and variables for linear solutions.
 (a) - Case 1, Constant N ; (b) - Case 2, symmetrical mixed layer structure; (c) - Case 3, general two layer structure; (d) - Case 4, asymmetrical mixed layer structure.

As Case 2, take a symmetrical three-layer density structure, with equal height top and bottom mixed layers and a constant value of N in the interior. The boundary conditions are:

$$w = 0 \text{ at } z = 0 \text{ and } z = D$$

$$w_{\text{upper}} = w_{\text{lower}} = \frac{\partial \eta}{\partial t} \text{ at each interface} \quad (\text{A. 7})$$

and

$$\frac{\partial w_{\text{upper}}}{\partial z} = \frac{\partial w_{\text{lower}}}{\partial z} \text{ at each interface}$$

The solutions are:

Dispersion relation -

for odd modes (1, 3, ...),

$$\frac{m}{k} \tanh kH = \cot mh \quad (\text{A.8})$$

and for even modes (2, 4, ...),

$$\frac{m}{k} \tanh kH = \tan mh \quad (\text{A.9})$$

Kinematical relationship -

$$a = \frac{u_{bi}}{\omega} \frac{k}{m} = \left(\frac{\cosh kH}{\sin mh}, - \frac{\cosh kH}{\cos mh} \right) \quad (\text{A.10})$$

For (odd, even) modes

Vertically integrated energy density -

$$E_{\text{tot}} = \frac{\rho}{2} u_{bi}^2 \left[\left(\frac{N^2}{N^2 - \omega^2} \right) \frac{\cosh^2 kH}{(\sin^2 mh, \cos^2 mh)} \cdot h + \left(\frac{N^2}{N^2 - \omega^2} \right) \frac{\sinh \frac{2kH}{2}}{2k} \right] \quad (\text{A.11})$$

for (odd, even) modes.

Group velocity must be calculated numerically.

As Case 3, take the classic 2 layers of neutral stratification with a density jump at the interface = $\Delta\rho$. The additional boundary conditions at the interface are:

$$w_u - w_b = \frac{\partial \eta}{\partial t}, \quad \omega^2 \left(\frac{\partial w_u}{\partial z} - \frac{\partial w_b}{\partial z} \right) = \frac{g}{\rho_0} \Delta\rho \frac{\partial^2 w}{\partial x^2} \quad (\text{A.12})$$

The solutions are:

Dispersion relation -

$$\omega^2 = \frac{g \Delta\rho}{\rho_0} \frac{k}{\coth kH_u + \coth kH_b} \quad (\text{A.13})$$

Kinematical relationship -

$$a = \frac{u_{bi}}{w} \sinh kH_b \quad (\text{A.14})$$

Vertically integrated energy density -

$$E_{\text{tot}} = \frac{1}{2} \rho_0 u_{bi}^2 \frac{\sinh^2 kH_b}{\omega^2} \frac{g \Delta\rho}{\rho_0} \quad (\text{A.15})$$

Group velocity -

$$c_g = \frac{1}{2} \frac{\omega}{k} \left(1 + \frac{kH_u \sinh^2 kH_b + kH_b \sinh^2 kH_u}{\sinh kH_u \sinh kH_b \sinh kD} \right) \quad (\text{A.16})$$

As Case 4, take an asymmetrical **three-layer** density structure, with upper mixed layer height H_u , interior of constant N and thickness H_I , and lower mixed layer of height H_b . The boundary conditions are the same as for Case 2. The solutions are:

Dispersion relation -

$$\tan mH_I = \frac{mk (\coth kH_u + \coth kH_b)}{m^2 - k^2 \coth kH_u \coth kH_b} \quad (A.17)$$

Kinematical relationship -

$$a = \frac{u_{bi}}{\omega} \left(\frac{1 + k^2 \coth^2 kH_b}{m^2} \right)^{1/2} \sinh kH_b \quad (A.18)$$

Vertically integrated energy density is not calculated, and group velocity must be calculated numerically.

Finally, a specific form of the solitary wave solution due to Benjamin (1966) is calculated as Case 5. The problem geometry and variables are defined in Figure A.2. Briefly, η is the undisturbed streamline coordinate, y is the disturbed streamline, $\epsilon \zeta$ is the wave height, ϕ_n is the vertical structure function, f is the wave form, and ϵ is a small parameter related to wave steepness. Thus,

$$\zeta = f(X) \phi_n(\eta) \text{ , where } X = \epsilon^{1/2} x \quad (A.19)$$

The solution for ϕ_n is the same as the solution for the long wave, no rotation limit of the equivalent linear problem.

The governing equation for f is:

$$I \left(\frac{df}{dX} \right)^2 = Jf^2 - Kf^3 \quad (A.20)$$

where

$$I = \int_0^D \rho C^2 \phi_n^2 d\eta = \rho_0 C^2 I_0 \quad (A.21)$$

$$J = \int_0^D \frac{\rho (C^2 - C_n^2)}{\epsilon} \left(\frac{d \phi_n}{d \eta} \right)^2 d \eta = \frac{\rho_0 (C^2 - C_n^2)}{\epsilon} J_0 \quad (\text{A.22})$$

$$K = \int_0^D \rho C_n^2 \left(\frac{d \phi_n}{d \eta} \right)^3 d \eta = \rho_0 C_n^2 K_0 \quad (\text{A.23})$$

where C is the solitary wave speed and C_n is the linear wave speed. The solution for $\epsilon \zeta$ is a transcendental equation -

$$\epsilon \zeta = y - \eta = \frac{\phi_n(y)}{\phi_{n \max}} \cdot A \cdot \text{sech}^2 \left(\frac{x}{T} \right) \quad (\text{A.24})$$

$$\text{where } A = \left(\frac{C^2 - C_n^2}{C_n^2} \right) \frac{J_0 \phi_{n \max}}{K_0} \text{ is the maximum wave height} \quad (\text{A.25})$$

$$\text{and } l = 2 \left[\left(\frac{C^2}{C^2 - C_n^2} \right) \frac{I_0}{J_0} \right]^{1/2} \text{ is a wavelength parameter} \quad (\text{A.26})$$

The horizontal velocities must be solved for iteratively, except at the boundaries, where $\eta = y$ and

$$u(y=0) = - C \Delta \text{sech}^2 \left(\frac{x}{T} \right) \quad (\text{A.27})$$

$$u(y=D) = \frac{C \Delta a}{\phi_{n \max}} \text{sech}^2 \left(\frac{x}{T} \right) \quad (\text{A.28})$$

where

$$a = \frac{HB}{H_u} \cdot \cos m H_I + \frac{1}{m H_u} \sin m H_I \quad (\text{A.29})$$

Note that $u(y=0)$ is in the direction of wave propagation for a wave

Figure A.2: Definition of geometry and variables for solitary wave solution.

of elevation ($A > 0$); it is written as negative because the problem was formulated for a wave propagating in the $-x$ direction. The integrated energetics are not calculated and group velocity is equal to phase velocity, c , for a (long) solitary wave.

APPENDIX B:

CALCULATIONS BASED ON TEMPERATURE MEASUREMENTS AT SITE C3

Introduction: The temperature data used in the present calculations are **from** VMCM current meters deployed on two moorings at site C3 during CODE I. The meters were deployed by two **groups** at Scripps, C. **Winant's** (denoted by the prefix **W**) and R. Davis' (denoted by the prefix **D**). They were arranged as in Table B.1, five on the surface mooring **C3A** and seven on the subsurface mooring **C3S**, which were separated by about 2 km. The data from C3A are incoherent with the data from C3S in the internal wave frequency band, though slower (subinertial) fluctuations are usually coherent. The temperature measurements from the **D** meters were contaminated by noise during the **A/D** conversion process, so that the data cannot be used for high frequency analysis. In most cases the noise is adequately **removed** by filtering, so that the data can be used in determining **slower** temperature variations. Details of the data selection and adjustment procedures used follow in the next section.

Two types of information are desired from the available data. The first is the slow time and space variation of the low frequency (subinertial) background temperature field. An implicit assumption is that the temperature gradient field is related to the density **gradient** field in a roughly constant way. This assumption is probably good, **especially** over time periods of a **week** or two (J. Irish, personal communication), and is discussed further at the end of **this** appendix. Experience indicates that a **common** structure is surface and bottom mixed

Current Meter Number	Depth (m)	Height (m) Above Bottom	Trend Correct. (°C/day)	Total Offset Trend Correct.	Average Temp (°C)	Absolute Error (± °C)	Comment	Δθ Mean (°C)	ΔZ (m)	Percent error in Δθ/ΔZ	E _{inertial} final (m)
M64	4	86	0	+ 0.07	8.71	0.01					
D04	9	81	0	+ 0.03	8.68	0.03	not used	0.12	10	27	1.1
M50	14	76	0	+ 0.11	8.59	0.01		0.12	10	27	1.1
M53	24	66	0	+ 0.07	8.47	0.01		0.07	5	67	4.3
D02	29	61	-0.0006	- 0.22	8.40	0.03					
D03	35	55	-0.001	- 0.01	8.33	0.03					
M61	39	51	0	+ 0.06	8.28	0.01	2	0.05	4	90	(16) 5
D11	45	45	-0.00075	+ 0.23	8.21	0.03		0.07	6	67	5.2
M59	55	35	0	+ 0.07	8.12	0.01		0.09	10	54	4.9
D06	65	25	-0.00025	- 0.01	8.05	0.03	not used	0.14	20	24	(1.9) 3.8
M66	75	15	0	+ 0.04	7.98	0.01					
D15	83	7	- .0004	+ 0.16	7.93	0.03		0.05	8	90	(32) 10

1. Constant offsets from CTD corrections are: + 0.08 (C3A); + 0.05 (C3S)
2. Used for internal wave band isotherm **estimation**

Table B.1: Results of temperature readjustment and mean error estimates for temperature data from moorings C3A and C3S

layers with a stratified interior region. The existence of mixed layers is important to questions of boundary layer stratification, local mixing vs. advection, and the relationship of interior internal wave energy to bottom HKE. Therefore, the procedure followed is to first calculate and store the subinertial temperature gradient field for future reference, then to use this field to obtain an estimate of the best fit to a mixed layer/stratified interior model. The second kind of information desired is an estimate of isotherm fluctuations in the internal wave frequency band from high-pass filtered temperature fluctuations, using the derived gradient field. The rest of this appendix describes adjustment and selection of usable temperature time series, calculation of the background gradient field, calculation of a best fit mixed layer/constant stratification interior structure, calculation of internal wave band isotherm height time series, and calculation of estimated error for each of these quantities.

Adjustment and selection of usable temperature time series:

Examination and intercomparison of temperature records indicates that the usable accuracy of the Scripps VMCM temperature measurements may be improved over the nominal calibration accuracy by careful adjustment based on operational characteristics and physical constraints to be described below.

In addition to the A/D noise problems of the D meters, which lead to a greater uncertainty in calibration, the D temperature circuitry may have been subject to long term drift during CODE I. The W meters did not have noise problems, nor were they likely to be subject to much drift (the maximum observed difference from pre- to post-cruise

calibration with the same circuitry in CODE II was 0.03°C). These considerations lead to the following initial readjustment criteria. A least squares linear trend is calculated thru all of the temperature records from C3, covering the ten week period of April 12 to June 21, 1981. Then, since the relative temperature is most important, the D trend slopes are adjusted to fall on an interpolation curve fit (by eye) to the W trend slope values. Two types of temperature plots are then produced with the necessary D slope adjustments subtracted out; these are ten week plots of subinertial temperature (low-pass running mean filtered at ten hrs and decimated at 1.67 hrs) and two week time series of hour averaged temperature from 26 April to 10 May, when OSU CTD coverage at C3 is good and generally reveals a stable temperature structure and a well developed pattern of mixed layer heights (Fleischbein et al. 1982).

Interior temperatures are adjusted relative to each other using the two week temperature time series, subject to three constraints. Offsets are determined by a compromise between avoiding temperature inversion as much as possible, mirroring as closely as possible the measured (CTD) mixed layer structure, and avoiding adjustment of the W meters by more than 0.1°C (A. Batkovich, private communication). These offsets are checked with a stricter non-inversion criterion using the ten week time series. At site C4, J. Irish has used CTD comparison to adjust temperature chain data with calibration problems, and has found that the low frequency temperature structure is indeed highly stable (CODE I: Moored Array and Large Scale Data Report, 1983). The time series at the top of C3 is not required to fit the time series at the bottom of CA

due to the separation of the moorings. The records from meters D04 (at 9 m depth) and D06 (at 65 m. depth) are excluded as containing physically unreasonable fluctuations relative to neighboring records.

Finally, the entire suite of OSU CTD casts at C3 is used to adjust the absolute temperatures of the uppermost and lowest records. Temperatures at 4 m depth and 7 m above the bottom are taken from only those CTD casts that reveal respective mixed layers at least 15 m thick and compared to the corresponding subinertial temperature plots. The current meter records are adjusted to give zero mean difference. It is believed that thick surface and bottom mixed layers reduce the likelihood of spatial temperature variation and buffer the instantaneous CTD temperatures from the effect of higher frequency fluctuations. This is not to deny the possibility of real differences in temperature between CTD and current meter, but is the most consistent way to reduce the differences.

The results of the temperature readjustment are presented in Table B.1. The necessary relative offsets are extremely close to those derived without correcting for drift by A. Dorkins at Scripps, lending credence to the procedure.

Calculation of the background gradient field: Adjusted, subinertial low-pass filtered temperature time series from each mooring are fit with a cubic spline in the vertical dimension at each time step, assuming that the bottom or surface current meter is in a mixed layer so that the gradient is zero at each boundary, and that the curvature is zero in the middle of the water column for each mooring. To the extent that a vertical density structure model with boundary mixed layers and

an interior with constant stratification fits the data (e.g., Figure 2.1a) these assumptions are reasonable. Errors resulting from these assumptions are minimal a short distance from the end points, and the use of assumed boundary conditions ensures that the fit in the center of each mooring string is as independent of the boundaries as possible, given the small number of data points in the vertical. The resultant temperature gradient at each input data point is stored for future reference.

Calculation of the best fit mixed layer structure: Using the stored matrix of low-pass filtered temperature gradient vs. depth, the following calculation is made at each time step. A linear variation in temperature gradient between current meters is assumed. Starting at each boundary and working toward the interior, the gradient profile is integrated over depth, giving the local total change in temperature relative to the boundary. The first criterion for setting the mixed layer height is that the total temperature change within the mixed layer must be less than some fixed value. The second criterion is that the gradient in the mixed layer never exceed a fixed fraction of a representative interior gradient. The depth averaged gradient over the center-most three points is used as an initial representative gradient; the corresponding mixed layer heights and depth-averaged gradient over the interior region are then calculated; this new depth averaged gradient is then used as a representative gradient and the calculation is repeated. After all time steps have been completed, the mixed layer height and interior gradient time series are resmoothed by performing a ten hr. running mean on each (20 hr. halfpower point lowpass filter).

The final criteria for a mixed layer are defined to be a total temperature change of not more than 0.05°C and a local gradient of not more than 0.5 times the representative interior gradient. Experimenting with the calculation shows that the results are fairly insensitive to the precise values chosen, and the above values seem to give the "best" agreement with low-passed isotherm contour plots. In any case, the definition of a mixed layer height must be somewhat subjective, depending on the exact criteria used for the definition of its extent.

Calculation of internal wave band isotherm height time series: The equation relating temperature fluctuations in the internal wave band to isotherm height fluctuations, and assumedly to isopycnal fluctuations, is:

$$\eta(t) = \frac{\theta(t)}{\left(\frac{\partial\theta}{\partial z}\right)_{\text{ave}}(z, \text{slow } t)} \quad (\text{B.1})$$

where θ is the high-passed (less than ten hr. period) temperature signal from a particular current meter and $\left(\frac{\partial\theta}{\partial z}\right)_{\text{ave}}$ is an appropriately averaged space (vertical dimension) and slow time varying background temperature gradient at that current meter height. The high-passed temperature time series is from current meter W61 at 51 m above the bottom, or roughly in the center of the water column. An appropriate space and time varying background gradient is constructed from three depth averaged, subinertial temperature gradient time series representative of the average gradient in the upper, middle and lower thirds of the interior of the water column. It has been found by experimentation that depth averaging over three sets of three current

meters provides the most stable estimate of temperature gradient for the present purpose, while still preserving the essential vertical structure. The groups are: upper - W50,W53,D02; middle - D03,W61,D11; lower - D11, W69,W66.

An initial estimate of isotherm height is obtained by dividing the high-passed temperature time series by the center gradient alone. Assuming a linear variation in gradient between the middle and extreme values and assuming that the extreme values extend to the boundaries, an average gradient between the center and the initial position estimate is calculated, which is then used to calculate a second isotherm height estimate. This procedure is carried out at each time step. It should be pointed out that a stable, noise-free isotherm time series results only if the gradient field has been sufficiently smoothed, wherefore the elaborate calculation scheme.

Estimation of error - procedure: The formulae used to calculate the worst case errors in estimated temperature gradient and estimated isotherm height are: Temperature gradient between two current meters-

$$\frac{\partial \theta}{\partial Z} \sim \frac{\Delta \theta}{\Delta Z} [1 \pm (\theta_r + Z_r)] \quad (B.2)$$

where $\Delta \theta$ is the measured temperature difference, ΔZ is the nominal difference in height, θ_r is the total fractional error in temperature, and Z_r is the total fractional error in height. Set $Z_r = 0.1$ (rather arbitrarily) and calculate θ_r by

$$\theta_r = \frac{E_{\theta, \text{upper}} + E_{\theta, \text{lower}}}{\Delta \theta} \quad (B.3)$$

where E_{θ} is the absolute error in temperature at the respective

current meter. Thus the total estimated error in temperature gradient is

$$\bullet \frac{\Delta\theta}{\Delta Z} \left(\frac{E_{\theta, \text{upper}} + E_{\theta, \text{lower}}}{\Delta\theta} + 0.1 \right) \quad (\text{B.4})$$

Using the formula for isotherm height presented previously, (B.1), the worst case error in the subinertial isotherm height can be written as

$$E_{\eta} = E_{\theta} \left[\frac{\Delta\theta}{\Delta Z} \left(1 - \frac{E_{\theta, \text{upper}} + E_{\theta, \text{lower}}}{\Delta\theta} - 0.1 \right) \right]^{-1} \quad (\text{B.5})$$

For the error in high-passed isotherm height, replace E_{θ} by P_{θ} , the precision error of the respective current meter. For current meter W61, $P_{\theta} = 0.004^{\circ}\text{C}$ (CODE I: Moored Array and Large Scale Data Report). Also replace $(E_{\theta, \text{upper}} + E_{\theta, \text{lower}})/\Delta\theta$ by some averaged measure of fractional error in keeping with the averaging schemes used in the internal wave band calculation procedure.

All values of $\Delta\theta$ and ΔZ are based on the average temperature profile for the ten week period and the nominal current meter heights (see Table 8.1).

Estimation of error - Instrumental error: Although the nominal accuracy of the Scripps VMCM temperature records from laboratory calibration is 0.1°C , the adjustment technique described above is believed to be robust enough to substantially reduce the absolute error in a usable sense, although it is not reasonable to make the error bars smaller than the precision of the instrument. The adjustment procedure has led to the conclusion that shifting the W temperature records by more than 0.01°C in either direction from their adjusted level produces

unacceptable discrepancies; this is taken as the new estimated absolute error for the W instruments. The adjustment error for the D instruments is apparently only slightly larger, but the precision of the instruments is 0.03°C ; this is taken as the new estimated absolute error for the D instruments. The precision of the W meters is 0.004°C , which is used in determining high-passed isotherm height error bars.

Estimation of error - results: The calculated errors between individual current meters are presented in Table B.1. Some representative average values are: for the interior region (without D15 and W64, which are assumed to be in their respective mixed layers), the average percent error in temperature gradient is 60 percent; the average temperature gradient is 0.01°C per meter; the average estimated error in low-passed isotherm height is ± 4 m; and based on these values, the error in high-passed isotherm height is ± 1 m. Including the extreme current meters makes virtually no difference in the estimated interior errors. Discussions of individual error estimates follow.

Within a mixed layer, the average gradient in temperature should be very low, so the uncertainty in location for any isotherm is very high; however, in establishing mixed layer height variations we are interested in the height of the isotherm just above the top of the mixed layer. Therefore, the uncertainty in mixed layer height is taken to be only slightly larger than the uncertainty in interior isotherm height, or about ± 5 m on average. When the interior gradient is lower the uncertainty is higher, and vice versa.

The percentage error in a depth averaged gradient is smaller than the percentage error between any two current meters, since its

calculation cannot be considered as a worst case; **i.e.**, an over-estimation of gradient between one pair of current meters will be necessarily balanced by an underestimation between neighboring pairs. Thus, though the method of calculation is not the same, the temperature difference between uppermost and **lowest** current meters can be used to estimate average gradient error. This estimation gives about 15 percent to 20 percent average error. Again, the error varies inversely **with** the magnitude of the gradient.

The error calculations indicate an uncertainty of ± 1 m in the high-passed isotherm height time series. This is probably a good estimate for the higher frequencies, except when in the presence of the strained density **field** of a very large **lower** frequency internal wave, when any additional uncertainty would be very difficult to estimate. The **lowest** frequency internal waves (tidal to inertial) may have a greater **uncertainty** due to the imperfect nature of splitting the density field into background and internal wave components; the presence of a large nonlinear internal wave will have a measureable effect on the background field, since straining due to its passage will not average to zero.

The **formulae** presented assume a **linear** variation in temperature gradient between instruments, which is probably valid locally in the interior and mixed **layers**, but becomes doubtful as the instrument separation increases. Regions of substantial expected curvature, such as the neighborhood of the mixed **layer** edges, are particularly suspect. This is unfortunately the **location** of the largest vertical separation between current meters (**W66-W69**; 20 m); the error estimate from this

current meter pair is thus doubled. Note that the formulae presented are worst case estimates based on average values. They do not account for improved estimates of low-passed temperature gradient due to the incorporation of neighboring data through a spline fit, nor do they account for time variations in the background field. Thus, when the gradient is higher than average, the error is lower, and vice versa. Finally, error bars larger than instrument spacing are limited to approximate instrument spacing, since there is a singularity in the error calculation formula as the average temperature difference between two instruments approaches the total possible error.

Discussion - Correspondence between temperature gradient and density gradient: J. Irish at UNH has compared time varying temperature to time varying density for the C-T chain at site C4 during CODE I and found a close to linear relationship with a slope that varies slowly over time. Occasional lapses from this simple behavior probably are associated with the passage of fronts (e.g., during relaxation events; J. Irish, personal communication). It seems reasonable to assume that this behavior will be replicated inshore at C3, giving a slowly varying, locally constant relationship between temperature gradient and density gradient. In any case, there are insufficient data to check the behavior at C3 directly. The timing and magnitude of the inevitable lapses at C3 relative to C4 are open to question. For the present purpose, it is assumed for simplicity that there is a non-varying constant relationship between temperature gradient and density gradient. This assumption should not affect calculations of relative isotherm (isopycnal) heights; it will enter into calculations which

include an estimate of Brunt-Vaisala frequency from a temperature gradient as slow variations about an expected value with occasional large errors, as in Figure 3.9.

APPENDIX C

STATISTICAL CALCULATION PROCEDURES

Procedural detail: This appendix presents the details of the calculation procedures followed in deriving auto- and cross-spectral time series for the analysis of Chapter 3. Procedures followed and techniques used are standard, but details and choices of parameters are unique to the present work, and are presented for reference.

The superinertial internal wave spectrum is split into four frequency bands, which are numbered from high frequency to low. Band 1 is 0.5 to 2.0 cph, covering the neighborhood of the "knee" in the ten week average spectra; band 2 is 0.2 to 0.5 cph; band 3 is 0.1 to 0.2 cph, including the first harmonic of the semidiurnal tide; band 4 is 0.067 to 0.1 cph, and is essentially tidal. Bands 3 and 4 have been designed to have a spectral line as close as possible to 6 and 12 hrs, respectively. Lengths of sample data records are chosen to be as small as possible, with the caveat that there be at least one averaged resolved frequency in the frequency band of interest. Each record is a power of two long for fast processing using an in-house integer FFT routine designed by K. Prada. The sampling interval is chosen as the next smallest even fraction or multiple of one day, for ease of later comparison, so that succeeding energy points are based on data records that overlap by between 20 percent and 40 percent. Table C.1 summarizes the processing parameters for each frequency band.

Data records are prepared for fourier transformation by bandpass filtering and application of a cosine bell to 5 percent of the points on

Frequency Band No.	Frequency Range/ Period Range	Filter Bandwidth	Record Length	Sample Interval	Degrees of Freedom	No. of Points in Energy Time Series	Filter Correction Factor
1	0.5-2 cph/ 0.5-2 hrs.	0.5-3.75 cph	8.53 hrs.	6 hrs.	20	280	1.76
2	0.2-0.5 cph/ 2-5 hrs.	0.2-1.88 cph	34.13 hrs.	24 hrs.	20	70	2.38
3	0.1-0.2 cph/ 5-10 hrs.	0.1-0.54 cph	59.73 hrs.	48 hrs.	10	35	2.89
4	0.067-0.1 cph/ 10-15 hrs.	0.067-0.54 cph	119.5 hrs.	96 hrs.	10	17	3.37

Table C.1: Processing parameters for spectral time series calculations

either end of the record. It has been found that the low frequency halfpower point must be equal to the low frequency limit of the band of interest to avoid leakage of information from lower frequencies; this is because the spectrum is "red" (discussed in section 3.4). Lowpass filtering is done with a running mean, decimating at half the averaging period. Highpass filtering is done by subtracting an undecimated running mean. The running mean filter is not the most efficient, but is easy to implement, intuitive, and leaves the phase of the signal unaffected (course notes, D. Aubrey).

After Fourier transformation, the following calculations are carried out on each pair of records (i.e., η - u , η - v , and u - v) from a given frequency band and time period classification. Calling the two complex valued Fourier transforms $X(f)$ and $Y(f)$, at each frequency

$$\begin{aligned} S_{xx} &= \text{Re } (X)^2 + \text{Im } (X)^2 \\ S_{yy} &= \text{Re } (Y)^2 + \text{Im } (Y)^2 \\ CO_{xy} &= \text{Re } (X) \text{Re } (Y) + \text{Im } (X) \text{Im } (Y) \\ QU_{xy} &= \text{Im } (X) \text{Re } (Y) + \text{Re } (X) \text{Im } (Y) \end{aligned} \tag{C.1}$$

where S_{ij} is the autospectrum, CO_{ij} is the cospectrum, and QU_{ij} is the quadrature spectrum (Kanasewich 1975), Chapt. 10). The raw records are smoothed by frequency band averaging, corrected for tapering effects, and divided by the fundamental frequency interval in Hz , to produce energy density estimates. The coherence (sometimes called