WHOI - 80-62

COPY 1

Woods Hole Oceanographic Institution

CRITICAL LAYERS AND THE GARRETT-MUNK
SPECTRUM

by

Barry Ruddick

November 1980

TECHNICAL REPORT

*Prepared for the Office of Naval Research
under Contract N00014-76-C-0197; NR 083-
400.*

*Approved for public release; distribution
unlimited.*

WHOI-80-62

CRITICAL LAYERS AND THE GARRETT-MUNK SPECTRUM

by

Barry Ruddick

WOODS HOLE OCEANOGRAPHIC INSTITUTION
Woods Hole, Massachusetts 02543

November 1980

TECHNICAL REPORT

*Prepared for the Office of Naval Research under Contract
N00014-76-C-0197; NR 083-400.*

*Reproduction in whole or in part is permitted for any
purposed of the United States Government. In citing this
report in a bibliography, the reference given should be
to the "Journal of Marine Research, Volume 38, 1, 1980,
pp. 135-145".*

Approved for public release; distribution unlimited.

Approved for Distribution: Valentine Worthington, Chairman
Department of Physical Oceanography

Critical layers and the Garrett-Munk spectrum¹

by Barry Ruddick²

ABSTRACT

The effects of critical level absorption of oceanic internal waves by a mean flow are estimated using the Garrett and Munk (1975) model spectrum. The horizontal currents of the wave field are found to be more intense perpendicular to the mean flow than parallel to it. The cause of this anisotropy is preferential absorption of waves travelling with the mean flow. However, the current anisotropy is only half as large as would be necessary to explain Frankignoul's (1974) observations. The wave momentum flux lost to critical level absorption is found to be nearly proportional to the mean velocity. When the momentum flux is deposited throughout a 400 m thick shear zone, typical of the main thermocline in the North-west Atlantic, the observed stress-shear relationship would correspond to a wave-induced eddy viscosity of $-200 \text{ cm}^2 \text{ s}^{-1}$. The effect of the absorbed momentum on the mean flow is to cause a slow (5 m/day) downward phase propagation and slow broadening of the shear profile.

1. Critical layers and the Garrett-Munk spectrum

Critical layer absorption (Booker and Bretherton, 1967) is thought to be a potentially important mechanism of interaction between the oceanic internal wave field and low-frequency ("mean") currents. (Critical level absorption is the enhanced absorption of a wave which occurs when the intrinsic frequency $\omega_o = \omega - \mathbf{k} \cdot \bar{\mathbf{u}}$ of a wave becomes equal to $\pm f$. For weak mean shear, or large Richardson number as assumed in what follows, the wave is thought to be completely absorbed at a critical level and the impulse of the wave given up to the mean flow. At small Richardson number, some of the incident wave action is re-radiated in the form of harmonics. See Fritts, 1979). There have so far been no quantitative estimates of the effects of such an interaction on either the wave field or the mean flow. The Garrett and Munk (1972) series of model internal wave spectra provide the analytical expressions necessary for such estimates. The two most recent versions, GM75 (Garrett and Munk, 1975) and GM76 (Desaubies, 1976), have successfully modelled both the vertical and horizontal wave number structure of the deep ocean internal wave field. The major features of the models have been verified by direct measurement (Müller, Olbers, and Willebrand, 1978). The models give enough information about the dis-

1. Contribution number 4326 of the Woods Hole Oceanographic Institution.

2. Research School of Earth Sciences, The Australian National University, P.O. Box 4, Canberra, A.C.T.

tribution of wave energy as a function of phase speed to allow estimation of the effects of critical layer absorption. In this note we have estimated:

1. The maximum horizontal anisotropy induced in the wave field.
2. The maximum rate of absorption of internal wave momentum by the mean flow.
3. The effect of the absorbed wave momentum on the mean flow.

2. The model

We assume that individual wavegroups propagate in the vertical, (i.e., they must be rays, not modes) conserving wave action. Then it can be shown (see Müller and Olbers, 1975, for a more complete discussion) that the internal wave field in a steady vertical shear flow $\bar{u}(z)$ obeys a simplified form of the Radiation Balance Equation (we are thus ignoring the "relaxation" and "source" terms discussed by Müller and Olbers):

$$\frac{\partial \Omega}{\partial \beta} \frac{\partial \eta}{\partial z} - \frac{\partial \Omega}{\partial z} \frac{\partial \eta}{\partial \beta} = 0 \quad (1)$$

where

$$\eta(k) = \epsilon(\mathbf{k})/\omega_0 \quad \text{"action density spectrum"}$$

$\epsilon(\mathbf{k})$ is the energy density spectrum of the wave field

$$\omega_0 = \omega - \mathbf{k} \cdot \bar{\mathbf{u}}$$

$$\mathbf{k} = (k_1, k_2, k_3) = (\alpha \cos \phi, \alpha \sin \phi, \beta)$$

$$\omega = \Omega(k; z) = \mathbf{k} \cdot \bar{\mathbf{u}} + (N^2 \alpha^2 + f^2 \beta^2)/|\mathbf{k}|^2$$

Note that Ω is the functional form of the dispersion relation $\omega = \Omega(\mathbf{k}; z)$. Equation (1) has a simple general solution:

$$\eta(\mathbf{k}; z) = F(\alpha, \phi, \Omega(\mathbf{k}; z), s) \quad (2)$$

We match the spectrum (2) to the GM75 and GM76 model spectra at a level of no motion, where $\bar{u} = 0$. The functional form (2) becomes, after transformation from $\mathbf{k}$ - space to (α, ϕ, ω) space, where $V_3^{-1} = \frac{\partial \beta}{\partial \omega}$ is the Jacobian of the transformation:

$$\begin{aligned} \eta(\alpha, \phi, \omega, s, z) &= F(\alpha, \phi, \omega, s)/V_3 \\ &= q(s) \frac{E_0 N(z) f(\omega^2 - f^2)}{N_0 \pi^2 \omega^3} \cdot \frac{A \left(\frac{\alpha}{\alpha_*} \right) \omega_0}{\alpha_* (\omega_0^2 - f^2)^{3/2}} \end{aligned} \quad (3)$$

where:

$$s = \text{sign}(\beta)$$

$$f = (4\pi/24\text{hr.}) \cdot \sin(\text{latitude}) = \text{inertial frequency}$$

$$N(z) = \left(\frac{g}{\rho_0} \frac{d\rho}{dz} \right)^{\frac{1}{2}} = \text{buoyancy frequency}$$

$$N_0 = 3 \text{ cph}$$

$$E_0 = 30 \text{ cm}^2 \text{ s}^{-2}$$

$$\alpha_* = j_* \pi (\omega^2 - f^2)^{\frac{1}{2}} / (N_0 b)$$

$$b = 1.3 \text{ km.}$$

$$A(x) = \begin{cases} \frac{\pi/2}{1+x^2} & \text{GM76 } (j_* = 3) \\ \frac{1.5}{(1+x)^{2.5}} & \text{GM75 } (j_* = 6) \end{cases}$$

$q(s)$ is the (unknown) vertical asymmetry of the

$$\text{wave field such that } \sum_{s=1}^2 q(s) = 1$$

$$V_s = \frac{\partial \Omega}{\partial \beta} \text{ is the vertical group velocity}$$

As in Garrett and Munk (1975), the spectrum has been normalized so that $\int d\omega \int d\alpha \int d\phi \omega_0 \cdot \eta(\alpha, \phi, \omega) = E_0$. Consistent with the hydrostatic approximation in the GM model, we freely make use of the approximations $\omega^2 \ll N^2$ and $f^2 \ll N^2$. The notation in eq. 3 follows that of Garrett and Munk (1975).

3. Current anisotropy

Frankignoul (1974) studied the anisotropy of horizontal internal wave currents in the presence of a mean flow. Especially at high frequencies and large mean flow, he found a tendency for the wave currents to be stronger in a direction normal to the current than parallel to it. He suggested that critical layer absorption could cause this by removing from an otherwise horizontally isotropic spectrum those waves which encounter a critical level in their travels. The remaining waves tend to have horizontal velocity components which are stronger in a direction transverse to the mean current, giving the anisotropic currents observed.

We ask: could the current anisotropy caused by critical level absorption be strong enough to account for Frankignoul's observations? To answer this, we assume that all waves which encounter a critical level are absorbed, and *not* replaced at other levels. This yields the maximum possible current anisotropy. We then use eq. 3 to estimate the current spectra, in the manner suggested by Frankignoul. The velocity spectra are (c.f. Garrett and Munk, 1975):

$$P_{11}(\omega) = \int_0^{2\pi} d\phi \int_0^{\alpha_M} d\alpha U^2 \eta \omega_0 \quad (4)$$

$$P_{22}(\omega) = \int_0^{2\pi} d\phi \int_0^{\alpha_M} d\alpha V^2 \eta \omega_0$$

where:

$\phi = 0$ is taken in the direction of the mean flow

$$\alpha_M = \begin{cases} \frac{\omega - f}{\bar{u}_M \cos \phi} & \cos \phi > 0 \\ \frac{\omega - N}{\bar{u}_M \cos \phi} & \cos \phi < 0 \end{cases}$$

$$U^2 = \cos^2 \phi + \frac{f^2}{\omega_0^2} \sin^2 \phi$$

$$V^2 = \frac{f^2}{\omega_0^2} \cos^2 \phi + \sin^2 \phi.$$

$$\bar{u}_M = \text{Max. in } z \text{ of } \bar{u}(z).$$

At the depth where $\bar{u}(z) = \bar{u}_M$, the V_3^{-1} behavior of η causes a piling up of waves which are near a critical level. This causes an unrealistically singular cusp at the integration boundary where $\omega_0 = f$. Dissipation in the real ocean would remove this singularity, so we "soften" the cusp by pre-weighting the integrands with $(\omega_0^2 - f^2)/(\omega^2 - f^2)$. Removing an isotropic portion of the spectrum in this way causes the resulting anisotropy estimates to be slightly larger than they would have been otherwise. We are thus estimating the maximum possible anisotropy.

U^2 and V^2 are geometric factors describing the elliptical particle motions as a function of ω_0/f such that $U^2 = V^2$ at $\omega_0/f = 1$. Except for these factors, and the abovementioned V_3^{-1} behavior, the integrands vary as the energy density $\eta\omega_0$, and are described in GM75.

For the GM75 spectrum, the integrals (4) had to be evaluated numerically. The anisotropy factor:

$$A.F. = \frac{P_{11} - P_{22}}{P_{11} + P_{22}} \quad (5)$$

thus computed is shown for several frequencies in Figure (1a), for $\bar{u}_M = 1.5 \bar{u}(z)$. Note that the anisotropy is smaller in magnitude than -0.2 , less than half that found by Frankignoul.

In the above numerical work, it was found that the Doppler shift terms were negligible for $\bar{u}(z) < \bar{u}_M/(1.5)$. In this case, the integrals are nearly independent of $\bar{u}(z)$ for $0 \leq \bar{u}(z) < \bar{u}_M/1.5$ and so the approximation $\omega_0 = \omega - \mathbf{k} \cdot \bar{\mathbf{u}}(z) \approx \omega$ is valid. (The regions where Doppler shift is important do not contribute significantly to the integrals). For $\bar{u}(z) > \bar{u}_M/1.5$, the Doppler terms caused ω_0 to tend toward

Figure 1. Wave current anisotropy factor as defined in equation 5 computed for

- a) the GM 75 spectrum, with $\bar{u}_M = 1.5 \bar{u}_L$
- b) the GM 76 spectrum, with doppler shift neglected ($\bar{u}_L \approx 0$).

f , decreasing the computed anisotropy. When this approximation is made for the GM76 spectrum, the integrals become:

$$\begin{aligned} P_{11} - P_{22} &\cong -2K(\omega) \left[H \left(\frac{\omega - f}{\bar{u}_M \alpha_*} \right) + H \left(\frac{N - \omega}{\bar{u}_M \alpha_*} \right) \right] \\ P_{11} + P_{22} &\cong \frac{\omega^2 + f^2}{\omega^2 - f^2} K(\omega) \left[G \left(\frac{\omega - f}{\bar{u}_M \alpha_*} \right) + G \left(\frac{N - \omega}{\bar{u}_M \alpha_*} \right) \right] \end{aligned} \quad (6)$$

where:

$$\begin{aligned} H(x) &= \left(x \sqrt{1+x^2} \ln \left(\frac{\sqrt{1+x^2} + 1}{x} \right) \right) - x \\ G(x) &= \frac{\pi^2}{2} - 2x \int_0^{\pi/2} \frac{u' \sin u' du'}{\cos^2 u' + x^2} \\ K(\omega) &= \frac{1.5 E_0 N(z) f \cdot \bar{u} \alpha_*}{8\omega (N^2 - f^2)} \end{aligned}$$

$G(x)$ could not be evaluated analytically; instead, asymptotic approximations to G were found for large and small x :

$$G(x) \cong \begin{cases} \frac{\pi^2}{2} - \frac{2}{x} & x \geq 2 \\ 2x \left(1 - \ln \frac{x}{2} \right) & x \leq 2 \end{cases} \quad (7)$$

The two approximations (eq. 7) are compared to numerical estimates of $G(x)$ in Figure 2. It is seen that in their respective ranges, the approximations are accurate

Figure 2. $G(x)$, the function defined below eq. 6 in the text. Shown are the approximations for small x (solid line), large x (dashed line), and several numerical estimates (+).

to a few per cent, and asymptotically approach the true values as x becomes small or large. The anisotropy factor computed for the GM76 spectrum using expressions 6 and 7 is shown as a function of $\bar{u}_M$ for several frequencies in Figure 1b. As with the GM75 results, the behavior of the critical level induced anisotropy factor is similar to that measured by Frankignoul but is only half as large. We therefore conclude that the anisotropy measured by Frankignoul cannot be caused entirely by critical level absorption.

4. Momentum flux

As described by Müller (1976), the effect of an internal wave on a geostrophic shear flow is given by a combination of Reynolds stress and buoyancy flux:

$$\begin{aligned}
 F_{13} &= \overline{u_1' u_3'} + \frac{f g}{N^2 \rho_0} \overline{\rho' u_2'} \\
 &= \overline{u_1' u_3'} \left(1 - \frac{f^2}{\omega_o^2} \right) \\
 &= \int \int \int d\mathbf{k} k_1 V_3 \eta
 \end{aligned} \tag{8}$$

The quantity (8) was interpreted by Jones (1967) and by Müller (1976) in terms of the vertical flux of mean angular momentum. Bretherton (1969) interpreted (8), measured in an Eulerian manner, as the mean stress exerted across a material, wavy, surface; i.e., as the Reynolds stress measured in a Lagrangian manner. He did, however, conclude that in a rotating system it is appropriate to regard (8) as the vertical momentum flux. This view was adopted in the measurements of Ruddick and Joyce (1979). They showed that as ω_o approaches f , the particle paths become nearly horizontal circles, and so the buoyancy flux in (8) almost cancels the Reynolds stress, leaving very little contribution to the effective stress F_{13} . An inertial wave has

much smaller effect on the mean flow than one would estimate from the Reynolds stress alone. The quantity F is thus a more accurate measure than the Reynolds stress of the effective flux of momentum from the wave field to the mean flow.

To compute the momentum flux, we will assume that waves which are absorbed at a critical level are replaced (say, by nonlinear interaction) at the rate at which they are absorbed. Then the momentum flux into a critical level is limited by the rate of vertical propagation of the waves toward a critical level. We also assume the wave field to be fully asymmetric; all waves are heading into a critical layer, not out of it. This involves an implicit assumption that the shear does not change sign in the region of interest. We take $\frac{\partial \bar{u}}{\partial z} > 0$. These assumptions ensure that our estimate is of the maximum possible momentum flux.

From eqs. 8 and 3, using $V_3 = \left(\frac{\omega_0^2 - f^2}{\omega_0 N \alpha} \right)^{3/2}$:

$$\begin{aligned} F_{13} &= \int d\omega \int d\alpha \int d\phi \frac{E_0 f}{\pi^2 N_0} \frac{\omega^2 - f^2}{\omega^3} A \left(\frac{\alpha}{\alpha_*} \right) \alpha_*^{-1} \cos \phi \\ &\cong \frac{E_0 f}{\pi^2 N_0} \int d\omega \int d\alpha \int d\phi \omega^{-1} A \left(\frac{\alpha}{\alpha_*} \right) \alpha_*^{-1} \cos \phi \quad (9) \\ &\quad (\text{for } \omega \geq 2f) \end{aligned}$$

We now compute the momentum flux *lost* to the mean flow through critical layer absorption; that part of the integral (9) for which $\omega - \mathbf{k} \cdot \bar{\mathbf{u}} \leq f$, for the GM76 spectrum:

$$\begin{aligned} F_{13}^{\text{LOST}} &\equiv \\ &\frac{2E_0 f}{\pi^2 N_0} \int_f^N \frac{d\omega}{\omega} \frac{2}{\pi} \int_{\frac{\omega-f}{\bar{u}_1}}^{\infty} \left[\frac{d\alpha}{\alpha_* \left(1 + \frac{\alpha^2}{\alpha_*^2} \right)} \right] \int_0^{\cos^{-1} \left(\frac{\omega-f}{\bar{u}_1 \alpha} \right)} \left[d\phi \cos \phi \right] \quad (10) \end{aligned}$$

For moderate mean velocities (less than 50 cm s^{-1}) the dominant contribution to the α and ϕ integrals is from the high wavenumber/slow phase speed tail of the spectrum. After the ϕ and α integrations are carried out, an examination of the ω integrand reveals an integrable rise to a finite value at $\omega = f$, and an approach to ω^{-1} behavior at large ω . Except for a slight increase at $\omega = f$, the momentum flux loss occurs in proportion to the total momentum flux at each frequency.

Evaluating the integrals, we find:

$$F_{13} \Big|_{\text{LOST}} = \frac{2E_0 f}{\pi^2 N_0 U} \left[\begin{aligned} & 2\sqrt{1-U^2} \left(\arctan \sqrt{\frac{U^2}{1-U^2}} - \arctan \sqrt{\frac{y^2+U^2}{1-U^2}} \right) \\ & + \sqrt{1+U^2} \left(\ln \frac{1 + \sqrt{\frac{y^2+U^2}{1+U^2}}}{1 - \sqrt{\frac{y^2+U^2}{1+U^2}}} - \ln \frac{1 + \sqrt{\frac{U^2}{1+U^2}}}{1 - \sqrt{\frac{U^2}{1+U^2}}} \right) \\ & + 2 \arctan y - \ln \frac{1+y}{1-y} \end{aligned} \right] \quad (11)$$

for $U < 1$, and

$$F_{13} \Big|_{\text{LOST}} = \frac{2E_0 f}{N_0 U} \left[\begin{aligned} & \sqrt{U^2-1} \left(\ln \frac{1 + \sqrt{\frac{y^2+U^2}{U^2-1}}}{1 - \sqrt{\frac{y^2+U^2}{U^2-1}}} - \ln \frac{1 + \sqrt{\frac{U^2}{U^2-1}}}{1 - \sqrt{\frac{U^2}{U^2-1}}} \right) \\ & + \sqrt{U^2+1} \left(\ln \frac{1 + \sqrt{\frac{y^2+U^2}{U^2+1}}}{1 - \sqrt{\frac{y^2+U^2}{U^2+1}}} - \ln \frac{1 + \sqrt{\frac{U^2}{U^2+1}}}{1 - \sqrt{\frac{U^2}{U^2+1}}} \right) \\ & + 2 \arctan y - \ln \frac{1+y}{1-y} \end{aligned} \right] \quad (12)$$

for $U > 1$, where:

$$y = \frac{1 - \frac{f}{N(z)}}{\sqrt{1 - \frac{f^2}{N^2(z)}}}$$

$$u_* = \frac{N_0 b}{j_* \pi} = 72.2 \text{ cm s}^{-1}$$

$$U = \bar{u}_1 / u_*$$

The depth dependence $N(z)$ is due to waves which encounter a vertical turning point at $\omega_0 = \pm N(z)$. These waves are absent from the spectrum at depths below their turning point.

Figure 3. Momentum flux lost to critical layer absorption versus mean velocity, estimated for the GM 76 spectrum.

The momentum flux loss (11, 12) is plotted versus $\bar{u}$, in Figure 3. It is well-behaved, approaching (as $\bar{u} \rightarrow \infty$)

$$\frac{2E_0 f}{\pi^2 N_0} \ln \frac{N(z)}{f} \quad (13)$$

A value of $f = .04$ cph has been chosen for this graph. If a velocity difference $\Delta \bar{u}$ were imposed over a 400 m thick thermocline as is typical for the North-west Atlantic, the observed eddy viscosity, ν , would be

$$\nu = \frac{-F_{13}(\Delta \bar{u})}{(\Delta \bar{u}/400 \text{ m})} \approx -200 \text{ cm}^2 \text{ s}^{-1} \quad (14)$$

An interaction of this strength is important to the wave field, but very hard to measure in the ocean.

5. Effect on the mean flow

As discussed by Jones and Houghton (1971), the momentum flux lost from the wave field serves to accelerate the mean flow. Ignoring the effects of rotation as they did leads to a very simple equation for the mean flow:

$$\begin{aligned} \frac{\partial \bar{u}_1}{\partial t} &= \frac{\partial F_{13}}{\partial z}^{\text{lost}} \quad \left(\text{since } \frac{\partial F_{13}}{\partial z} = -\frac{\partial F_{13}}{\partial z}^{\text{lost}} \right) \\ \longrightarrow \frac{\partial \bar{u}_1}{\partial t} + \bar{V}(\bar{u}_1) \left| \frac{\partial \bar{u}_1}{\partial z} \right| &= 0 \\ \text{where } \bar{V}(\bar{u}_1) &= \frac{\partial F_{13}}{\partial \bar{u}_1}^{\text{lost}} \end{aligned} \quad (15)$$

Equation 15 can be solved by the method of characteristics (c.f. Whitham, 1974, pp. 19-23). An initial velocity profile moves downward with phase speed $\bar{V}$ ($\bar{u}_1$). since $\bar{V}$ is a decreasing function of $\bar{u}_1$ the regions of larger $\bar{u}$ will move somewhat more slowly than the regions of small $\bar{u}$. Thus a shear region will move downward (at about 5 m/day) and broaden. Conversely, for $\frac{\partial \bar{u}_1}{\partial z} < 0$, $\bar{u}_1 > 0$, a shear region will move upward and broaden. The fact that $\partial F / \partial \bar{u}_1$ is a decreasing function of $\bar{u}_1$ prevents the formation of "shocks" in the solution of eq. 15. Thus critical layer absorption of waves in the GM76 spectrum cannot cause the spontaneous formation of velocity fine structure.

These simple descriptions of the evolution of the mean flow are valid between the level where $\bar{u} = 0$ and the level where $\bar{u}(z)$ is a maximum. In order to extend the results to (say) a higher level, account must be taken of the shielding of that level by the lower levels with larger $\bar{u}$, as well as the positions of the wave sources. This was done by Plumb (1977) for the case of two discrete waves. Because the wave spectrum assumed by Plumb was discrete, he found that a shear zone will tend to sharpen as it moves downward, whereas we find for the continuous GM spectra that a shear zone will spread.

6. Conclusions

1. The maximum horizontal wave current anisotropy which could be induced by critical layer absorption is only half as large as was observed by Frankignoul (1974).
2. The maximum momentum flux induced by wave absorption in a shear across a 400m thick thermocline is equivalent to an eddy viscosity of $-200 \text{ cm}^2 \text{ s}^{-1}$.
3. The effect of the wave absorption on the shear flow is to cause a downward propagation of the shear zone at about 5m/day. The shear zone will also become more diffuse.

Acknowledgments. This work was carried out under United States Office of Naval Research Contract Number N 000 14-76-C-0197 NR 083-400. The author would like to thank the referees, whose comments were very helpful.

REFERENCES

- Booker, J. R. and F. P. Bretherton. 1967. The critical layer for internal gravity waves in a shear flow. *J. Fluid Mech.*, 27, 513-539.
- Bretherton, F. P. 1969. Momentum transport by gravity waves. *Quart. J. Roy. Met. Soc.*, 95, 213-243.
- Desaubies, Y. J. F. 1976. Analytical representation of internal wave spectra. *J. Phys. Oceanog.*, 6, 976-981.
- Frankignoul, C. J. 1974. Observed anisotropy of spectral characteristics of internal waves induced by low-frequency currents. *J. Phys. Oceanog.*, 4, 625-634.
- Fritts, D. C. 1979. The excitation of radiating waves and Kelvin-Helmholtz instabilities by the gravity wave-critical level interaction. *J. Atmos. Sci.*, 36, 12-23.

- Garrett, C. J. R. and Walter Munk. 1972. Space-time scales of internal waves. *Geophys. Fluid Dyn.*, 2, 225–264.
- . 1975. Space-time scales of internal waves: a progress report. *J. Geophys. Res.*, 80, 291–297.
- Jones, W. L. 1967. Propagation of internal gravity waves in fluids with shear flow and rotation. *J. Fluid Mech.*, 30, 439–448.
- Jones, W. L. and D. D. Houghton. 1971. The coupling of momentum between internal gravity waves and mean flow: A numerical study. *J. Atmos. Sciences*, 28, 604–608.
- Müller, P. 1976. On the diffusion of momentum and mass by internal gravity waves. *J. Fluid Mech.*, 77, 789–823.
- Müller, P. and D. J. Olbers. 1975. On the dynamics of internal waves in the deep ocean. *J. Geophys. Res.*, 80, 3848–3860.
- Müller, P., D. J. Olbers and J. Willebrand. 1978. The IWEX spectrum. *J. Geophys. Res.*, 83, 479–500.
- Plumb, R. A. 1977. The interaction of two internal waves with the mean flow: implications for the theory of the quasi-biennial oscillation. *J. Atmos. Sci.*, 34, 1847–1858.
- Ruddick, B. R. and Joyce, T. M. 1979. Observations of interaction between the internal wave-field and low-frequency flows in the North Atlantic. *J. Phys. Oceanog.*, 9, 498–517.
- Whitham, G. B. 1974. *Linear and Nonlinear Waves*. John Wiley & Sons, New York, 636 pp.

Printed in U.S.A. for the Sears Foundation for Marine Research,
Yale University, New Haven, Connecticut, 06520, U.S.A.
Van Dyck Printing Company, North Haven, Connecticut, 06473, U.S.A.

MANDATORY DISTRIBUTION LIST

FOR UNCLASSIFIED TECHNICAL REPORTS, REPRINTS, AND FINAL REPORTS
PUBLISHED BY OCEANOGRAPHIC CONTRACTORS
OF THE OCEAN SCIENCE AND TECHNOLOGY DIVISION
OF THE OFFICE OF NAVAL RESEARCH

(REVISED NOVEMBER 1978)

1	Deputy Under Secretary of Defense (Research and Advanced Technology) Military Assistant for Environmental Science Room 3D129 Washington, D.C. 20301	12	Defense Documentation Center Cameron Station Alexandria, VA 22314 ATTN: DCA
	Office of Naval Research 800 North Quincy Street Arlington, VA 22217		Commander Naval Oceanographic Office NSTL Station Bay St. Louis, MS 39522
3	ATTN: Code 483	1	ATTN: Code 8100
1	ATTN: Code 460	1	ATTN: Code 6000
2	ATTN: 102B	1	ATTN: Code 3300
1	CDR J. C. Harlett, (USN) ONR Representative Woods Hole Oceanographic Inst. Woods Hole, MA 02543	1	NODC/NOAA Code D781 Wisconsin Avenue, N.W. Washington, D.C. 20235
	Commanding Officer Naval Research Laboratory Washington, D.C. 20375		
6	ATTN: Library, Code 2627		

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER WHOI-80-62	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) CRITICAL LAYERS AND THE GARRETT-MUNK SPECTRUM		5. TYPE OF REPORT & PERIOD COVERED Technical
		6. PERFORMING ORG. REPORT NUMBER WHOI CON. NO. 4326
7. AUTHOR(s) Barry Ruddick		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0197;
9. PERFORMING ORGANIZATION NAME AND ADDRESS Woods Hole Oceanographic Institution Woods Hole, MA 02543		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 083-400
11. CONTROLLING OFFICE NAME AND ADDRESS NORDA National Space Technology Laboratory Bay St. Louis, MS 39529		12. REPORT DATE November 1980
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES Reprinted from the "Journal of Marine Research, Volume 38, 1, 1980, pp. 135-145".		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) 1. Critical layer absorption of internal waves 2. Internal wave-mean flow interaction 3. Garrett-Munk model internal wave spectrum		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The effects of critical level absorption of oceanic internal waves by a mean flow are estimated using the Garrett and Munk (1975) model spectrum. The horizontal currents of the wave field are found to be more intense perpendicular to the mean flow than parallel to it. The cause of this anisotropy is preferential absorption of waves travelling with the mean flow. However, the current anisotropy is only half as large as would be necessary to explain Frankignoul's (1974) observations. The wave momentum (Cont. on		

flux lost to critical level absorption is found to be nearly proportional to the mean velocity. When the momentum flux is deposited throughout a 400 m thick shear zone, typical of the main thermocline in the North-west Atlantic, the observed stress-shear relationship would correspond to a wave-induced eddy viscosity of $-200 \text{ cm}^2 \text{ s}^{-1}$. The effect of the absorbed momentum on the mean flow is to cause a slow (5 m/day) downward phase propagation and slow broadening of the shear profile.

Woods Hole Oceanographic Institution WHOI-80-62	<p>CRITICAL LAYERS AND THE GARRETT-MUNK SPECTRUM by Barry Ruddick. Reprinted from the "Journal of Marine Research, Volume 38, 1, 1980, pp. 135-145". Nov. 1980. Prepared for the Office of Naval Research under Contract N00014-76-C-0197; NR 083-400.</p> <p>The effects of critical level absorption of oceanic internal waves by a mean flow are estimated using the Garrett and Munk (1975) model spectrum. The horizontal currents of the wave field are found to be more intense perpendicular to the mean flow than parallel to it. The cause of this anisotropy is preferential absorption of waves travelling with the mean flow. However, the current anisotropy is only half as large as would be necessary to explain Frankignoul's (1974) observations. The wave momentum flux lost to critical level absorption is found to be nearly proportional to the mean velocity. When the momentum flux is deposited throughout a 400 m thick shear zone, typical of the main thermocline in the North-west Atlantic, the observed stress-shear relationship would correspond to a wave-induced eddy viscosity of $\sim 200 \text{ cm}^2 \text{ s}^{-1}$. The effect of the absorbed momentum on the mean flow is to cause a slow (5 m/day) downward phase propagation and slow broadening of the shear profile.</p>	<p>1. Critical layer absorption of internal waves</p> <p>2. Internal wave-mean flow interaction</p> <p>3. Garrett-Munk model internal wave spectrum</p> <p>I. Ruddick, Barry</p> <p>II. N00014-76-C-0197; NR 083-400</p>	<p>1. Critical layer absorption of internal waves</p> <p>2. Internal wave-mean flow interaction</p> <p>3. Garrett-Munk model internal wave spectrum</p> <p>I. Ruddick, Barry</p> <p>II. N00014-76-C-0197; NR 083-400</p>
Woods Hole Oceanographic Institution WHOI-80-62	<p>CRITICAL LAYERS AND THE GARRETT-MUNK SPECTRUM by Barry Ruddick. Reprinted from the "Journal of Marine Research, Volume 38, 1, 1980, pp. 135-145". Nov. 1980. Prepared for the Office of Naval Research under Contract N00014-76-C-0197; NR 083-400.</p> <p>The effects of critical level absorption of oceanic internal waves by a mean flow are estimated using the Garrett and Munk (1975) model spectrum. The horizontal currents of the wave field are found to be more intense perpendicular to the mean flow than parallel to it. The cause of this anisotropy is preferential absorption of waves travelling with the mean flow. However, the current anisotropy is only half as large as would be necessary to explain Frankignoul's (1974) observations. The wave momentum flux lost to critical level absorption is found to be nearly proportional to the mean velocity. When the momentum flux is deposited throughout a 400 m thick shear zone, typical of the main thermocline in the North-west Atlantic, the observed stress-shear relationship would correspond to a wave-induced eddy viscosity of $\sim 200 \text{ cm}^2 \text{ s}^{-1}$. The effect of the absorbed momentum on the mean flow is to cause a slow (5 m/day) downward phase propagation and slow broadening of the shear profile.</p>	<p>1. Critical layer absorption of internal waves</p> <p>2. Internal wave-mean flow interaction</p> <p>3. Garrett-Munk model internal wave spectrum</p> <p>I. Ruddick, Barry</p> <p>II. N00014-76-C-0197; NR 083-400</p>	<p>1. Critical layer absorption of internal waves</p> <p>2. Internal wave-mean flow interaction</p> <p>3. Garrett-Munk model internal wave spectrum</p> <p>I. Ruddick, Barry</p> <p>II. N00014-76-C-0197; NR 083-400</p>
Woods Hole Oceanographic Institution WHOI-80-62	<p>CRITICAL LAYERS AND THE GARRETT-MUNK SPECTRUM by Barry Ruddick. Reprinted from the "Journal of Marine Research, Volume 38, 1, 1980, pp. 135-145". Nov. 1980. Prepared for the Office of Naval Research under Contract N00014-76-C-0197; NR 083-400.</p> <p>The effects of critical level absorption of oceanic internal waves by a mean flow are estimated using the Garrett and Munk (1975) model spectrum. The horizontal currents of the wave field are found to be more intense perpendicular to the mean flow than parallel to it. The cause of this anisotropy is preferential absorption of waves travelling with the mean flow. However, the current anisotropy is only half as large as would be necessary to explain Frankignoul's (1974) observations. The wave momentum flux lost to critical level absorption is found to be nearly proportional to the mean velocity. When the momentum flux is deposited throughout a 400 m thick shear zone, typical of the main thermocline in the North-west Atlantic, the observed stress-shear relationship would correspond to a wave-induced eddy viscosity of $\sim 200 \text{ cm}^2 \text{ s}^{-1}$. The effect of the absorbed momentum on the mean flow is to cause a slow (5 m/day) downward phase propagation and slow broadening of the shear profile.</p>	<p>1. Critical layer absorption of internal waves</p> <p>2. Internal wave-mean flow interaction</p> <p>3. Garrett-Munk model internal wave spectrum</p> <p>I. Ruddick, Barry</p> <p>II. N00014-76-C-0197; NR 083-400</p>	<p>1. Critical layer absorption of internal waves</p> <p>2. Internal wave-mean flow interaction</p> <p>3. Garrett-Munk model internal wave spectrum</p> <p>I. Ruddick, Barry</p> <p>II. N00014-76-C-0197; NR 083-400</p>