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FRICTIONAL SECONDARY CIRCULATION NEAR
AN UPWELLED THERMOCLINE

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Note: The material of this report was presented ten years ago at a symposium, but on account of its speculative nature, remained unpublished. Several colleagues were, however, kind enough to quote it on occasion (e.g. Garrett, 1982) and some have requested copies. In order to make it more readily available, it has now been re-issued as a WHOI report.

ABSTRACT

A first order, linear analysis of thermocline upwelling shows that a quasigeostrophic baroclinic jet accompanies the upwelling. The secondary circulation induced in such a flow field in a meridional plane by turbulent friction is investigated. A double Ekman layer is found to form at the thermocline, of a few meters in thickness. The suction of this Ekman layer destroys the primary flow by the "spin-up" mechanism in a period of which the order of magnitude is tentatively estimated at 100 hrs. Thus the primary flow must be reestablished by successive, reasonably frequent wind-stress impulses to maintain a quasi-stationary upwelling.

The lower side Ekman layer on the thermocline is fed from a free shear layer (turbulent "Stewartson layer") underneath the free surface-thermocline intersection, which thus becomes an upwelling zone. The width of the turbulent Stewartson layer is estimated to be the velocity difference divided by the Coriolis parameter, i.e. typically of the order of 1 km.

INTRODUCTION

The dynamical conditions near the intersection of the free surface and a thermocline* are difficult to elucidate mainly because of the complex interrelationships between turbulent friction and stratification. Although these interrelationships are qualitatively understood, we lack the quantitative relationships on which a general theory of similar phenomena could be based. Our current understanding of upwelling dynamics (Smith, 1968, O'Brien and Hurlburt, 1972) is therefore derived from piecemeal evidence, built up using theoretical models in which either friction or stratification or at least their interaction is ignored. Different assumptions in such models produce insight into various different aspects of our problem. The present paper is a further contribution in this category, focussing at least on certain well established features of the stratification/turbulent friction interaction.

Starting from one of the simplest models, we may examine small vertical displacements in a stratified fluid (two-layer in the simplest case, continuously stratified in a somewhat more realistic approach) which are produced by the sudden imposition of wind stress at the surface (Charney, 1955, Csanady, 1968, 1972a). We find a tendency for an upwelling to occur on the left-hand shore of the wind (in the Northern Hemisphere), quite soon and for quite moderate winds, but of course the small displacement theory breaks down long before the thermocline actually

*We shall designate a thermocline intersecting the free surface an "upwelled" one. This is more descriptive although dynamically perhaps less appropriate than the term "oceanic front" used by Collins (1964).

intersects the surface. Nevertheless, such an analysis suggests that a "coastal jet" nearly in geostrophic equilibrium develops as the upwelling begins to form, and that in the adjustment process potential vorticity is conserved. Extending the same two assumptions to finite thermocline movements (Csanady, 1971) one arrives at a first order, inviscid model of upwelled thermoclines, applying also to such relatively unusual situations as the "spring" thermocline in the Great Lakes (confined to the warm shore zone only). The same dynamical principles underlie Yoshida's (1967) frictionless upwelling model and Stommel's (1965, p. 109) constant potential vorticity model of the Gulf Stream. A crude consideration of turbulent friction (using constant eddy coefficients) does not modify the essential features of the upwelling, as derived from the first order model (Yoshida, 1967, O'Brien and Hurlburt, 1972).

While the first order model satisfactorily describes the gross features of upwelled thermoclines, it does not yield much information on more subtle occurrences, such as any slow vertical (upward) motions which are of crucial biological importance. Of course, vertical motions of considerable amplitude occur as an upwelling first becomes established, as was clearly explained already by Sverdrup (1938). However, when an upwelled thermocline becomes a "stationary oceanic front", to use the phrase of Collins (1964), other types of vertical motions may also be expected to occur, in which turbulent friction plays a crucial role. One may legitimately regard such motions superimposed on a first-order geostrophically balanced flow as secondary circulations. We would like to know the flow pattern of such circulations and, in particular, zones and magnitudes of upward directed velocities.

The frictionless secondary circulation associated with upwellings in a plane transverse to a coast has been the subject of several studies, notably those of Leetmaa (1971), Blumsack (1972), Garvine (1971), Hsueh and O'Brien (1971) and Hsueh and Kenney (1972). While these studies offer valuable insight, they cannot be regarded as directly modelling the motion near an upwelled thermocline for various reasons, one common difficulty being that they are all based on the assumption of spatially constant eddy viscosities. If we know anything about turbulent friction in a stratified fluid, it is that the eddy viscosity depends strongly on the vertical density gradient. Also, the eddy viscosity is not an independent external parameter, but is determined by other properties of the flow.

We may expect to improve our understanding of the phenomena under discussion if, in our theoretical model, we take into account at least in a very crude way two well established features of turbulent friction:

(1) The eddy viscosity in the immediate neighborhood of the thermocline is very low. A careful analysis of thermocline movements in Lake Ontario by Sweers (1969) suggests, for example, a turbulent transport coefficient at the thermocline of $0.1 - 1.0 \text{ cm}^2/\text{sec}$. Moreover, the recent studies of Woods (1971, and earlier references there) suggest that this coefficient is a result of intermittent bursts of turbulence, depending in a complex way on baroclinic modes of motion. If it were not for this complication, one would be tempted to postulate that turbulent friction in the thermocline zone depends only on the differences in velocity Δu and effective gravity g_e , so that if an eddy viscosity were used this would have to be proportional to $\Delta u^3/g_e$. Given that other physical parameters are involved, how-

ever, we are forced to adopt an empirical low value of the eddy coefficient.

(2) The eddy viscosity in the (nearly) homogeneous layers above and below the thermocline is fairly high and is scaled by appropriate velocity differences and lengths. Within a wind-driven mixed layer, for example, the eddy viscosity was found to be about $0.05 u^* h$, where u^* is friction velocity at the surface and h is mixed layer depth (Csanady, 1972c). Similar scaling laws may be expected to apply to other turbulent momentum transfer problems.

In the following, we shall discuss a very simple two-layer model, in which an upwelled thermocline has been produced by some prior event. The first-order dynamics will be assumed to be described by geostrophic balance and conservation of potential vorticity. We will then infer the secondary circulation produced in such a system by turbulent friction, modelling such friction according to the above two principles.

THE PRIMARY FLOW

Most of our following arguments will be independent on the exact nature of the first-order or primary flow, except that this will be assumed to be in geostrophic equilibrium. The understanding is, however, facilitated by adopting a reasonably specific model, so that we shall use the results of an earlier paper (Csanady 1971) in which the equilibrium shape of a "summer" thermocline was found from conservation of potential vorticity. Let the undisturbed depths of top and bottom layers respectively be h_0 and h_0' , densities ρ and ρ' so that $g\epsilon$ is effective gravity with $\epsilon = (\rho' - \rho)/\rho'$. Laying the y-coordinate perpendicular to shore we find the distribution of actual top-layer depths and top and bottom alongshore velocities after adjustment to geostrophic balance:

$$\begin{aligned} h &= h_0 (1 - e^{-y/R}) \\ u &= -c e^{-y/R} \quad (y \geq 0) \\ u' &= \frac{h_0}{h_0'} c e^{-y/R} \end{aligned} \tag{1}$$

where c is celerity of long internal waves

$$c = \sqrt{g\epsilon \frac{h_0 h_0'}{h_0 + h_0'}} \tag{2}$$

and R is radius of deformation:

$$R = \frac{c}{f} \tag{3}$$

(f = Coriolis parameter). These distributions are illustrated in Fig. (1), the origin $y = 0$ having been chosen to coincide with the

thermocline - free surface intersection. The abrupt change in velocities at $y = 0$ is of course an unrealistic feature of the inviscid solution, as is the discontinuity in velocity at the thermocline surface. Our analysis with friction will have to remove these discontinuities.

The illustrations in Fig. (1) have a grossly exaggerated vertical scale. In an actual situation the ratio of h_0/R is quite small, of order 10^{-2} or so. Therefore the thermocline surface is very nearly horizontal everywhere: momentum transfer across such a surface may be analyzed as if it was horizontal, just as in its undisturbed state.

DOUBLE EKMAN LAYER AT THE THERMOCLINE

Frictional effects at a (nearly) horizontal thermocline may be analyzed regardless of their application to the situation sketched in Fig. 1. While it is possible to construct more sophisticated models involving the diffusion of heat as well as that of momentum, the principal facts emerge from an analysis of a simple model in which the density is regarded unchanged above and below the thermocline. We therefore assume that the density is ρ at $z > 0$, ρ' for $z < 0$.

In the absence of friction, the velocities (u, v) and (u', v') above and below the thermocline respectively are constant with depth and may be calculated from equations of motion applying to the two-layer system. With friction present, the velocities may be separated into a "general" depth-independent current and a "frictional" one, the general component satisfying the same equations as in the absence of friction (Csanady, 1972c). The frictional current, on the other hand, obeys the classical Ekman layer equations

$$\begin{aligned} -fv_f &= \frac{1}{\rho} \frac{\partial \tau_x}{\partial z} \\ fu_f &= \frac{1}{\rho} \frac{\partial \tau_y}{\partial z} \end{aligned} \tag{4}$$

where (u_f, v_f) are x and y components of the supposedly steady frictional current, $(\tau_x$ and $\tau_y)$ are similar components of the total stress in horizontal planes and f is Coriolis parameter.

It is convenient to introduce complex vectors for stresses and velocities:

$$\begin{aligned}w_g &= u_g + iv_g \\w_f &= u_f + iv_f \\T &= \tau_x + i\tau_y\end{aligned}\tag{5}$$

Here (u_g, v_g) are components of the general current which is in equilibrium with pressure gradients and local accelerations. Surface wind stress is supposed negligible for simplicity.

Friction at the thermocline is assumed to be described by a constant eddy viscosity K . The magnitude of this coefficient is expected to be of order 0.1 to 1.0 cm²/sec, yielding an Ekman depth $D = \pi \sqrt{2K/f}$ of the order of a few metres at most. Except where the thermocline is very close to the surface or to the bottom, we can therefore ignore the presence of these other surfaces and calculate the frictional velocity distribution as if the depth was infinite both above and below the thermocline. This is the method we shall adopt: in the immediate neighborhood of a thermocline - free surface or - bottom intersection our simple approach is likely to break down in any case.

The eddy viscosity hypothesis may be expressed as

$$T = \rho K \frac{\partial w}{\partial z}\tag{6}$$

where

$$w = w_g + w_f\tag{7}$$

is the total velocity, although of course w_g is depth - independent.

Substituting now into Eq. (4) we find

$$\frac{d^2 w_f}{dz^2} - \frac{1}{K} w_f = 0 \quad (8)$$

This equation holds separately for w_f and w_f' (above and below thermocline) the boundary conditions at infinity being

$$\begin{aligned} w_f &= 0 & z \rightarrow \infty \\ w_f' &= 0 & z \rightarrow -\infty \end{aligned} \quad (9)$$

Solutions of (8) satisfying (9) are

$$\begin{aligned} w_f &= A \exp \left[-\sqrt{\frac{f}{2K}} (1+i)z \right] \\ w_f' &= B \exp \left[\sqrt{\frac{f}{2K}} (1+i)z \right] \end{aligned} \quad (10)$$

with A, B constants, in general complex. Boundary conditions at the interface $z = 0$ are the continuity of stress and velocity:

$$\begin{aligned} w_o &= w_f|_o + w_g = w_f'|_o + w_g' \\ T_o &= \rho K \frac{\partial w}{\partial z} = \rho K \frac{\partial w'}{\partial z} \quad (z = 0) \end{aligned} \quad (11)$$

From these conditions we find at once

$$A = -B = \frac{w_g' - w_g}{2} \quad (12)$$

"Pure" baroclinic motions in a two-layer sea or lake are characterized by top and bottom layer "general" current velocities being oppositely directed, and of such a magnitude as to make the total transport

vanish (the specific case discussed in the previous section is also of this kind). Laying the real axis along the direction of the general current we have in such cases

$$\begin{aligned} w_g &= u \\ w_g' &= u' = -\frac{h}{h'} u \end{aligned} \tag{13}$$

where h and h' are depths of top and bottom layers respectively. The velocity distribution in the double Ekman layer at the thermocline in such a case is illustrated by Fig. 2, for $\frac{h}{h'} = \frac{1}{4}$. We observe an antisymmetrical double Ekman spiral. The endpoints of velocity vectors are marked at fractions of the Ekman depth $D = \pi \sqrt{2K/f}$. The corresponding stress distribution appears in Fig. 3: this is symmetrical about $z = 0$. The stress at the thermocline is a maximum, directed at 45° to the general current, and its magnitude is

$$|T_o| = \rho(u - u') \sqrt{fK} \tag{14}$$

With $f = 10^{-4} \text{ sec}^{-1}$ and $K = 1.0 \text{ cm}^2/\text{sec}$ this gives a stress of 0.1 dynes/cm for a velocity difference of $u - u' = 10 \text{ cm/sec}$. Such a result is not unreasonable, the calculated $|T_o|$ being just one order of magnitude less than the "typical" surface wind stress.

Of greatest interest to our further discussions is the volume transport by the frictional current. As one sees at once from Fig. (2) this has a strong component in a plane perpendicular to the general current. We find in fact without difficulty, for one side of the double Ekman layer

$$\int_0^{\infty} w_f dz = \frac{u - u'}{8} \sqrt{\frac{2K}{f}} (1 - i) \quad (15)$$

The component along the imaginary axis is clearly

$$V = - \frac{u - u'}{8\pi} D \quad (16)$$

pointing to the left of the current in the top layer. An equal and opposite transport is found in the bottom layer, which therefore also points to the left of u' . For the "typical" values $u - u' = 10$ cm/sec, $K = 1$ cm²/sec, $f = 10^{-4}$ one finds $V = 177$ cm²/sec, say 200 cm²/sec. The maximum velocity component along the y-axis in the case illustrated in Fig. 2 is then about 2.7 cm/sec ($u = 13.3$ cm/sec, $u' = - 3.3$ cm/sec).

THE INFERRED SECONDARY CIRCULATION

We return now to the upwelled thermocline model illustrated in Fig. 1. The scale width R of the inclined portion corresponds to some 10^3 Ekman depths. It is therefore legitimate to apply our theory developed in the previous section point by point along the inclined thermocline, excepting the region near the free surface intersection. For the moment we leave open the question whether the existence of such a singular region is likely to modify the flow in any essential way.

For the slow motions involved in the secondary circulation it is appropriate to rely on the linearized theory of rotating fluids and regard this flow as being induced by Ekman layer suction. On the "cold" side of the thermocline, Ekman transport in the boundary layer adjacent to the thermocline is directed away from the coast. Using the first order theory of the primary flow (Eq. 1) and our result on Ekman transport on one side of the double Ekman layer (Eq. 16) we find

$$V = \frac{cD}{8\pi} \frac{h_o' + h_o}{h_o'} e^{-y/R} \quad (17)$$

This expression is valid for $y > 0$; for $y < 0$ the transport is zero. The first order theory thus yields an abrupt increase in Ekman transport at $y = 0$ from zero to its maximum value. Beyond this, the transport gradually decreases to zero again.

The abrupt increase in Ekman transport acts on the interior flow as a concentrated sink placed at the boundary. Indeed the problem is very similar to the case of rotating laminar flow between concentric

disks discussed by Stewartson (1957), see also Greenspan (1968, p. 100). In view of the nearly horizontal position of the thermocline surface, the same mathematical analysis applies to a high degree of approximation, if we represent eddy friction by a constant eddy kinematic viscosity K . In this situation a free vertical shear layer ("Stewartson layer") forms just below the concentrated sink, at the boundary between the fluid moving with an alongshore velocity u' , and the stagnant fluid outside (at $y < 0$). The abrupt change in the alongshore velocity is made continuous in an "outer" frictional layer of a thickness proportional to Ekman number E to the $1/4$ power. An "inner" layer of thickness proportional to $E^{1/3}$ also forms and permits a vertical mass flux to the sink at the top. Further below we return to the question how wide such a layer is in the turbulent flow case.

We observe also that along the bottom surface at $y \geq 0$ another horizontal Ekman layer forms, of the "conventional" type. The transport in this layer is also away from the coast (towards increasing y), abruptly increasing from zero to a maximum at $y = 0$. The "inner" Stewartson layer also supplies the fluid for this lower Ekman layer. The inferred circulation on the "cold" side of the thermocline is sketched in Fig. 4. We shall discuss the effects of this circulation on the first order flow further below.

On the warm side of the upwelled thermocline we find rather different conditions. Ekman transport along the thermocline is shore-ward, gradually increasing as one approaches the free surface intersection. The Ekman layer suction here acts as a distributed sink. However, at the intersection there can be no concentrated source, because there is

nowhere for the fluid to go: no Ekman or Stewartson layer to accept the flow and an interior region of vanishing thickness. Similar situations are not unknown in rotating systems, e.g. in the corner regions of turbo-machine blading (see for example Csanady, 1964). Low energy fluid is observed to accumulate in such locations. By analogy, we would expect fluid which is slower (and colder, although we have neglected this aspect of the problem) than the rest of the top layer to accumulate in the singular region near $y = 0$. The reduced Coriolis force acting on the low speed fluid is no longer enough to balance the hydrostatic pressure gradient and the thermocline surface tends to move back toward the shore. The accumulation of low energy fluid is thus clearly a part of the dissipative mechanism which in time destroys the coastal currents and restores static equilibrium.

An order of magnitude estimate of the time scale of this accumulation process is of some interest. Let us use the "typical" value of Ekman transport of $V = 200 \text{ cm}^2/\text{sec}$ calculated in the previous section, a thermocline slope of 10^{-2} and let us stipulate that the accumulation of low speed fluid becomes "significant" when it fills a band 1 km. wide on the offshore side of the thermocline. A simple calculation gives the time required to fill up such a band: it is $2.5 \cdot 10^5$ sec. or less than 4 days (6 inertial periods, say).

The Ekman layer suction (or discharge, in the bottom layer) also influences the first order flow in much the same manner as in the well known "spin-up" problem. The simplest way to estimate such effects is to consider the vorticity balance of the core flow. In the top layer,

Ekman layer suction at the thermocline "stretches" fluid columns and produces cyclonic vorticity. The vorticity of the first order current is anticyclonic, so that, as is to be expected, the Ekman suction tends to destroy the top layer jet. The converse is the case in the bottom layer: fluid is being discharged from both Ekman layers, at the thermocline and at the bottom, "squashing" fluid columns and generating anticyclonic vorticity, i.e. again vorticity of opposite sign as compared to the vorticity of the first order current.

The relaxation time ("spin-up" time) of this process is easily calculated. The vertical velocity is given by the divergence of the Ekman transport, hence in the top layer by

$$w = - \frac{dV}{dy} = \frac{fD}{8\pi} \cdot \frac{h_o' + h_o}{h_o'} \cdot e^{-y/R} \quad (18)$$

having used Eq. (17). The rate at which vorticity is being generated is therefore

$$\frac{\partial \omega}{\partial t} = \frac{f}{h} w = \frac{f^2 D}{8\pi h} \frac{h_o' + h_o}{h_o'} e^{-y/R} \quad (19)$$

The original vorticity is

$$\omega = - \frac{\partial u}{\partial y} = - f e^{-y/R} \quad (20)$$

Therefore at any location y the vorticity decays initially as e^{-t/t_s} where the "spin-up" time t_s is

$$\begin{aligned} t_s = \frac{w}{\partial \omega / \partial t} &= \frac{8\pi}{fD} \frac{h_o h_o'}{h_o + h_o'} (1 - e^{-y/R}) \\ &= 4 T_p \frac{h}{D} \cdot \frac{h_o'}{h_o' + h_o} \end{aligned} \quad (21)$$

Here $T_p = 2\pi/f$ is the half pendulum day. The factor multiplying $4 T_p$ in this result is nearly h/D and the result itself is not valid unless $h > D$, at least. As $y \rightarrow \infty$, h/D becomes relatively large (as we pointed out in the previous section) say of order 10. Therefore the "spin-up" time is typically between 4 and 40 inertial periods, for varying distances y . At least reasonably close to the free surface intersection this is very much as the time scale of low-energy fluid accumulation, as may be expected. A very similar time scale may be calculated for the bottom layer, using the Ekman suction of the thermocline only. The bottom Ekman layer alone would produce a spin-up time which may be calculated assuming a drag coefficient. The Ekman transport is there

$$V_b = \frac{u^{*2}}{f} \quad (22)$$

where u^* is friction velocity, assumed to be related to u' as

$$u^{*2} = c_d u'^2 \quad (22)$$

where c_d = drag coefficient, typically perhaps 2.10^{-3} . Carrying out essentially the same calculations as above we find

$$t_s = \frac{h_o' (h_o' + h_o)}{2h_o c_d c} \quad (24)$$

a "typical" value of which is again 100 hrs. The actual spin-up time of the bottom layer is the harmonic mean of the two values given separately by the thermocline and bottom Ekman layer discharges, or rather less than the above value, say 3 inertial periods in order of magnitude, instead of 6.

These quantitative estimates are of course very crude and no undue significance should be attached to the difference between 3 and 6 inertial periods. The agreement between our various estimates, however, encourages belief that the first order flow would be destroyed by Ekman layer suction in a period of the order of 10 inertial periods. A new wind-stress impulse is needed within this time if an upwelling is to be maintained.

To conclude this section we shall attempt to estimate the width of the turbulent free shear layer underneath the thermocline - free surface intersection. This is a turbulent flow problem with remarkably few external parameters: at high enough Reynolds numbers, if there is an equilibrium thickness L attained by such a layer, it can only depend on the velocity difference u' across the layer, and the Coriolis force, i.e. by dimensional reasoning:

$$L = \text{const.} \frac{u'}{f} \quad (25)$$

This result is of very much the same form as the scale relationship applying to a neutral equilibrium Ekman layer over a solid surface. A crude physical explanation why such a layer should form is obtained by considering a fluid particle of velocity u' , displaced to the shoreward side of the flow: there is no pressure gradient there to balance the Coriolis force and the particle is deflected back toward the high speed zone. If we brought the fluid impulsively to a velocity u' at time zero (at $y \geq 0$ only), a shear layer known as a "mixing layer" (observable at the edge of a free jet, Townsend 1956, p. 172) would at first form. The thickness of this layer would increase in time much as it increases with

distance x downstream from a jet (where its thickness is about $L = 0.15 x$).

In our case therefore the initial growth would be according to

$$L = 0.15 u' t \quad (26)$$

In a period comparable to the half pendulum day an equilibrium thickness would be obtained, which would therefore be

$$L = 0.15 u' T_p \approx \frac{u'}{f} \quad (27)$$

giving the estimate of 1.0 for the constant in Eq. (25). Of course a reliable value of this constant could only be obtained by experiment. Nevertheless our crude estimate shows L to be typically of order 10^5 cm or 1 km. Note that the laminar-theory result that the Stewartson layer thickness is proportional to $E^{1/3}$ and $E^{1/4}$ is just as irrelevant as the similar estimate ($E^{1/2}$) for the thickness of the horizontal turbulent boundary layer over a solid surface, because the Ekman number is itself an output variable of the turbulent flow.

DISCUSSION

There is not sufficient information available to subject the above theoretical conclusions to comprehensive experimental checks. Some piecemeal evidence may, however, be offered in support.

(1) There should be little doubt about the existence of a skewed shear layer at the thermocline, similar to the double Ekman layer we have arrived at. However, an important element in our subsequent discussions was the estimate that this shear layer is only a few metres thick. That this at least can be so is demonstrated in Fig. 5. This figure shows the velocity distribution in the thermocline region of Lake Ontario, 14 km from shore, observed on an occasion when the thermocline was quite sharp and when higher order baroclinic motions did not complicate matters. Also, the velocities were high enough below the thermocline to obtain at least a rough picture of their distribution in the course of the "coastal jet" project (Csanady and Pade 1971, see also Csanady, 1972b). The accuracy of direction readings was only $\pm 10^\circ$, of speed readings ± 2 cm/sec except that below 5 cm/sec the readings rapidly worsened. Also, readings were taken only at every 2 metres, providing only a few vectors in the shear zone proper.

With all these shortcomings, Fig. 5 still shows clearly that (a) the thickness of the shear zone is only a few metres and (b) there is an Ekman transport away from shore below the thermocline. The opposite transport above is not apparent in this figure, but this may be due to the inaccuracy of the current direction readings. The magnitude of the volume transport V in this instance is almost an order of magnitude greater than our estimate of $200 \text{ cm}^2/\text{sec}$ given above, which is perhaps the reason why

the spiral could be easily identified in this instance by direct observation.

(2) Our theoretical results have shown pronounced upward motions in the free shear layer underneath the free surface-thermocline intersection. When an upwelled thermocline first becomes established, the upward flow occurs mainly in a band close to shore, as demonstrated from observational evidence by Sverdrup (1938). Thus there should be two regions where nutrient enrichment of surface waters is most pronounced: at the shore and at the thermocline outcropping. Some observational evidence in favor of this conclusion may be found in a paper by Armstrong et al (1967). The observations were taken off the coast of Peru and show an outer "rich" zone a few km wide, clearly separated from the coast, and at the thermocline outcropping.

(3) Our theoretical discussion implies a periodic regeneration of thermocline upwelling and of its associated quasi-geostrophic currents by wind-stress impulses, and a subsequent decay with a time scale of the order of 100 hrs. In the Great Lakes, at any rate, the generation of upwelling episodes by appropriate wind stress impulses is easily demonstrated. What their time scale of decay is, is not so easily decided because opposing or reinforcing windstress impulses normally obscure any decay. On the basis of present evidence, however, one would be inclined to estimate the decay period to be rather longer than 100 hrs. The action of wind stress in this context may therefore legitimately be described as a "nutrient pump", an expression used by Steven (1971) in connection with the Gulf of St. Lawrence upwelling.

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LEGENDS FOR ILLUSTRATIONS

- Fig. 1. Elementary summer thermocline model with uptilt (upwelling) showing also associated velocity distribution.
- Fig. 2. Double Ekman spiral at thermocline, calculated with constant eddy viscosity, velocity vector distribution. Depths are marked in fraction of the Ekman depth.
- Fig. 3. As Fig. 2, stress distribution.
- Fig. 4. Inferred secondary circulation near an upwelled thermocline.
- Fig. 5. Observed velocity distribution in a sharp thermocline zone. Lake Ontario, September 1971.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5.

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16. Abstract (Limit: 200 words) <p>A first order, linear analysis of thermocline upwelling shows that a quasigeostrophic baroclinic jet accompanies the upwelling. The secondary circulation induced in such a flow field in a meridional plane by turbulent friction is investigated. A double Ekman layer is found to form at the thermocline, of a few meters in thickness. The suction of this Ekman layer destroys the primary flow by the "spin-up" mechanism in a period of which the order of magnitude is tentatively estimated at 100 hrs. Thus the primary flow must be reestablished by successive, reasonably frequent wind-stress impulses to maintain a quasi-stationary upwelling.</p> <p>The lower side Ekman layer on the thermocline is fed from a free shear layer (turbulent "Stewartson layer") underneath the free surface-thermocline intersection, which thus becomes an upwelling zone. The width of the turbulent Stewartson layer is estimated to be the velocity difference divided by the Coriolis parameter, i.e. typically of the order of 1 km.</p>			
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