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Woods Hole Oceanographic Institution

Synthetic Aperture Image Holography

by

Tomamasa Sato and Jules S. Jaffe

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Technical Report

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A handwritten signature in dark ink, appearing to read 'Robert C. Spindel', is written over the 'Approved for Distribution' text.

Robert C. Spindel, Chairman
Department of Ocean Engineering

STRUCTURE OF THE PAPER

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NOTATIONS

j	imaginary unit
N	number of transmitting elements
M	number of receiving elements
c	ultrasound velocity
ω	angular frequency
$\underline{r}_T$	the position vector of the transmitter T = $[\xi_T, 0]$
$\underline{r}$	the position vector of the point P on the surface of the target = $[x, a]$
$\underline{r}_\ell$	the position vector of the point Q on the surface of acoustic lens = $[\xi_\ell, 0]$
$\underline{r}_r$	the position vector of the point R on the receiving plane = $[x_r, -b]$
$\underline{r}_R$	the position vector of the image reconstruction point = $[x_R, a_R]$
$s_T(t)$	transmitting ultrasound signal
$S_T(\omega)$	Fourier spectrum of $s_T(t)$
$s_\ell(t)$	refracted ultrasound signal evaluated at the point Q
$S_\ell(\omega)$	Fourier spectrum of $s_\ell(t)$
$s_r(t)$	received signal at the point R on the receiving plane
$S_r(\omega)$	Fourier spectrum of $s_r(t)$
$\rho(\underline{r})$	the complex reflectivity of the target at the position of $\underline{r}$
a	the distance between target plane and the acoustic lens
b	the distance between receiver and the acoustic lens
f_ℓ	the focal length of acoustic lens
$IH(\underline{r}_T, \underline{r}_r)$	image hologram
$FI(\underline{r}_R)$	synthesized final image
$O(\underline{r}_R)$	image obtained by conventional acoustic holography
$H(\underline{r}_T, \underline{r}_R)$	Hologram from which $O(\underline{r}_R)$ is reconstructed

ABSTRACT

This paper describes the underwater imaging method of "Synthetic Aperture Image Holography" which has real-time imaging capability. This method utilizes image holography and computer synthetic aperture techniques. The imaging system consists of the acoustic lens, ultrasound receiving and transmitting arrays, and the computer. The diameter of the acoustic lens is smaller than the aperture of the transmitting array. By transmitting the ultrasound from the elements of the transmitting array one by one, the phase and amplitude of the lower resolution images (image holograms), focused by the acoustic lens, are detected by the receiving array. After obtaining all of the image holograms, the final image is calculated by synthesizing all of these image holograms in the computer. The results of one-dimensional underwater experiments are shown. The conceptual design of a real-time imaging system that is based on this method is also presented.

1 INTRODUCTION

Acoustic computer holography is the imaging method in which the information of the target wavefront is stored once in the memory as a hologram, and after that, the image is reconstructed by computer calculations. The advantage of this method is the ease with which the image quality is improved by introducing the technique of synthetic aperture or utilizing multi-frequency information. However, the improvement of the image quality requires a large amount of data processing. This has prevented the realization of a real-time imaging system.

To shorten this image reconstruction time, K. Mano and K. Nitadori introduced a technique of using a phased array along with acoustic holography [1]. They executed the image reconstruction process by forming the transmitting array. Since only the image reconstruction from the data of receiving array is done by the computer, the imaging system was able to realize a frame rate of 2 frames/sec. The disadvantage of this method is the complexity of the electric circuit of the phased array.

In this paper, the author proposes another imaging method which also reduces the amount of image reconstruction calculations by using an acoustic lens [2]. This imaging method is a combination of acoustic image holography using an acoustic lens, and the synthetic aperture technique.

The imaging system consists of the acoustic lens, ultrasound receiving and transmitting arrays, and the computer. By transmitting the ultrasound from the elements of the transmitting array one by one, the phase and amplitude of the lower resolutional images (image holograms), focused by the acoustic lens, are detected by the receiving array. After obtaining all the image holograms, the image is calculated by synthesizing all these image holograms in the computer. The advantage of this method is that the system has a simpler circuit than that of Mano's because it does not utilize the beam-formed ultrasound transmission. Another advantage is that the system doesn't need a

large diameter acoustic lens because the lateral resolution is not limited by the diameter of acoustic lens like other focused imaging systems [3, 4]. In this system, the resolution is determined only by the aperture length of transmitting array.

2. IMAGING PROCEDURES

Figure 1 shows an overview of the imaging method which is based on Synthetic Aperture Image Holography. Figure 2 shows the coordinates of the imaging system. The imaging method consists of the following two steps as shown in Section 2-1 and 2-2.

2-1 Recording Step of Image Holograms

One transmitter T at $\underline{r}_t$ projects ultrasound to the target. The reflected ultrasound from the target is then focused by an acoustic lens. The amplitude and phase of this image is detected by the receiving array. Since this image has information about both the phase and the amplitude, it is an image hologram. [This image (image hologram) is a lower resolution image than the synthesized one obtained by the synthetic aperture calculations over the transmitting array, because it is focused by the acoustic lens, the diameter of which is smaller than the aperture of the transmitting array.] By changing the transmitters from T_1 to T_N , N image holograms are obtained.

(a) The transmitting ultrasound signal $s_T(t)$ has an angular frequency which is ω . Its Fourier spectrum is given by:

$$S_T(\omega) = \int_t s_T(t) \exp(-j\omega t) dt \quad (1)$$

(b) Let $\rho(\underline{r})$ be the complex reflectivity of point P on the target where $\underline{r}$ is the position vector of point P . The reflected signal from the target arrives to the point Q ($\underline{r}_\rho$ is the position vector of Q) which is on the surface of the acoustic lens. The reflected signal $s_\rho(t)$ evaluated at the point Q can be approximated as follows, neglecting the ultrasound absorption.

$$s_{\ell}(t) = k_1 \int_{\underline{r}} \rho(\underline{r}) \cdot s_T(t - \frac{|\underline{r}-\underline{r}_T| + |\underline{r}-\underline{r}_{\ell}|}{c}) \quad (2)$$

where k_1 is a constant.

By setting $t' = t - (|\underline{r}-\underline{r}_T| + |\underline{r}-\underline{r}_{\ell}|)/c$, equation (2) can be rewritten as follows in the Fourier domain.

$$s_{\ell}(\omega, \underline{r}_{\ell}) = k_2 \int_{\underline{r}} S_T(\omega) \rho(\underline{r}) \cdot \exp[-\frac{j\omega}{c} (|\underline{r}-\underline{r}_T| + |\underline{r}-\underline{r}_{\ell}|)] d\underline{r} \quad (3)$$

where k_2 is a constant.

(c) The acoustic lens causes the phase shift of $\exp(j\omega |\underline{r}_{\ell}|^2 / 2cf_{\ell})$ where f_{ℓ} is the focal length of the acoustic lens. The phase shifted ultrasound constructs the image of the target at the receiver plane. The receiver signal $s_r(t)$ and its Fourier spectrum $S_r(\omega)$ at the point R can be expressed as follows, because $\omega |\underline{r}_{\ell} - \underline{r}_r| / c$ is the phase shift of ultrasound travelling between acoustic lens and the receiver.

$$s_r(t) = k_3 \int_{\underline{r}_{\ell}} e^{\frac{j\omega |\underline{r}_{\ell}|^2}{2cf_{\ell}}} s_{\ell}(t - \frac{|\underline{r}_{\ell}-\underline{r}_r|}{c}) d\underline{r}_{\ell} \quad (4)$$

$$S_r(\omega, \underline{r}_r) = \frac{k_2}{A_{\ell}} \int_{\underline{r}_{\ell}} S_{\ell}(\omega) \cdot \exp(\frac{j\omega |\underline{r}_{\ell}|^2}{2cf_{\ell}}) \cdot \exp(-\frac{j\omega |\underline{r}_{\ell}-\underline{r}_r|}{c}) d\underline{r}_{\ell} \quad (5)$$

where k_2 is a constant and A_{ℓ} is the diameter of acoustic lens.

(d) The phase and amplitude of the received signal measured by using the original transmitting signal $s_T(t)$ of expression (1) as the reference signal construct the image hologram. The Fourier domain expression of this image hologram $IH_i(\underline{r}_r)$ at the point (vector $\underline{r}_r$) can be written as follows:

$$IH(\underline{r}_T, \underline{r}_R) = \frac{S_r(\omega)}{S_T(\omega)} \quad (6)$$

By substituting equations (3) & (5) into (6), the image hologram can be expressed as follows:

$$IH(\underline{r}_T, \underline{r}_R) = \frac{k_4}{A_\ell} \int_{\underline{r}_\ell} \int_{\underline{r}} \rho(\underline{r}) \cdot \exp[-j \frac{\omega}{c} (|\underline{r}-\underline{r}_T| + |\underline{r}-\underline{r}_\ell| - \frac{\omega |\underline{r}_\ell|^2}{2cf_\ell} + |\underline{r}_\ell - \underline{r}_R|)] d\underline{r} d\underline{r}_\ell \quad (7)$$

where k_4 is a constant.

2-2 Synthesis Step of Image Holograms

After N image holograms are obtained, the final image is calculated by synthesizing all of these holograms. The image is calculated to be in the target plane but now at a position $\underline{r}_R$. These calculations contain a phase correction term which is due to the transmitter and the receiver position. Since this final image $FI(\underline{r}_R)$ is synthesized by the larger aperture transmitter array, it has a higher resolution than that of the image hologram.

$$FI(\underline{r}_R) = \frac{1}{A_\ell A_T} \int_{\underline{r}_T} IH(\underline{r}_T, \underline{r}_R) \cdot \exp[\frac{\omega}{c} (|\underline{r}_R - \underline{r}_T| + |\underline{r}_R - \underline{r}_T|)] d\underline{r}_T \quad (8)$$

where A_T is the length of the transmitting array. The exponential term represents the phase correction term which corresponds to the delay time of τ . τ is the elapsed time of ultrasound to travel from transmitter to receiver via target. τ is expressed as follows:

$$\tau = \frac{|\underline{r}_R - \underline{r}_T| + |\underline{r}_R - \underline{r}_r|}{c} \quad (9)$$

Substituting (7) into (8), we can summarize the imaging processes as follows:

$$FI(\underline{r}_R) = \frac{k_5}{A_\ell A_T} \int_{\underline{r}_\ell} \int_{\underline{r}_T} \int \rho(\underline{r}) \cdot \exp(j\omega\Phi) d\underline{r} d\underline{r}_\ell d\underline{r}_T$$

where k_5 is a constant and

$$\begin{aligned} \Phi &= -\Phi_T - \Phi_\ell + \tilde{\Phi}_T + \tilde{\Phi}_r \\ \Phi_T &= \frac{\omega}{c} |\underline{r} - \underline{r}_T| \\ \Phi_\ell &= \frac{\omega}{c} (|\underline{r} - \underline{r}_\ell| - \frac{|\underline{r}_\ell|^2}{2f_\ell} + |\underline{r}_\ell - \underline{r}_r|) \\ \tilde{\Phi}_T &= \frac{\omega}{c} |\underline{r}_R - \underline{r}_T| \\ \tilde{\Phi}_r &= \frac{\omega}{c} |\underline{r}_R - \underline{r}_r| \end{aligned} \quad (10)$$

$-\Phi_T$ and $-\Phi_\ell$ represent the phase shift term due to the disturbance between the transmitter and target, and target and receiver through the acoustic lens.

$\tilde{\Phi}_r$ and $\tilde{\Phi}_T$ represent the phase compensation in image reconstruction time between transmitter and image pixel and between receiver and image pixel, respectively.

3. PROPERTY OF IMAGING METHOD

3-1 Image Quality Analysis

To avoid complexity, the following discussion is done using the one-dimensional, continuous imaging model shown in Figure 2. Substituting the following approximations (11) into (10), the Φ can be approximated as expression (12).

$$\begin{aligned}
|\underline{r} - \underline{r}_T| &= \sqrt{(x - \xi_T)^2 + a^2} = a + (x - \xi_T)^2/2a \\
|\underline{r} - \underline{r}_\ell| &= \sqrt{(x - \xi_\ell)^2 + a^2} = a + (x - \xi_\ell)^2/2a \\
|\underline{r}_\ell - \underline{r}_r| &= \sqrt{(\xi_\ell - x_r)^2 + b^2} = b + (\xi_\ell - x_r)^2/2a \\
|\underline{r}_R - \underline{r}_T| &= \sqrt{(x_R - \xi_T)^2 + a_R^2} = a_R + (x_R - \xi_T)^2/2a \\
|\underline{r}_R - \underline{r}_r| &= \sqrt{(x_R - x_r)^2 + (a_R + b)^2} = a_R + b + (x_R - x_r)^2/2(a+b)
\end{aligned} \tag{11}$$

$$\begin{aligned}
\Phi = j \frac{\omega}{c} [& 2(a_R - a) + \frac{x_R^2}{2a} + \frac{(x_R - x_r)^2}{2(a+b)} - \frac{x^2}{a} - \frac{x_r^2}{2b} + (\frac{x}{a} - \frac{x_R}{a}) \xi_T \\
& - \frac{1}{2} (\frac{1}{a} + \frac{1}{b} - \frac{1}{f_\ell}) \xi_\ell^2 + (\frac{x}{a} + \frac{x_r}{b}) \xi_\ell]
\end{aligned} \tag{12}$$

By using the following conditions

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f_\ell} \quad \text{and} \quad x_r = \frac{b}{a} x_R \tag{13}$$

We can obtain

$$\Phi = j \frac{\omega}{c} [2(a_R - a) + \frac{x_R^2 - x^2}{a} + \frac{x - x_R}{a} \xi_T + \frac{x - x_R}{a} \xi_\ell] \tag{14}$$

Calculating the integration in (10) over ξ_ℓ ($-\frac{A_\ell}{2} \leq \xi_\ell \leq \frac{A_\ell}{2}$) and ξ_T ($\frac{A_T}{2} \leq \xi_\ell \leq \frac{A_T}{2}$), the following expression (15) can be obtained:

$$FI(x_R, a_R) = k_6 \exp \left\{ j \frac{\omega}{c} \left[2(a_R - a) + \frac{x_R^2 - x^2}{a} \right] \right\} \times \int_{\underline{r}} \rho(r) \operatorname{sinc} \left[\frac{\omega}{ca} A_T (x_R - x) \right] \operatorname{sinc} \left[\frac{\omega}{2ca} A_\ell (x_R - x) \right] d\underline{r} \quad (15)$$

where A_T is the length of the transmitting array. The length between the peak and the first zero point of the synthesized image Δ_x is given as follows, because $A_T > A_\ell$.

$$\Delta_x = \frac{2\pi ca}{\omega A_T} \quad (16)$$

This expression (16) represents that the lateral resolution x is determined by the length of the transmitter aperture. Due to this fact, the imaging method does not need a large diameter lens to obtain lateral resolution.

3-2 Amount of Calculations

Consider the conventional acoustic holography imaging method illustrated in Figure 3. The detected hologram $H(\underline{r}_T, \underline{r}_R)$ can be expressed as follows, because $\omega (|\underline{r} - \underline{r}_T| + |\underline{r} - \underline{r}_R|)/c$ is the phase shift caused by the delay in ultrasound transmission between the transmitter and the receiver.

$$H(\underline{r}_T, \underline{r}_R) = k_7 \int_{\underline{r}} \rho(\underline{r}) \exp \left[-j \frac{\omega}{c} (|\underline{r} - \underline{r}_T| + |\underline{r} - \underline{r}_R|) \right] d\underline{r} \quad (17)$$

Where k_7 is a constant.

Since the image reconstruction is regarded as the phase compensating process of the above phase shift: i.e., image reconstruction is the inverse transformation of the expression (17), the reconstruction image $O(r_R)$ can be shown to be equal to the following expression.

$$O(r_R) = \frac{k_7}{A_\ell A_T} \int_{r_T} \int_{r_r} H(r_T, r_r) \cdot \exp\{j\frac{\omega}{c}(|r_R - r_t| + |r_R - r_r|)\} dr_r dr_T \quad (18)$$

where k_7 is a constant

By substituting (18) into (17), the imaging procedure can be summarized as:

$$O(r_R) = \frac{k_7}{A_\ell A_T} \int_{r_T} \int_{r_r} \rho(r) \cdot \exp(j\Phi') dr_r dr_T$$

where $\Phi' = -\Phi'_T - \Phi'_r + \tilde{\Phi}'_T + \tilde{\Phi}'_r$

$$\begin{aligned} \Phi'_T &= \frac{\omega}{c} |r - r_T| \\ \Phi'_r &= \frac{\omega}{c} |r - r_r| \\ \tilde{\Phi}'_T &= \frac{\omega}{c} |r_R - r_T| \\ \tilde{\Phi}'_r &= \frac{\omega}{c} |r_R - r_r| \end{aligned} \quad (19)$$

As shown in Appendix A, the imaging method of Figure 1 and Figure 3, or the expression (10) and expression (19) represent the same thing. Since these two imaging processes are the same, consider the amount of calculations in both imaging methods. The conventional acoustic holography imaging method of expression (18) can be rewritten as follows (introducing a discrete representation of the data).

$$\begin{aligned}
O_{\ell}(\underline{r}_{R\ell}) &= \frac{k_8}{MN} \sum_{k=1}^M \exp \left[j \frac{\omega}{c} (\underline{r}_{R\ell} - \underline{r}_{T_k}) \right] \\
&\times \sum_{i=1}^N H(\underline{r}_{T_k}, \underline{r}_{r_i}) \cdot \exp \left[j \frac{\omega}{c} (\underline{r}_{R\ell} - \underline{r}_{r_i}) \right] \\
&(\ell = 1, \dots, N)
\end{aligned} \tag{20}$$

where k_8 is a constant.

If a data table of the exponential terms is prepared before image reconstruction time, the amount of multiplication and addition between two complex numbers in the above expression becomes $NM(1+N)$. Since in the Synthetic Aperture Imaging Method of expression (10) the integration calculation of the phase term concerning the acoustic lens is realized by the physical acoustic lens, the necessary calculation is only the integration concerning the transmitting array expression (8). Since $\underline{r}_R$: $\underline{r}_r = a:b$, the imaging procedure of (8) becomes as follows in case of sampled data.

$$FI(\underline{r}_{R\ell}) = \frac{k_9}{M} \sum_{k=1}^M \exp \left\{ j \left(\frac{\omega}{c} |\underline{r}_{R\ell} - \underline{r}_{T_k}| \right) \right\} IH(\underline{r}_{T_k}, \underline{r}_{r_i}) \quad (\ell=1, \dots, N, i=1, \dots, N) \tag{21}$$

Where k_9 is a constant.

If the exponential is given beforehand, the amount of calculation becomes NM . Comparing the case where all calculations are done in the computer, the acoustic lens reduces the number of calculations by

$$1/(1 + N) \cong 1/N \text{ (if } N \gg 1) \quad (22)$$

3-3 Number of Receivers and the Number of Synthesized Image Pixels

The lateral resolution of the image holograms is lower than that of the final one. So, the sampling distance of an image hologram is governed by the resolution of the hologram which is determined by the lens aperture, not the transmitter array aperture. However, to synthesize the final image, interpolation should be done. In this paper, the author explores three methods. The first one is based on the point spread function of an acoustic lens. This method can be implemented by using the FFT algorithm. In this method, the Fourier spectrum of the image hologram is calculated first. Next, after filling zero data into the higher frequency components, the inverse Fourier transformation gives us the interpolated image hologram data. As shown in expression (15), the image hologram is band limited by the acoustic lens. This interpolation gives us the exact interpolated image hologram if there is no additional noise.

The second interpolation method is based on average calculations. The interpolated data is obtained by linearly interpolating the adjacent pixels. The third method is calculated by using the same data repeatedly, or by pixel replication. Since this method contains no arithmetic calculations, the calculation time is the shortest.

4. A ONE-DIMENSIONAL IMAGING EXPERIMENT

4-1 Overview of the Experiment

Figure 4 shows the schematic diagram of experimental imaging system. The imaging target (0.78 ϕ mm wire), the acoustic lens and the transmitting and receiving array are placed in the water tank. The direction of the

straight wire and the direction of the linear receiving array are adjusted so that they are perpendicular to each other. The pulse generator and digital oscilloscope are placed near the tank.

4-2 Signal and Information Flow of the Experiment

The signal flow of the experiment is as follows:

(1) First, a 1.036 MHz continuous square wave generated by the pulse generator is divided into two parts. One of the divided signals is fed to channel A of the digital oscilloscope as a reference wave.

The other divided signal is converted to a pulse signal by the time gate circuit and is fed to the ultrasound amplifier. The duration of the pulse is 36.0 microseconds. The output signal of the amplifier is used to drive the first transmitting element T_1 . Thus, the ultrasound illuminates the target. The reflected ultrasound from the target is focused by the acoustic lens, and the image of the target is detected by the receiving element R_i ($i=1,2,\dots,14,17,\dots,32$).

The detected signal is fed to channel B of the digital oscilloscope and is converted to a digital format to be compared with the reference signal. The amplitude ratio and the phase difference between the reflected ultrasound signal and the reference signal is calculated by the LSI-11/2 micro-computer. These results are sent to the VAX11/750 and stored as the data of one pixel of the image hologram of the target. By repeating these procedures, one image hologram IH_1 can be obtained. Since there are 30 receiving elements, 30 pulses of ultrasound transmission and reception will form one image hologram.

(2) Another three time repetition of procedure (1) with respect to the transmitting elements T_2, T_3, T_4 yields the image holograms IH_2, IH_3, IH_4 .

(3) After obtaining four image holograms, the synthesis of these image holograms is done by computer calculations in the VAX11/750. The algorithm of this synthesis is described in expression (24). The synthesized final image is then displayed on a graphics terminal.

4-3 Experimental System

Figure 5 shows the picture of the experimental system.

(a) Transmitting and Receiving Array

The transmitting array has four transmitting elements. The receiving array consists of 30 receiving elements. The dimensions of these arrays are shown in Figure 6. The spacing of receiving elements has been chosen so that they are less than the resolution of individual image holograms and are greater than the resolution of synthesized image. Table 1 contains the detailed specifications of these elements.

(b) Acoustic Lens

Figure 6 also shows the shape of the acoustic lens and the geometrical configurations of the arrays and lens. The lens is constructed of polyethylene. Table 1 also contains the specifications of the lens.

(c) Computer System

Two computers are used for the experiment. The LSI-11/2 controls the transmitting multiplexer, the receiving elements, and the digital oscilloscope. This computer also calculates the amplitude ratio and the phase difference between the reference signal and the refracted ultrasound. The VAX11/750 is used to store the image data and calculate the image synthesis. The interactive execution of experiments is realized by the small monitor on the VAX11/750.

5. RESULTS OF THE EXPERIMENT

5-1 Results

Figure 7 A ~ D shows the measured four image holograms: IH1, IH2, IH3, IH4, respectively. The upper graph of each figure represents the power (square of amplitude) distribution of the image hologram; the lower graph shows the phase of the hologram. The transverse axis represents the position of the image pixel, which has a sampling distance of 3.13 mm.

Figure 8 A ~ C shows the interpolated image hologram (left) and the synthesized image of the target (right) by three interpolation methods. The real position of the target is located at an x location of 35 in that figure. The upper graph of each figure represents the power (square of amplitude) distribution of the synthesized image; the lower graph shows the phase of it. The traverse axis represents the position of the image pixel, the sampling distance is 1.57 ($=3.13/2$) mm.

5-2 Considerations

Table 2 summarizes the lateral resolutions of the obtained images. The lateral resolution is obtained by measuring the distance between -3.92 DB points from the maximum power point, which gives the equivalent distance of first zero point in sinc function. It shows good correspondence between experimental and theoretical resolution. By comparing the resolution of the image hologram and the synthesized image, the lateral resolution can be improved by using the synthetic aperture technique. In other words, the lateral resolution is determined by the aperture length of the transmitting array. This allows us to utilize a smaller diameter acoustic lens. Figure 8 also shows that the three interpolation methods can be effectively used to obtain the synthesized image.

Although these initial results are promising, we are not completely satisfied with the correlation between our experimental results and the theoretical computer model. In particular, the synthetic image does not show as high resolution as we expected. Further computer simulations will be pursued to resolve the differences and improve the image quality.

6. CONCEPTUAL DESIGN OF A REAL-TIME IMAGING SYSTEM

In this section, the author shows the conceptual design of a real-time imaging system using this method. The imaging system is supposed to have a 16x16 element receiving array and a 4x4 element transmitting array, as shown in Figure 8. Let the frequency of ultrasound be 2 MHz. It is also assumed that 15 μ s pulse duration and all receivers are active simultaneously and measure the image hologram at the same time. By transmitting pulse signals every 20 ms, the imaging system can monitor the distance from 1.1 cm ($= 0.5 \times 1500 \text{ m/s} \times 15 \mu\text{s}$) to 15m ($= 0.5 \times 1500 \text{ m/s} \times 20 \text{ ms}$). In this case, one image hologram can be obtained every $(20 \text{ ms} + 15 \mu\text{s}) = 20.015 \text{ ms}$. The process of obtaining the final image contains complex interpolations and image synthesis calculations. If we utilize the linear interpolation method, the interpolated pixel of image hologram PI between two measured image hologram pixels P1 and P2 can be calculated by the following algorithm.

$$PI = r1 \cdot P1 + r2 \cdot P2 \quad (r1 + r2 = 1) \quad (23)$$

This can be realized three complex multiply-and-sum calculations. If we denote the computing time of a complex multiply-and-sum as T_{ms} , and apply interpolation calculations to all image pixels of the interpolated image holograms (and say $N \times M$ pixels), the computation time T_i can be evaluated by $T_i = 3 \times M \times N \times T_{ms}$. Furthermore, as shown in the expression (24), the synthesis calculation is realized by one complex multiply-and-sum computation for one transmitter-pixel pair, if we prepare the table for phase compensation. The computation time T_s to synthesize the image is $T_s = M \times N \times T_{ms}$. Therefore, the

necessary computing time becomes $T_i + T_s = 4 * M * N * T$. By using a computer which has $T_{ms} = 1.0 \mu s$ capability, the synthesized image can be obtained within 16 ms ($= 4 * 4 * 4 * 16 * 16 * 1.0 \mu s$). This synthesizing time enables us to obtain synthesized images before the next transmission time. Since the synthesizing calculation can be completed before the next transmission, the image quality becomes finer along with the number of transmissions. By this consideration, a synthesized final image can be obtained every 320 ms ($= 4 * 4 * 20 ms$). In other words, this conceptual system has a frame rate of 3/s. Considering that the frame rate of 50 frame/s ($= 1 / (20 ms)$) of the image hologram can be used as a rough image of the target, and the underwater environment, this conceptual imaging system can be regarded as a real-time imaging system. In this case, the number of pixels in the final image can be $128 * 128$, because each pixel of the synthesized image contains real and imaginary components.

7. SUMMARY

A new underwater imaging method for real-time ultrasound imaging has been proposed. This method combines image holography techniques via an acoustic lens and synthetic aperture techniques via computer. The imaging system based on this method consists of an acoustic lens, an ultrasound transmitting and receiving array, and a computer.

One of the advantages of this method is that some of the image reconstruction calculations are done by the lens. The result of using this lens is a reduction in the number of calculations of order $1/N$, where N is the number of receiving elements.

Several points were confirmed by the one dimensional underwater experiments. The lateral resolution is limited by the size of the aperture of the transmitting array and not by the aperture of the acoustic lens. This is because the resolution of the system is increased by use of the synthetic

aperture over the normal resolution limit of the lens by itself. Therefore, a smaller diameter acoustic lens can be used than previously considered for a given resolution. Another advantage of the method is that the number of necessary receiving elements has been reduced. This is because the received image hologram is band limited, and therefore, intermediate values can be interpolated.

An additional advantage of the imaging method is the simplicity of the circuitry. Available computer peripherals can be easily utilized and there is no need for the complexities associated with phased array circuitry.

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APPENDIX A

EQUIVALENCE OF HOLOGRAPHY AND SYNTHESIS APERTURE IMAGE HOLOGRAPHY METHODS

We seek to compare both imaging procedures expressed by equations (10) and (19). Since $\tilde{\Phi}_T = \tilde{\Phi}'_T$, $\tilde{\Phi}_T = \tilde{\Phi}'_T$ and $\tilde{\Phi}_r = \tilde{\Phi}'_r$

Compare the phase shift caused by the acoustic lens $-\Phi_\ell$ and the phase shift term $-\Phi'_r$.

By substituting the approximation of equations (11) into $-\Phi_\ell$, this term becomes the following:

$$\begin{aligned} -\Phi_\ell &= \frac{\omega}{c} |\underline{r} - \underline{r}_\ell| - \frac{|\underline{r}_\ell|^2}{2f_\ell} + |\underline{r}_\ell - \underline{r}| \\ &= \frac{\omega}{c} \left\{ a+b + \frac{x^2}{2a} + \frac{x^2}{2b} - \frac{\xi_\ell^2}{2} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{f_\ell} \right) - \left(\frac{x}{a} + \frac{x}{b} \right) \right\} \end{aligned}$$

Substituting the imaging condition (13) into this,

$$\begin{aligned} -\Phi_\ell &= \frac{\omega}{c} \left(a+b + \frac{x^2}{2a} + \frac{x^2}{2b} \right) \\ &= \frac{\omega}{c} \left\{ a+b + \left(\frac{a}{2b^2} + \frac{b}{2b^2} \right) x^2 \right\} = \frac{\omega}{c} \left\{ a+b + \frac{(a+b)}{2b^2} x^2 \right\} \end{aligned}$$

On the other hand,

$$-\Phi_r = \frac{\omega}{c} |\underline{r} - \underline{r}_r| = \frac{\omega}{c} \sqrt{(x-x_r)^2 + (a+b)^2} = \frac{\omega}{c} \left\{ a+b + \frac{(x-x_r)^2}{2(a+b)} \right\}$$

by substituting the imaging condition (13)

$$-\Phi_r = \frac{\omega}{c} \left\{ a+b + \frac{(a+b)^2}{2b^2} x^2 \right\}$$

Since $-\Phi_\ell$ equals $-\Phi_r$, the imaging method of Holography and Synthetic Aperture Image Holography or the expression (10) and expression (19) are the same.

TABLE I

Specification of Experimental Components

Transmitting Elements:

Number of elements		4
resonance frequency:		1.0 MHz
diameter:		19.5 mm
position (ξ_T, z)	1:	(-79.8, 264.5)
	2:	(-56.3, 264.7)
	3:	(56.3, 264.7)
	4:	(79.8, 264.5) mm

Receiving elements

number of elements:	30
resonance frequency:	1.5 MHz
spacing:	3.13 mm

Acoustic lens

material:	polyethylene
sound velocity in lens:	2350 m/s
density:	1.05 g/cm
focal length:	15 cm
minimum thickness:	10 mm
radius:	$r_1 = 162$ $r_2 = 81$ mm

TABLE II
Results of Image Quality

Experimental Environment

water temperature:		18.8°C
ultrasound velocity:		1476.07 m/s
ultrasound frequency:		1.036 MHz
ultrasound wave length:		0.1424 cm
diameter of acoustic lens:	A_ℓ	10 cm
aperture length of transmitting array:	A_t	15.96 cm
distance between lens and receiving element:	b	25 cm
sampling distance of image hologram:		3.12 mm
sampling distance of final image		1.56 mm

Image Quality

Theory		Experiment
(first zero)		(-3.92 dB)
Image Hologram:	$\lambda b/A_\ell = 3.6$	$1.25 \times 3.13 = 3.9 \text{ mm}$
Final Image:	$\lambda b/A_t = 2.2$	$1.5 \times 1.57 = 2.4 \text{ mm}$

Figure 1: An Overview of Synthetic Aperture Image Holography Method

Figure 2: Coordinates of the Synthetic Aperture Image Holography System

Figure 3: Coordinates of the Acoustic Holography Imaging Method

Figure 4: A Schematic diagram of Experimental Imaging System

Figure 5: The Underwater Part of Experimental Imaging System

Figure 6: The Configuration of Experimental Imaging System

Figure 7: Measured Image Holograms

Figure 8: Interpolated Holograms (Left) and Synthesized Final Images (Right) for (A) Sinc Function Interpolated, (B) Linearly Interpolated, and (C) Pixel Replicated Data

Figure 9: A Conceptual Two-Dimensional Imaging System

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