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RESPONSE CHARACTERISTICS OF THE
NEUTRALLY BUOYANT FLOAT

Arthur D. Voorhis

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WOODS HOLE OCEANOGRAPHIC INSTITUTION
Woods Hole, Massachusetts

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RESPONSE CHARACTERISTICS OF THE
NEUTRALLY BUOYANT FLOAT

by

Arthur D. Voorhis

December 1971

TECHNICAL REPORT

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Ferris Webster, Chairman
Department of Physical Oceanography

Abstract

The response characteristics of the neutral buoyant vertical current meter are discussed primarily from a theoretical point of view. Computations are presented to illustrate the useful range and limitations of present instruments. Three aspects of the response are considered separately: rotational, vertical, and tilting due to vertical shear.

The rotational response at low frequencies is poor only for very weak vertical currents due to viscous drag. At high frequencies the response is uniformly limited by the moment of inertia of the instrument.

The vertical response is best in neutrally stable water. In highly stratified water, on the other hand, the response deteriorates for all frequencies because of the small relative flow past the instrument. In all cases it is limited at high frequencies because of instrument inertia.

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Response Characteristics of the Neutrally Buoyant Vertical Current Meter

I Introduction

The vertical component of current in the sea is an important feature of a number of processes of interest to the physical oceanographer. Among these are turbulence, upwelling, sinking, and internal wave generation. Unfortunately, it has been difficult to measure because it is usually much weaker than the horizontal current which threatens to contaminate the measurement. This is particularly true for moored instruments where great care must be taken to either align the instrument with the vertical or to make precise measurements of the angle of tilt. In 1965 D. Webb developed a neutrally-buoyant vertical current meter which is capable of measuring vertical currents directly at all depths. Using this instrument Webb and Worthington (1968) tried in 1966 to detect small vertical convection currents in the nearly neutrally stable water of the Cayman Basin with inconclusive results. However, vertical currents were observed having oscillations with a period of 8 hours and were probably due to internal waves. Since then Voorhis (1968) has used these instruments to measure the vertical kinetic energy spectrum of internal waves in the Slope water off New England, in the Sargasso Sea, and in the Gulf Stream. Most recently Voorhis and Webb (1970) measured with the same technique large vertical currents in a winter sinking region of the northwestern Mediterranean Sea.

The vertical current meter is still evolving as an oceanographic instrument and a precise calibration of its complex response characteristics has not been made. As a preliminary step this report discusses in some detail what we consider to be the most important features of its response, using numerical data when possible. From it one can estimate the capabilities and limitations of the instrument.

II Description of the Instrument

Shown in Figures 1 and 2 are photographs of the earliest and latest versions, respectively, of the neutrally buoyant vertical current meter (hereafter termed a VCM). Simply stated, these instruments are Swallow floats (Swallow, 1955) fitted with vanes. They are constructed with sufficient buoyancy and with less compressibility than sea water so that when they are properly ballasted and thrown overboard they sink to a predetermined depth and drift with their axis vertical in stable equilibrium. Fastened around the outside of the VCM is an array of tilted vanes which twist the instrument about its axis when water is advected vertically past it. After launch the VCM rotates as it sinks to its preset depth. At this depth rotation ceases if there are no vertical currents*. On the other hand, rotation will continue if there are vertical currents, such as those due to internal waves. This occurs primarily because the compressibility of the instrument is different than that of sea water. This rotation,

*This also assumes the absence of vertical vorticity. This is discussed by Voorhis (1970).

which for our VCM's is approximately once per meter of vertical water displacement, is detected relative to an internal magnetic compass and is used to measure the vertical flow. In the earliest instruments the rotations were transmitted acoustically to a nearby surface vessel as they occurred and the VCM was expendable. Later models contained a recording circuit which kept a detailed account of number of revolutions as well as of temperature and pressure. They also contain a clock which commands the VCM to drop a weight and surface after a preset time, and a flashing light that helps locate it at night in order to recover the recorded data. All VCM's are equipped with an underwater pinger so that they can be tracked by surface vessel as they drift.

The complete coupled set of six dynamical equations governing the motion of the VCM in a moving stratified sea are too complex to solve. Instead we shall assume the total response can be broken down into three main parts, each considered separately. These are:

- a) rotational response
- b) vertical response
- c) horizontal and tipping response

Part a) describes how well the instrument rotates about a vertical axis in response to the relative vertical current past its vanes. Part b) considers the relation between the relative vertical current and the true vertical current. These are usually not the same because the instrument itself moves vertically. Finally, part c) describes how well the VCM follows or keeps up with the horizontal current and to what extent it tilts due to vertical shear. The latter is important because tilting can introduce unwanted float rotation due to the horizontal flow.

III Rotational Response

The VCM rotates in response to the vertical current past the vanes projecting outward from the instrument. If the angular velocity of the instrument is Ω its rate of change of angular momentum is given by,

$$I, \frac{d\Omega}{dt} = \mathcal{T}_v - \mathcal{T}_e \quad (1)$$

where $I,$ is the moment of inertial of the instrument about a vertical axis, $\mathcal{T}_v$ is the driving torque, and $\mathcal{T}_e$ is the drag torque exerted on the instrument tending to slow it down.

3.1 Driving torque

The driving torque is due to the tangential component of the "lift" generated by the relative vertical flow past the slanted vanes which can change continuously in both magnitude and direction*. The nature of the lift depends on the ratio of the vertical displacement Z_R to the chord ℓ (or width) of the vanes.

3.1.1 Large vertical displacement

When $Z_R > 5\ell$ the flow is of sufficient duration to set up a circulation around the vanes. The lift generated by this circulation is non-linear and when integrated over the area of the vanes gives the following approximate driving torque

$$\mathcal{T}_v = \frac{1}{2} C_L \rho m R_v A_v \cos \alpha \left(W_R - \frac{1}{2\pi} \Omega \right) |W_R| \quad (2)$$

*We are ignoring here the driving torque due to vertical water vorticity.

Here ρ is the water density, n the number of vanes, R_v the mean vane radius, A_v the area of each vane (one side), α the inclination of the vanes to the horizontal, and W_R is the relative vertical water velocity past the instrument, that is,

$$W_R = \frac{dZ_R}{dt} \quad (3)$$

In (2) the quantity λ is a constant length and C_L is a numerical lift coefficient. If we assume thin, flat, rectangular vanes having outer radius r_1 , inner radius r_0 , then

$$\lambda = \frac{4\pi}{3} \frac{r_1^3 - r_0^3}{r_1^2 - r_0^2} \tan \alpha \quad (4)$$

and, from the data summarized by Briggs and Dryden (1926),

$$C_L \approx \frac{6}{1 + 2.3 \frac{t}{r_1 - r_0}} \quad (5)$$

This value for the lift coefficient is only valid provided the driving torque does not exceed the condition

$$\tau_r < \frac{1}{3} \frac{C_L \rho n R_v A_v}{\sin \alpha} \left(\frac{\lambda}{2\pi} \right)^2 \Omega^2 \quad (6)$$

For larger torques the coefficient decreases rapidly due to stalling at the vanes.

3.1.2 Small vertical displacement

When $Z_R < 5l$ the vertical flow in either direction is not long enough to set up a circulation around the vanes. Instead, the tangential lift is primarily impulsive and linear. One finds in this case that the driving torque is given, approximately, by

$$\mathcal{T}_v = \frac{\pi}{8} \rho m l A_v R_v \sin 2\alpha \left(\frac{dW_R}{dt} - \frac{1}{2\pi} \frac{d\Omega}{dt} \right) \quad (7)$$

3.2 Drag torque

The drag torque in (1) depends on the shape of the instrument and on the magnitude and variability of the rotation rate.

3.2.1 Long vertical cylinder

A long circular cylinder rotating about its principal axis generates, through viscosity, a simple cylindrical vortex in the surrounding water with no non-linear angular accelerations. We can represent the drag torque as follows

$$\mathcal{T}_\theta = m a^2 \left[\mathcal{J}_\theta(\omega) \frac{d\Omega}{dt} + \sigma_\theta(\omega) \Omega \right] \quad (8)$$

where m is the instrument mass (assumed to be neutrally buoyant), and a is the cylinder radius. In

(8) $\mathcal{J}_\theta$ is the ratio of the water mass rotated in phase

with the instrument to its own mass, and σ_θ is a reciprocal damping time. Both $\mathcal{J}_\theta$ and σ_θ depend on the frequency of rotation ω according to the theoretical expressions given in Appendix 6.1. They are also plotted as a function of the dimensionless frequency $\omega a^2 / \mu$ in Figure 3, where μ is the kinematic viscosity. Note that for steady rotation ($\omega = 0$) the torque is given simply by

$$\mathcal{J}_\theta = 4\mu m \Omega \quad (9)$$

3.2.2 Other shapes

The drag torque for shapes other than a long cylinder is usually complicated by the presence of non-linear accelerations. Assuming an oscillatory rotation rate of angular frequency ω the torque depends on the magnitude of a rotational Strouhal number,

$$\mathcal{S}_\theta = \frac{\omega}{|\Omega|} = \frac{1}{2\pi |\mathcal{N}|} \quad (10)$$

and a rotational Reynolds number,

$$R_\theta = \frac{a^2 |\Omega|}{\mu} = 2\pi |\mathcal{N}| \frac{a^2 \omega}{\mu} \quad (11)$$

where $\mathcal{N}$ is the number of revolutions turned per cycle, and a is a mean radius. With our instruments in the ocean the Reynolds number is usually much greater than unity.

Assuming this to be true then the drag torque for large rotations, so that $\mathcal{J}_\theta < 1$, can be expressed as,

$$T_0 = C m \mu R^{\alpha-1} \Omega \quad (12)$$

where C, α are constants which depend on the shape of the instrument. For a sphere rotation pumps water from the poles to the equator and Bowden and Lord (1963) found $C = .84$, $\alpha = 3/2$. For very rapid rotation rates with real instruments the torque is more likely to be due to the form drag of various small knobs and protruberances which destroy its radial symmetry. In this case $\alpha = 2$ and C depends on the area swept out by these imperfections.

If R_0 is much greater than unity but the instrument undergoes fractions of a rotation per cycle so that $S_0 > 1$, then the drag torque is confined to a thin oscillating boundary layer of thickness $\sqrt{\frac{2\mu}{\omega}}$ on the surface of the instrument. Regardless of its shape the torque can be represented by (8) where

$$\begin{aligned} T_0(\omega) &= \frac{A_w}{V} \sqrt{\frac{\mu}{2\omega}} \\ T_0(\omega) &= \frac{A_w}{V} \sqrt{\frac{\omega \mu}{2}} \end{aligned} \quad (13)$$

where A_w is the setted surface area, and V the volume of the instrument.

3.3 Response for the long cylinder

For the sake of further discussion we shall assume a long cylindrical instrument, approximated by that in Figure 2. Then

substituting the drag torque given by (8) and the driving torque given by (2) or (7) into (1) one obtains the following equation for the rotational response,

$$\left[1 + \frac{a^2}{R^2} \sigma_{\theta}(\omega)\right] \frac{d\Omega}{dt} + \frac{a^2}{R^2} \sigma_{\theta}(\omega) \Omega = \begin{cases} \frac{2\pi |W_R|}{\Gamma^2} \left(W_R - \frac{\lambda}{2\pi} \Omega\right) & Z_R > 5\ell \\ \frac{2\pi}{\lambda'} \left(\frac{dW_R}{dt} - \frac{\lambda}{2\pi} \frac{d\Omega}{dt}\right) & Z_R < 5\ell \end{cases} \quad (14)$$

Here Γ and λ' are constant lengths defined by

$$\Gamma^2 = \frac{4\pi I_1}{c_L \rho m R_v A_v \cos \alpha} \quad (15)$$

$$\lambda' = \frac{16 I_1}{\rho m \ell A_v R_v \sin 2\alpha} \quad (16)$$

and R is the radius of gyration of the instrument about its vertical axis, i.e.,

$$R^2 = \frac{I_1}{m} \quad (17)$$

3.3.1 Steady rotation

An important situation is the case of steady rotation due to a constant vertical velocity past the vanes. If we define the pitch P_0 of the VCM as the steady relative vertical displacement per revolution η , where $\Omega = 2\pi \frac{d\eta}{dt}$, one finds that

$$\begin{aligned} P_0 &\equiv 2\pi \frac{W_R}{\Omega} = 1 + \frac{\Gamma^2}{T |W_R|} \\ &= \frac{1}{2} + \sqrt{\frac{1}{4} + 2\pi \frac{\Gamma^2}{T |\Omega|}} \end{aligned} \quad (18)$$

where T is the steady rotation damping time given by

$$T = \frac{I_1}{4\mu m} = \frac{R^2}{4\mu} \quad (19)$$

We have plotted P_0 as a function of W_R in Figure 4 for the "cylindrical" instrument shown in Figure 2. The relevant parameters for the calculation were

$$\Gamma = 0.28 \text{ m} \quad \lambda = 1.10 \text{ m} \quad T = 23 \text{ min} \quad (20)$$

Note that the pitch for vertical currents greater than about 1 mm/sec is practically constant and equal to λ .

3.3.2 Response for variable rotation

The rotational frequency response for oscillating vertical flows is complex because of the non-linear character of the driving term in (14). This causes the production of small higher order harmonics in the rotation. If we assume

$$W_R = \int_{-\infty}^{+\infty} \hat{W}_R e^{i\omega t} d\omega$$

$$\Omega = \int_{-\infty}^{+\infty} \hat{\Omega} e^{i\omega t} d\omega$$
(21)

and neglect these harmonics one finds

$$\frac{\hat{W}_R}{\hat{\Omega}} = \frac{p(\omega)}{2\pi} e^{i\phi(\omega)}$$
(22)

where $p(\omega)$ is a frequency dependent pitch given by

$$p = \begin{cases} \frac{\Gamma^2}{|W_R|} \sqrt{\left[\frac{\lambda |W_R|}{\Gamma^2} + \frac{a^2}{R^2} \sigma_\Theta \right]^2 + \omega^2 \left[1 + \frac{a^2}{R^2} \sigma_\Theta \right]^2} & \overline{|Z_R|} > 5\ell \\ \frac{\lambda'}{\omega} \sqrt{\frac{a^4}{R^4} \sigma_\Theta^2 + \omega^2 \left[1 + \frac{1}{\lambda'} + \frac{a^2}{R^2} \sigma_\Theta \right]^2} & \overline{|Z_R|} < 5\ell \end{cases}$$
(23)

and $\varphi(\omega)$ is the phase angle by which the rotation lags the vertical flow, given by

$$\tan \varphi = \begin{cases} \omega \left[\frac{1 + \frac{a^2}{R^2} \mathcal{J}_\Theta}{\frac{\lambda |W_R|}{R^2} + \frac{a^2}{R^2} \sigma_\Theta} \right] & \overline{|Z_R|} > 5\ell \\ -\frac{1}{\omega} \left[\frac{\frac{a^2}{R^2} \sigma_\Theta}{1 + \frac{1}{\lambda'} + \frac{a^2}{R^2} \mathcal{J}_\Theta} \right] & \overline{|Z_R|} < 5\ell \end{cases} \quad (24)$$

In the above $\overline{|W_R|}$ and $\overline{|Z_R|}$ are the mean vertical speed and distance respectively.

The contribution to the rotational response due to higher order harmonics at large amplitudes is small and of the order of $\frac{a^2}{\lambda^2}$ times that given by (23).

3.3.3 Numerical calculations of response

As an example, for the instrument in Figure 2 we have computed $p(\omega)$, given by (23), as a function of frequency for the rather artificial situation where the vertical displacement is monochromatic with constant amplitude, that is,

$$Z_R = Z_{R0} \cos \omega t \quad (25)$$

Calculations were done for five different values of Z_{R0} , namely, 50, 10, 5, 1, 0.1 meters and the results are shown in

Figure 5. In addition to (20) the following parameters were assumed in the calculation,

$$\begin{aligned} a &= 8.6 \text{ cm} & b &= 13 \text{ cm} & l' &= 0.80 \text{ m} \\ R &= 7.4 \text{ cm} \end{aligned} \quad (26)$$

Because of the magnitude of b the small amplitude approximation in (23) was used for $Z_{R0} = 0.1 \text{ m}$ and the large amplitude approximation for the other values of Z_{R0} . For the latter we also assumed the mean vertical speed to be

$$\overline{|W_R|} = \frac{2}{\pi} \omega Z_{R0} \quad (27)$$

In the ocean the vertical motion is due predominantly to internal waves whose mean absolute vertical displacement usually is big enough to ensure that the large amplitude excitation in (24) is applicable. These waves, instead of being monochromatic, are distributed over a range of frequencies. In most cases the rotational response at the lower frequencies where most of the wave energy resides is given with sufficient accuracy by the d.c. response for our instruments. It then is a relatively simple matter to estimate the mean vertical speed from the magnitude of the observed angular velocity Ω . For example, assuming (18),

$$\overline{|W_R|} = \frac{\Omega}{2\pi} \left(\frac{1}{2} + \sqrt{\frac{1}{4} + 2\pi \frac{l'^2}{T\Omega}} \right) \quad (28)$$

With this value of $\overline{W_R}$ one can then estimate the rotational response at the higher frequencies using (23). As an example we have computed the frequency dependent pitch and phase lag of the instrument in Figure 2 for $\overline{W_R}$ equal to 1, .5, .1, .05, and .01 cm/sec which covers most of the range of interest. The results are shown in Figures 6 and 7. In general, the higher the pitch the poorer the response. The response is best at low frequencies where the pitch approaches its d.c. value (Figure 4). At high frequencies the rotation is damped out by the low frequency flow (through the non-linear excitation) and the response deteriorates. The phase lag at low frequencies is small and increases gradually to 90° at the higher frequencies.

IV Vertical Response

The VCM rotates in response to the relative vertical flow past its vanes as described in the previous section. To determine the true vertical current it is necessary to correct for the vertical motion of the instrument itself. This is due, primarily, to two factors: the advection of positive and negative buoyancy past the instrument, and vertical drag. Clearly, the best way to determine this bobbing motion is from precise pressure measurements at the instrument. Without them it is still possible and also instructive to estimate this motion from the water structure and the relative flow itself.

Assume that the VCM has been launched and has sunk to the vicinity of its equilibrium depth. The sea water is compressible and is locally stratified. The instrument is also compressible

(less than the water, however) and has a thermal coefficient of expansion. For simplicity assume a two dimensional fixed frame of reference with X horizontal and Z positive upwards where the equilibrium depth is given by $Z = 0$. Let X_F and Z_F be the horizontal and vertical coordinates of the VCM as it moves relative to the fixed frame. They are functions of time t only. The equation of vertical motion for the VCM is given by,

$$m \frac{d^2 Z_F}{dt^2} = P - mg + \mathcal{V} - \mathcal{D} \quad (29)$$

where m is the instrument mass and g the acceleration due to gravity. The quantity P is the net vertical force due to water pressure on the surface of the instrument, $\mathcal{V}$ is the vertical thrust generated by the rotating vanes as they accelerate or decelerate water up and down, and $\mathcal{D}$ is the vertical tangential skin drag due to viscosity.

Relative to the fixed frame let u, w be the horizontal and vertical components of the water velocity and X_w, Z_w the corresponding components of water displacement. The components, then, of water velocity and displacement relative to the moving instrument are simply given by,

$$\begin{aligned} X_R &= X_w - X_F, & Z_R &= Z_w - Z_F \\ U_R &= u - \frac{dX_F}{dt}, & W_R &= w - \frac{dZ_F}{dt} \end{aligned} \quad (30)$$

The problem posed in this section is to relate W_R or Z_R , which drive the rotation as discussed in the previous section, to ω or Z_N which are of scientific interest. To simplify the following discussion we shall decouple the vertical response from the horizontal response (considered in the next section) by assuming the instrument always follows the horizontal component of flow, that is

$$U_R = Z_R = 0 \quad (31)$$

Actually, the time required to adjust to a change in horizontal flow is small. It is given approximately by the transit time required by the current change to pass the instrument, which is usually less than 1 minute.

4.1 Pressure force

The pressure force P in (29) can be broken into two parts

$$P = P_0 + P'$$

where

$$P_0 = -\vec{\gamma} \cdot \int_{S_F} p d\vec{S}_F \quad (32)$$

$$P' = -\vec{\gamma} \cdot \int_{S_F} p' d\vec{S}_F$$

Here $d\vec{S}_F$ is the inward vector normal to a small surface area element dS_F and $\vec{\gamma}$ in the upward

unit vector. In the above p is the water pressure which would occur at the instrument if the latter were not present, and p' is the pressure reaction which results because the vertical flow is modified locally as it passes around the instrument.

4.1.1 Unmodified pressure force

We shall assume that the spatial scale of the unmodified flow at the VCM is much larger than the size of the instrument. It is then easy to show that

$$P_o = -V \frac{\partial p}{\partial z} \quad (33)$$

where V is the instrument volume and $\frac{\partial p}{\partial z}$ is the local unmodified vertical pressure gradient. From the vertical balance of water momentum, however,

$$\frac{\partial p}{\partial z} = -\rho_w \left(g + \frac{Dw}{Dt} \right) \quad (34)$$

where

$$\frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} \quad (35)$$

is the vertical water acceleration at the VCM following a parcel of water and ρ_w is the local water density. However,

what we really want is the acceleration as one moves with the instrument. This is,

$$\frac{dw}{dt} = \frac{\partial w}{\partial t} + w_x \frac{dZ_F}{dt} + w_y \frac{dZ_F}{dt} \quad (36)$$

Combining (36) with (35) and assuming (30) and (31) one finds

$$\frac{\partial p}{\partial z} = -g\rho_w - \rho_o \left(\frac{d^2 Z_w}{dt^2} + \frac{dZ_R}{dt} \frac{\partial w}{\partial z} \right) \quad (37)$$

where ρ_o is the average water (and float) density at the equilibrium depth. Defining the instrument density by

$$\rho_F = \frac{m}{V} \quad (38)$$

one can express the unmodified pressure force as follows

$$\rho_o = mg + m \left[g \frac{\rho_w - \rho_F}{\rho_o} + \frac{d^2 Z_w}{dt^2} + \frac{\partial w}{\partial z} \frac{dZ_R}{dt} \right] \quad (39)$$

The first term on the right of (39) is the hydrostatic force. In the case of no motion it is the only force acting on the instrument.

The next term which involves the density difference between water and float drives the relative flow. This difference, itself, depends on the vertical displacements of both the water and the instrument and on their mechanical and thermal characteristics. Assume that the VCM is compressible and has a thermal coefficient of expansion. Assume further that its temperature remains equilibrated with the water temperature*. The rate of change of its effective density as it moves vertically in the water's temperature gradient is,

$$\frac{1}{\rho_0} \frac{d\rho_F}{dt} = -\beta_F \frac{dz_F}{dt} - \alpha_F \frac{dT_w}{dt} \quad (40)$$

where T_w is the water temperature at the instrument, and β_F , α_F are the compressibility and thermal expansion coefficients, respectively, of the instrument, which are assumed constant. If there is no thermal dissipation in the water the rate of change of water temperature at the VCM as one moves with a parcel of water is given by

$$\begin{aligned} \frac{DT_w}{Dt} &= \frac{\partial T_w}{\partial t} + u \frac{\partial T_w}{\partial x} + w \frac{\partial T_w}{\partial z} \\ &= -w \left| \frac{d\theta}{dz} \right| \end{aligned} \quad (41)$$

*The thermal relaxation time for heat conduction through the instrument wall varies from several minutes for the glass spheres (Fig. 1) to several seconds for the aluminum cylinders (Fig. 2).

where $\frac{d\Theta}{dz}$ is the adiabatic temperature gradient. However, we want the rate of change at the VCM as one moves with instrument. This is,

$$\frac{dT_w}{dt} = \frac{\partial T_w}{\partial t} + \frac{\partial T_w}{\partial x} \frac{d\bar{x}_F}{dt} + \frac{\partial T_w}{\partial z} \frac{d\bar{z}_F}{dt} \quad (42)$$

Hence, assuming (33)

$$\frac{dT_w}{dt} = -\left(\omega - \frac{d\bar{z}_F}{dt}\right) \frac{\partial T_w}{\partial z} - \omega \left|\frac{d\Theta}{dz}\right| \quad (43)$$

and substituting into (40) one finds for the rate of change of VCM density,

$$\begin{aligned} \frac{1}{\rho_0} \frac{d\rho_F}{dt} = & -\left(\beta_F - \alpha_F \left|\frac{d\Theta}{dz}\right|\right) \frac{d\bar{z}_w}{dt} \\ & + \left(\beta_F + \alpha_F \frac{\partial T_w}{\partial z}\right) \frac{d\bar{z}_R}{dt} \end{aligned} \quad (44)$$

In a very similar way one can compute the rate of change of the water density at the VCM in terms of the local density gradients.

$$\frac{1}{\rho_0} \frac{d\rho_w}{dt} = -\beta_w \frac{d\bar{z}_w}{dt} - \frac{1}{\rho_0} \frac{\partial \rho_w}{\partial z} \frac{d\bar{z}_R}{dt} \quad (45)$$

where $\partial \rho_w$ is the adiabatic compressibility of the water. Hence, the rate of change of the density difference between instrument and its surroundings is

$$\begin{aligned} \frac{1}{\rho_0} \frac{d}{dt} (\rho_F - \rho_w) = & \\ & (\partial \rho_w - \partial \rho_F + \alpha_F \left| \frac{d\theta}{dz} \right|) \frac{dZ_w}{dt} \\ & + \left(\frac{1}{\rho_0} \frac{\partial \rho_w}{\partial z} + \partial \rho_F + \alpha_F \frac{\partial T_w}{\partial z} \right) \frac{dZ_R}{dt} \end{aligned} \quad (46)$$

In equations (39) and (46) are coefficients containing the local vertical gradients of water density, temperature and vertical velocity (convergence). We shall assume these have constant means and ignore their fluctuations. For the sake of simplicity, although it is not necessary, we shall assume also that the mean convergence $\overline{\frac{\partial w}{\partial z}}$ is zero. Then, integrating (46) with respect to time and substituting the resulting density difference for the right side of (39) one obtains the following for the unmodified pressure force,

$$\begin{aligned} P_0 = m g & \\ & + m \left[\frac{d^2 Z_w}{dt^2} - (M^2 - N^2 + Q^2) Z_w + M^2 Z_R \right] \end{aligned} \quad (47)$$

where

$$\begin{aligned}
 M^2 &= g \left(\frac{1}{\rho_0} \left| \frac{\partial p_w}{\partial z} \right| - \alpha_F - \alpha_F \frac{\partial T_w}{\partial z} \right) \\
 N^2 &= g \left(\frac{1}{\rho_0} \left| \frac{\partial p_w}{\partial z} \right| - \alpha_p \right) \\
 Q^2 &= g \alpha_F \left(\frac{\partial T_w}{\partial z} + \left| \frac{d\theta}{dz} \right| \right)
 \end{aligned} \tag{48}$$

are all constant coefficients having the units of circular frequency.

In particular N is the local Brunt-Väisälä frequency.

4.1.2 Pressure reaction force

This contribution to the total pressure force depends on the ratio of local inertial to viscous effects, that is, on a vertical Reynolds Number

$$R_v = \frac{l}{\mu} \left| \frac{d\bar{z}_R}{dt} \right| \tag{49}$$

and on the ratio of acceleration to inertial effects, that is, on a vertical Strouhal number

$$S_v = \frac{\omega l}{\left| \frac{d\bar{z}_R}{dt} \right|} \approx \frac{l}{|\bar{z}_R|} \tag{50}$$

where l is the vertical dimension of the instrument. In the ocean with our instruments we find almost always that $R_v \gg 1$. Assuming this to be true the critical factor becomes the size of S_v .

4.1.2.1 Large relative displacements

Usually the relative displacement is considerably larger than the vertical extent of the instrument so that

$S_v < 1$. Because they are rather blunt ended the flow then is of sufficient duration to cause separation in the VCM's wake. In this situation the pressure reaction force P' can be represented by a square-law form drag, that is,

$$P' = \frac{m}{L_v} \left| \frac{dz_R}{dt} \right| \frac{dz_R}{dt} \quad (51)$$

where L_v is a drag length of the same order as l . This form drag is borne out quite well from measurements of the sinking rate of the VCM to its equilibrium depth after launch. We find L_v to equal 0.30 m and 1.20 m for the instruments in Figures 1 and 2 respectively.

4.1.2.2 Small relative displacements

Vertical currents are also observed which generate relative displacements small compared with l so that

$S_v > 1$. This can occur when the water is highly stratified or when the instrument is near the bottom or sea surface. In this case little or no flow separation occurs in the instrument's

wake. The reaction force becomes a linear function of the relative flow and is strongly influenced by the presence of a local density stratification. Symbolically it can be represented as follows,

$$P' = m \left[\mathcal{J}_1(\omega) \frac{d^2 Z_R}{dt^2} + \sigma_1(\omega) \frac{dZ_R}{dt} \right] \quad (52)$$

Here, $\mathcal{J}_1$ is the ratio of the local water mass carried with the instrument to the instrument's own mass, and σ_1 is the reciprocal of a damping time which occurs when there is stratification because the instrument will radiate its own internal waves. Both of these quantities depend on frequency and on the shape of the VCM. Theoretical estimates are given in the Appendix 6.2 for two idealized instrument shapes; a prolate ellipsoid having a vertical major axis, and a vertical right cylinder. The results for the ellipsoid are plotted in Figures 8 and 9 as a function of ω/N for various ratios of major to minor axis. Several features are apparent. When the frequency is equal to the Väisälä frequency N both $\mathcal{J}_1$ and σ_1 are equal to zero. At higher frequencies the former rapidly increases to the constant value given by potential flow theory ($\frac{1}{2}$ for a sphere, for example). The latter remains zero because no internal waves can be radiated. At frequencies less than N the damping coefficient first increases to a maximum, then decreases steadily. The effective mass steadily increases because the internal wave characteristics gradually become horizontal and the VCM carries more and more mass with it. In general, when the major to minor axis is large (as for

the instrument in Figure 2) both $\mathcal{J}$ and σ are small and the reaction force becomes negligible.

4.2 Vertical vane thrust

The thrust arises from the interaction between rotational and vertical motion and is given by the vertical component of the "lift" generated by the flow past the vanes (discussed in Section III). Symbolically one can express it, for practical purposes, as follows,

$$\mathcal{J} = m \left[\mathcal{J}_2(\omega) \frac{d^2 \bar{z}_R}{dt^2} + \sigma_2(\omega) \frac{d \bar{z}_R}{dt} \right] \quad (53)$$

Here, $\mathcal{J}_2$ is the ratio of the water mass accelerated up or down by the rotating vanes to the mass of the instrument, and σ_2 is the reciprocal of a damping time which arises from the rotational viscous damping. Both of these quantities are functions of frequency and the shape of the instrument. For the long cylindrical instrument discussed in Section III

$$\mathcal{J}_2(\omega) = \left(\frac{2\pi R}{\lambda} \right)^2 \left[1 + \frac{a^2}{R^2} \mathcal{J}_\Theta(\omega) \right] \quad (54)$$

$$\sigma_2(\omega) = \left(\frac{2\pi a}{\lambda} \right)^2 \sigma_\Theta(\omega)$$

where the quantities R, λ were defined in that

section and $\mathcal{J}_\theta$ and σ_θ are plotted as a function of frequency in Figure 3.

For other than a cylindrical shape one can approximate $\mathcal{J}_2$ and σ_2 at high frequency by

$$\begin{aligned}\mathcal{J}_2(\omega) &= \left(\frac{2\pi a}{1}\right)^2 \frac{A_w}{V} \sqrt{\frac{\mu}{2\omega}} \\ \sigma_2(\omega) &= \left(\frac{2\pi a}{1}\right)^2 \frac{A_w}{V} \sqrt{\frac{\omega\mu}{2}}\end{aligned}\tag{55}$$

provided $\frac{a^2\omega}{\mu} \gg 1$. In (55) a is the mean instrument radius and A_w is the wetted surface area.

4.3 Tangential viscous drag

Again this force can be expressed, symbolically, as follows,

$$\mathcal{D} = -m \left[\mathcal{J}_3(\omega) \frac{d^2 \bar{z}_R}{dt^2} + \sigma_3(\omega) \frac{d\bar{z}_R}{dt} \right] \tag{56}$$

Here, $\mathcal{J}_3$ is the ratio of the water mass dragged with the instrument to its own mass, and σ_3 is a reciprocal damping time. Both of these quantities are functions of frequency and the geometry of the VCM

Theoretical expressions for $\mathcal{J}_3$ and σ_3 are given in the Appendix 6.3 for a long cylindrical instrument. They are also plotted in Figure 10 as functions of the dimensionless

frequency $a^2 \omega / \mu$.

In most cases the drag is a significant factor only at high frequencies where $\omega > \mu / a^2$. In this range, regardless of instrument shape, the drag is confined to a boundary layer of thickness $\sqrt{\frac{\mu}{2\omega}}$ over its surface and the quantities $\mathcal{J}_3$ and σ_3 in (56) can be estimated from

$$\begin{aligned}\mathcal{J}_3 &= \frac{A_w}{V} \sqrt{\frac{\mu}{2\omega}} \\ \sigma_3 &= \frac{A_w}{V} \sqrt{\frac{\omega \mu}{2}}\end{aligned}\tag{57}$$

for any shape where A_w is the wetted area.

4.4 Response for slow motions

For slow vertical motions (how slow can be determined later) one can neglect the float acceleration in (29) and all relative accelerations and velocities in the expressions found for ρ , $\mathcal{V}$, and $\mathcal{D}$. As a result one finds that the instrument moves so as to equilibrate its density with the moving water. The vertical response becomes,

$$\begin{aligned}Z_w &= g_0 Z_R \\ Z_F &= \left(1 - \frac{1}{g_0}\right) Z_w\end{aligned}\tag{58}$$

where $\mathcal{F}_0$ is the slow motion attenuation coefficient given by

$$\mathcal{F}_0 = \frac{M^2}{M^2 - N^2 + Q^2} \quad (59)$$

This coefficient is a positive number (for a stable VCM) which depends through the quantities defined by (48) on the mechanical and thermal properties of the instrument and on the stratification of the surrounding water. Clearly, the response of the VCM is best for those conditions which maximize Z_R , and hence rotation, for a given Z_w . This means $\mathcal{F}_0$ cannot be too large. With our present instruments it assumes values between one and two for most oceanic conditions; hence, the instrument motion, assuming no drag, varies from zero to about one-half the water motion. The variation of $\mathcal{F}_0$ with depth at two locations is shown in Figure 11 as computed for the newest VCM (Figure 2). One location is at Site D in the slope water off New England and the other is in the southern Sargasso Sea. The computation assumes the following fixed parameters for the instrument,

$$\begin{aligned} \mathcal{H}_F &= 2.1 \times 10^{-6} / m \\ \alpha_F &= 8.0 \times 10^{-5} / ^\circ C \end{aligned}$$

and for the water,

$$\begin{aligned} \mathcal{H}_w &= 4.4 \times 10^{-6} / m \\ \left| \frac{d\Theta}{dz} \right| &= 1.5 \times 10^{-4} ^\circ C / m \end{aligned}$$

(60)

Figure 11 exhibits the following features. In the deep water and in the surface mixed layer the coefficient f_0 is practically unity. This is because of the low water stability ($N^2 \approx 0$). In the thermocline region f_0 exceeds unity because of the increased stability and reaches a maximum value of 2.3 in the near-surface thermocline of the Sargasso Sea.

There are regions of the ocean, especially near the surface and along coasts, with exceptionally strong density stratification. If $N^2 \gg g \rho_w$ the coefficient f_0 becomes much greater than unity and the response of the VCM is very poor. It becomes a better density follower than vertical current meter. Düing (1969) has, in fact, used neutral buoyant elements as isotherm followers in such regions.

It is conceivable that regions exist in the ocean where f_0 would be less than unity; where the instrument sinks as the water rises and vice versa. Warm, salty water must overlies cool, fresher water. The salinity and temperature gradients, however, must just counteract each other to produce low stability. As the water rises in such a region the VCM is not buoyed up but sinks from cooling and contraction.

4.5 Response for rapid motions

For more rapid motions it is no longer possible to neglect the effects of acceleration, drag, etc. on the vertical response. If one substitutes into (29) the expressions for P , V , and D given in Sections 4.1, 4.2, and 4.3 one obtains for the response equation

$$(1 + \mathcal{J}_T) \frac{d^2 \bar{Z}_R}{dt^2} + \sigma_T \frac{d\bar{Z}_R}{dt} + M^2 \bar{Z}_R = (M^2 - N^2 + Q^2) \bar{Z}_W \quad (61)$$

where

$$\mathcal{J}_T = \begin{cases} \mathcal{J}_2(\omega) + \mathcal{J}_3(\omega) & |\bar{Z}_R| > l \\ \mathcal{J}_1(\omega) + \mathcal{J}_2(\omega) + \mathcal{J}_3(\omega) & |\bar{Z}_R| < l \end{cases} \quad (62)$$

$$\sigma_T = \begin{cases} \frac{|W_R|}{L_v} + \sigma_2(\omega) + \sigma_3(\omega) & |\bar{Z}_R| > l \\ \sigma_1(\omega) + \sigma_2(\omega) + \sigma_3(\omega) & |\bar{Z}_R| < l \end{cases} \quad (63)$$

depend on frequency and instrument geometry as discussed previously.

For large relative amplitudes $|\bar{Z}_R| > l$ equation (61) is non-linear due to the presence of the form drag in σ_T . This introduces higher harmonics into the frequency response. In most cases, however, the principle contribution to $|W_R|$ in (63) is due to large motions at low frequencies where the response is essentially that given by (58). The production of higher harmonics is then quite small. If we assume

$$Z_W = \int_{-\infty}^{+\infty} \hat{Z}_W e^{i\omega t} d\omega$$

$$Z_R = \int_{-\infty}^{+\infty} \hat{Z}_R e^{i\omega t} d\omega \quad (64)$$

and neglect these harmonics one finds

$$\frac{Z_W}{Z_R} = f(\omega) e^{-i\phi(\omega)} \quad (65)$$

where $f(\omega)$ is a frequency dependent attenuation coefficient given by

$$f(\omega) = f_0 \sqrt{\left[1 - \frac{\omega^2}{M^2}(1 + \sigma_T)\right]^2 + \left(\frac{\sigma_T}{M}\right)^2 \frac{\omega^2}{M^2}} \quad (66)$$

and $\phi(\omega)$ is the phase angle by which $\hat{Z}_R$ lags $\hat{Z}_W$, given by

$$\tan \phi = \frac{\sigma_T}{M} \frac{\omega/M}{1 - \frac{\omega^2}{M^2}(1 + \sigma_T)} \quad (67)$$

In (66) and (67) the quantity $\mathcal{J}_T$ is the same as (62) but σ_T has been modified as follows,

$$\sigma_T = \begin{cases} \frac{\overline{|W_R|}}{L_v} + \sigma_2(\omega) + \sigma_3(\omega) & \overline{|Z_R|} > \rho \\ \sigma_1(\omega) + \sigma_2(\omega) + \sigma_3(\omega) & \overline{|Z_R|} < \rho \end{cases} \quad (68)$$

where $\overline{|W_R|}$ and $\overline{|Z_R|}$ are now the mean relative vertical speed and distance respectively.

The form of (61) and the attenuation (66) and phase lag (67) all indicate that the VCM responds to the oscillating water displacements Z_w like a harmonic oscillator with frequency dependent mass and damping. At low frequencies the response is the same as (58). At very high frequencies

$$\frac{\hat{Z}_R}{Z_w} \approx \frac{M^2}{g_0 \omega^2} \quad (69)$$

and the response becomes very poor. At intermediate frequencies a resonance may occur if σ_T is not too large.

4.6 Numerical calculations

To see the nature of the vertical response we have computed the attenuation coefficient and phase lag as functions of frequency, using (66) and (67), for the cylindrical instrument in Figure 2.

Computations were made for five values of the mean vertical speed

$\overline{|W_R|}$ equal to 5.0, 1.0, 0.5, 0.1, and 0 cm/sec, and are

shown in Figures 12 and 13. It was assumed that the relative mean displacement was large $\overline{|Z_R|} > \ell$ for the first four values and small $\overline{|Z_R|} < \ell$ for the last value. The calculation assumes that the instrument was at a depth of about 700 m near Site D (39°10'W 70°00'N) off the New England coast where $N = 6.28 \text{ rad./hr}$ corresponding to a period of one hour, and $M = 18 \text{ rad./hr}$ corresponding to a period of .35 hrs. The low frequency attenuation coefficient q_0 is approximately 1.06 from Figure 11. In the calculation the values (20), (26), and (60) were used and L_v was taken as 1.2 m. The parameters $\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3$ and $\sigma_1, \sigma_2, \sigma_3$ were obtained using Figures 3, 8, 9, 10. Both $\mathcal{J}_1$ and σ_1 were negligible at all frequencies for our length of cylinder.

The attenuation curve in Figure 12 shows a resonance for low amplitude response $\overline{|W_R|} \approx 0$ at a frequency of .235 cycles/hour. At this resonance the vertical displacement relative to the instrument is approximately 4 times the actual water displacement and lags the latter by 90°. This means the instrument is moving up and down with an amplitude approximately 4.1 times the water displacement and leading the latter with a phase angle of 76°.

The attenuation for large relative displacements shows that the resonance gradually disappears and the entire high frequency response deteriorates with increasing mean vertical speed. This is simply due to the increased damping provided by the form drag (51).

V Horizontal response and instrument tilting due to vertical shear

For the sake of discussion we shall assume the instrument is an upright, neutrally buoyant, circular cylinder which is drifting with respect to a fixed frame of reference (X horizontal, Z positive upwards). The vertical component of water motion, to simplify matters, is assumed to be zero. Hence, the center of gravity of the instrument remains at a constant depth ($Z = 0$) and moves along the X axis in response to the horizontal component of current. The latter is assumed to have a simple linear vertical shear, that is

$$u(z, t) = u_0(t) + S(t)z \quad (70)$$

where u_0 is the horizontal current at the center of gravity, and S is the shear which causes tilting of the instrument. Both of these quantities can vary with time.

The situation is pictured in Figure 14. Here $C.G.$ is the center of gravity and $C.B.$ the center of buoyancy. The latter is assumed to be at the midpoint of the cylinder, a distance $l/2$ from either end where l is the cylinder length. The distance h between the $C.B.$ and $C.G.$ is called the metacentric height. The radius of the cylinder is a .

The problem posed in this section is to relate the position $X_c(t)$ of the center of gravity and the tilt $\theta(t)$ to the horizontal current $u(z, t)$ given by (70). This is done using the equation for the rate of change of horizontal momentum

$$m \frac{d^2 \mathcal{I}_F}{dt^2} = P_H + D_H \quad (71)$$

and the equation for the rate of change of angular momentum about an axis perpendicular to Figure 1⁴ through the center of gravity,

$$I_2 \frac{d^2 \xi}{dt^2} = \mathcal{T}_T + \mathcal{T}_\xi - mg \xi h \quad (72)$$

where m is the instrument mass and I_2 the moment of inertia around this horizontal axis. In (71) P_H is the horizontal force exerted by the water pressure on the cylinder and D_H is the tangential viscous drag. In (72) $\mathcal{T}_T$ is the torque exerted by the pressure and $\mathcal{T}_\xi$ the viscous torque. The quantity $mg h \xi$ is the restoring torque of the buoyant force. We shall assume in what follows that the tilt is small enough that one can approximate $\sin \xi$ by ξ and $\cos \xi$ by 1.

5.1 Horizontal force

As in 4.1 the horizontal pressure force can be separated into two parts

$$P_H = P_{OH} + P_H' \quad (73)$$

where P_{OH} is the force which would occur at the instrument

if the latter were not present, and P_H' is the reaction pressure force due to the flow modification required in passing around the cylinder.

The unmodified force P_{OH} is given by the horizontal water pressure gradient integrated over the instrument volume. Assuming the spatial scale of the latter to be much larger than the diameter of the cylinder one finds

$$P_{OH} = m \frac{d u_m}{dt} \quad (74)$$

where u_m is the current at the midpoint $z = h$ of the cylinder.

The pressure reaction force P_H' depends on the magnitude of a horizontal Reynolds number

$$R_H = \frac{|U_R| a}{\mu} \quad (75)$$

where U_R is the horizontal current relative to the moving instrument given by,

$$U_R = \left(u_0 - \frac{d X_F}{dt} \right) + z \left(S - \frac{d \xi}{dt} \right) \quad (76)$$

We shall assume in the following that R_H is much greater than unity*. Accordingly, we shall represent P_H' as a form drag, that is

$$P_H' = \frac{m}{L_H l} \int_{-\frac{l}{2}+h}^{\frac{l}{2}+h} |U_R| U_R dz \quad (77)$$

and neglect the viscous drag in (71) that is,

$$D_H = 0 \quad (78)$$

In (77) L_H is a horizontal drag length related to the cylinder radius by

$$L_H = \frac{\pi a}{C_H} \quad (79)$$

where C_H is a drag coefficient having a value near unity for flow past a circular cylinder (Rouse, 1946). Hence, L_H is of the order of the instrument diameter.

5.2 Tilting and drag torque

In a similar way one can express the torque T in

*For the instrument in Figure 2 one finds $R_H \approx 100$ for a relative current of 1 mm/sec .

two parts

$$\mathcal{J}_T = \mathcal{J}_{OT} + \mathcal{J}'_T \quad (80)$$

due to the unmodified and modified flow. One finds for the former

$$\mathcal{J}_{OT} = m h \frac{du_m}{dt} + m \frac{l^2}{12} \frac{dS}{dt} \quad (81)$$

and for the latter, assuming (75) much greater than unity,

$$\mathcal{J}'_T = \frac{m}{L_H l} \int_{-\frac{l}{2}+h}^{\frac{l}{2}+h} |U_R| U_R z dz \quad (82)$$

and, in addition, the drag torque will be neglected, that is

$$\mathcal{J}_\xi = 0$$

5.3 The general response equations

With the expressions given for the horizontal force and torque we have the following equations for the motion of the center of gravity

$$\frac{d^2 \mathcal{F}}{dt^2} = \frac{du_m}{dt} + \frac{1}{L_H l} \int_{-\frac{l}{2}+h}^{\frac{l}{2}+h} |U_R| U_R dz \quad (83)$$

and for the tilt,

$$\begin{aligned} & \frac{d^2 \xi}{dt^2} + \omega_0^2 \xi \\ &= \frac{\omega_0^2}{g} \left[\frac{du_m}{dt} + \frac{l^2}{12h} \frac{ds}{dt} + \frac{1}{L_H l h} \int_{-\frac{l}{2}+h}^{\frac{l}{2}+h} |U_R| U_R z dz \right] \end{aligned} \quad (84)$$

In the above U_R is given by (76) and ω_0 is the circular frequency of free pendular oscillations of the cylinder given by

$$\omega_0^2 = \frac{mgh}{I_2} = \frac{gh}{R_2} \quad (85)$$

where R_2 is a radius of gyration.

5.3.1 Large relative current

We shall suppose that initially the current relative to the cylinder is sufficiently large that

$$|u_0 - \frac{dX_F}{dt}| \gg l |S - \frac{d\xi}{dt}| \quad (86)$$

Equations (83) and (84) then can be approximated by,

$$\frac{dU_{Rm}}{dt} = - \frac{1}{L_H} |U_{Rm}| U_{Rm} \quad (87)$$

and

$$\begin{aligned} \frac{d^2 \xi}{dt^2} + \frac{\omega_o^2 l^2 |U_{Rm}|}{6g L_H h} \frac{d\xi}{dt} + \omega_o^2 \xi \\ = \frac{\omega_o^2}{g} \left[\frac{dU_{Rm}}{dt} + \frac{l^2}{12h} \frac{dS}{dt} + \frac{1}{L_H} |U_{Rm}| U_{Rm} \right. \\ \left. + \frac{l^2}{6L_H h} |U_{Rm}| S \right] \end{aligned} \quad (88)$$

where we have introduced U_{Rm} , the relative current at the cylinder midpoint,

$$U_{Rm} = (u_0 - \frac{dI_F}{dt}) + L (S - \frac{d\xi}{dt}) \quad (89)$$

For the sake of argument we will assume that the horizontal current and its vertical shear are constant in time, that is,

$$u_m = u_{m0} = u_0 + h S_0 \quad (90)$$

and that initially the cylinder is motionless with no tilt, that is,

$$\frac{dI_F}{dt} = 0 \quad \xi = \frac{d\xi}{dt} = 0 \quad \text{at } t=0$$

The solution to (87) is then

$$U_{Rm} = \frac{u_{m0}}{1 + t/\gamma} \quad (91)$$

where

$$\gamma = L_F / u_{m0} \quad (92)$$

Hence, the cylinder catches up with the current in a time of the order of τ . The solution of (88) for the subsequent tilt is more complex. Typically, the tilt goes through a series of damped oscillations (having a frequency near ω_0) around a mean which decreases towards zero in a time which is again of the order of (92). During these oscillations the maximum tilt is bounded by

$$\xi_{\max} \leq \frac{2}{g L_H} \left(U_{mo}^2 + \frac{l^2}{6h} U_{mo} S_0 \right) \quad (93)$$

For the instrument in Figure 2 we have

$$\begin{aligned} l &= 1.20 \text{ m} & h &= 0.08 \text{ m} \\ L_H &= 0.30 \text{ m} & R_2 &= 0.36 \text{ m} \\ \omega_0 &\approx 2.5 \text{ rad/sec.} \end{aligned} \quad (94)$$

Hence, the catch up time τ is of the order of 3 sec for a starting current of 10 cm/sec and 30 sec for a current of 1 cm/sec . For a large oceanic shear of $.1 \text{ sec}^{-1}$ the maximum tilt is less than 1.5 degrees for a current of 10 cm/sec and 0.1 degrees for a current of 1 cm/sec .

5.3.2 Small relative current

After a time of the order of (92) the relative current becomes quite small and the approximations (87) and (88) are invalid.

Instead, one can show that

$$\frac{dU_{Rm}}{dt} + \frac{l}{2L_H} \left(S - \frac{d\xi}{dt} \right) U_{Rm} + l \frac{d^2 \xi}{dt^2} = 0 \quad (95)$$

and

$$\begin{aligned} \frac{d^2 \xi}{dt^2} + \omega_0^2 \xi = \omega_0^2 \left[\frac{1}{g} \frac{dU_m}{dt} + \frac{l^2}{12gH} \frac{dS}{dt} \right. \\ \left. + T_s^2 \left(S - \frac{d\xi}{dt} \right) \left(S - \frac{d\xi}{dt} \right) \right] \end{aligned} \quad (96)$$

where

$$T_s^2 = \frac{l^3}{32g L_H H} \quad (97)$$

Equation (96) can be greatly simplified if the fluctuation frequency ω of U_m and S is restricted according to,

$$\omega \ll \omega_0, \quad \frac{1}{2 T_s^2 |S|} \quad (98)$$

In that case (96) reduces to

$$\ddot{\xi} = \frac{1}{g} \left(\frac{dU_m}{dt} + \frac{l^2}{12L} \frac{dS}{dt} \right) + T_s^2 |S| S \quad (99)$$

and (95) becomes

$$\frac{dU_{Rm}}{dt} + \frac{l}{2L_H} |S| U_{Rm} = -L \frac{d^2 \xi}{dt^2} \quad (100)$$

For the instrument in Figure 2, using (94), one finds

$$T_s = 0.50 \text{ sec} \quad (101)$$

The pendular frequency ω_0 is about 1/2 cycle/sec and, assuming a large shear of $S \approx 10^{-1} \text{ sec}^{-1}$, one finds that the quantity $\frac{1}{2T_s^2 S}$ corresponds to a frequency of about 23 cycles/sec. Hence, (99) and (100) are good approximations for all frequencies less than a few cycles per second, which completely covers the useful range of the instrument.

5.3.2.1 Steady state

Assuming (90) equations (99) and (100) have the following steady state solution,

$$\xi = T_s^2 |S_0| S_0$$

and

$$U_{Rm} = 0$$

Thus the cylinder has a constant tilt proportional to the square of the shear, and it drifts in such a way as to make the relative current zero at its midpoint ($z = L$ in Figure 14).

For the instrument in Figure 2 one finds the steady tilt to be about $1/8$ of a degree for a steady shear of 0.1 sec^{-1} while the relative current at the top and bottom of the cylinder is 6 cm/sec .

5.3.2.2 Variable current and shear

If the current and shear are variable one can compute the tilt directly from (99). Using this tilt one can then obtain the relative current at the midpoint by solving (100). A useful approximation to the latter is

$$U_{Rm} = \begin{cases} \frac{-2 L_H L}{\rho |S|} \frac{d^2 \xi}{dt^2} & \omega \ll \frac{\rho \overline{|S|}}{2 L_H} \\ -L \frac{d\xi}{dt} & \omega \gg \frac{\rho \overline{|S|}}{2 L_H} \end{cases} \quad (103)$$

where $\overline{|S|}$ is the mean absolute shear and ω is the fluctuation frequency of the computed tilt.

If one assumes for $u_m(t)$ and $S(t)$ values which are typical for oceanic conditions one finds that U_{Rm} and ξ are extremely small for the instrument in Figure 2.

As an extreme situation suppose both u_m and S oscillate with a period of one minute and with amplitudes of 200 cm/sec and 0.1 sec^{-1} respectively. Using (99) one estimates that the maximum tilt is about one degree, and, using (100), that the relative current

at the cylinder's midpoint is less than 0.01 cm/sec .

5.4 Apparent vertical velocity induced by horizontal current and instrument tilt

Clearly if the cylinder in Figure 14 is equipped with vanes to measure vertical currents as discussed in Sections III and IV the component of relative horizontal current along the tilted cylinder axis will cause rotation and appear as a fictitious vertical current. Designating the latter as W_R' one has

$$W_R' = U_R \xi \quad (104)$$

For the linear vertical shear discussed in some detail here

$$W_R' = U_{Rm} \xi + (z - R) S \xi \quad (105)$$

After the cylinder has come to an approximate equilibrium with the current we have shown that the quantity $U_{Rm} \xi$ is typically very small ($2 \times 10^{-4} \text{ cm/sec}$ for the extreme example given in 5.3.2.2).

If the shear profile is not linear the situation is more complex. Referring to Figure 14 suppose the vertical shear has a discontinuity at $z = z_0$ between the top and bottom of the cylinder, that is,

$$u(z) = \begin{cases} u_1 + \Delta u & z > z_0 \\ u_1 & z < z_0 \end{cases} \quad (106)$$

If the shear is constant the steady state tilt is found to be

$$\xi = \frac{\rho \Delta u / \Delta u}{8g L_H h} \frac{1 - \eta^2}{1 + \sqrt{1 - \eta^2}} \quad (107)$$

where

$$\eta = 2 \frac{h - z_0}{l}$$

It is not possible to position the vanes to minimize the steady apparent vertical velocity. If they are at the cylinder midpoint one finds

$$W_R' = \pm \frac{\rho (\Delta u)^3}{16g L_H h} f(\eta) \quad \begin{array}{l} + \quad h > z_0 \\ - \quad h < z_0 \end{array}$$

(108)

where

$$f(\eta) = \frac{1 - \eta^2}{1 + \sqrt{1 - \eta^2}} \left(1 - \frac{|\eta|}{1 + \sqrt{1 - \eta^2}} \right)$$

If one assumes that the shear discontinuity can occur with equal probability between the top and bottom of the cylinder one finds for the mean and the r.m.s. apparent vertical velocity at the midpoint,

$$\overline{W_R'} = 0$$

$$\sqrt{\overline{W_R'^2}} \approx \frac{\rho (\Delta u)^3}{50 g L_H h} \quad (109)$$

Assuming $\Delta u = 10 \text{ cm/sec}$ one finds for the instrument in Figure 2 an rms apparent vertical velocity of 10^{-2} cm/sec .

VI Appendix

6.1 Theoretical expression for viscous torque acting on a submerged rotating cylinder

It is assumed that an infinitely long cylinder of radius a is submerged in water having kinematic viscosity μ and is rotating about its axis with an oscillatory angular velocity given by

$$\Omega = \Omega_0 e^{i\omega t}$$

where ω is the circular frequency of oscillation. From standard texts (Batchelor 1967) one can derive the drag torque per unit length of the cylinder as

$$\pi a^2 \rho [\omega \mathcal{J}_0(\omega) + \tau_0(\omega)] \Omega_0 e^{i\omega t} \quad (110)$$

where ρ is the water density and the quantities J_θ
and σ_θ are given by

$$J_\theta = \frac{2}{x} \frac{N_0(x)}{N_1(x)} \sin(\phi_0 - \phi_1 - \frac{\pi}{4})$$

(111)

$$\sigma_\theta = \frac{4\mu}{a^2} \left(1 + \frac{x}{2} \frac{N_0(x)}{N_1(x)} \cos(\phi_0 - \phi_1 - \frac{\pi}{4}) \right)$$

where $x = a \sqrt{\frac{\omega}{\mu}}$

and (N_0, ϕ_0) , (N_1, ϕ_1) are the modulus and phase of the zero and first order Kelvin functions respectively, that is,

$$N_0(x) e^{i\phi_0(x)} = \text{ker}_0 x + i \text{kei}_0 x$$

(112)

$$N_1(x) e^{i\phi_1(x)} = \text{ker}_1 x + i \text{kei}_1 x$$

For a long neutrally buoyant cylinder the total drag torque can be expressed as (8) using (110). The quantities (111) have been plotted in Figure 3 as functions of $a^2 \omega / \mu$ using tables for the

functions (112). Limiting forms are:

$$\left. \begin{aligned} \mathcal{J}_\theta &= 2 \left(\ln \frac{2}{a} \sqrt{\frac{\mu}{\omega}} - .577 \right) \\ \sigma_\theta &= \frac{4\mu}{a^2} \end{aligned} \right\} \frac{a^2 \omega}{\mu} \ll 1$$

$$\left. \begin{aligned} \mathcal{J}_\theta &= \frac{1}{a} \sqrt{\frac{2\mu}{\omega}} \\ \sigma_\theta &= \frac{1}{a} \sqrt{2\omega\mu} \end{aligned} \right\} \frac{a^2 \omega}{\mu} \gg 1 \quad (113)$$

6.2 Theoretical expressions for the pressure reaction force on various shaped submerged objects.

It is assumed that the object is submerged in a stably stratified fluid and is oscillating vertically with a relative displacement by

$$Z_R = Z_{R0} e^{i\omega t}$$

where ω is the circular frequency of oscillation. If the vertical displacement is small compared with the height of the object one can express the force acting on the object due to fluid pressure as follows

$$P' = \rho V [-\omega^2 \mathcal{J}_1(\omega) + i\omega \sigma_1(\omega)] Z_{R_0} e^{i\omega t} \quad (114)$$

where ρ is the fluid density, and V the volume of the object. If the latter is neutrally buoyant (114) is equivalent to (52). The quantities $\mathcal{J}_1$ and σ_1 depend on the frequency, the stratification, and the geometric shape of the object.

6.2.1 Oscillating sphere

Closed analytical expressions for $\mathcal{J}_1$ and σ_1 can be derived from Larsen's (1969) analysis of a closely related problem; the radiation damping of a neutral buoyant sphere which is released in a still stably-stratified frictionless fluid after being initially displaced from its equilibrium depth. One finds,

$$1 + \mathcal{J}_1(\omega) = \begin{cases} \frac{\frac{\pi^2 X^2}{4} + (\sqrt{1-X^2} + X^2 \cosh^{-1} \frac{1}{X}) \cosh^{-1} \frac{1}{X}}{\frac{\pi^2 X^4}{4} + (\sqrt{1-X^2} + X^2 \cosh^{-1} \frac{1}{X})^2} & X < 1 \\ \frac{\sin^{-1} \frac{1}{X}}{X^2 \sin^{-1} \frac{1}{X} - \sqrt{X^2 - 1}} & X > 1 \end{cases} \quad (115)$$

$$\sigma_1(\omega) =$$

$$\left\{ \begin{array}{ll} \frac{\pi N}{2} \frac{x \sqrt{1-x^2}}{\frac{\pi^2 x^4}{4} + (\sqrt{1-x^2} + x^2 \cosh^{-1} \frac{1}{x})^2} & x < 1 \\ 0 & x > 1 \end{array} \right.$$

where

$$x = \omega / N \quad (116)$$

and N is the circular Brunt Väisälä frequency. Both $\mathcal{J}_1$ and σ_1 are plotted in Figure 8 and 9 as functions of ω/N for the case $1/d = 1$ (sphere). Limiting forms are,

$$\left. \begin{array}{l} \mathcal{J}_1 = \ln \frac{2N}{\omega} - 1 \\ \sigma_1 = \frac{\pi \omega}{2} \end{array} \right\} \quad \omega \ll N \quad (117)$$

$$\left. \begin{array}{l} \mathcal{J}_1 = \frac{1}{2} \\ \sigma_1 = 0 \end{array} \right\} \quad \omega \gg N$$

6.2.2 Oscillating prolate spheroid having a vertical major axis

Closed analytical expressions for $\mathcal{J}_1$ and σ_1 were not obtainable. The following approximations may be used where X is given by (116) and d/l is the ratio of minor to major axis

1) $\frac{d}{l} X \ll \sqrt{1-X^2}$

$$\mathcal{J}_1 = \begin{cases} \left(\frac{d}{l}\right)^2 \left[-\ln\left(\frac{2l}{d} \sqrt{\frac{1}{X^2} - 1}\right) - 1 \right] & X < 1 \\ \left(\frac{d}{l}\right)^2 \left[-\ln\left(\frac{2l}{d} \sqrt{1 - \frac{1}{X^2}}\right) - 1 \right] & X > 1 \end{cases}$$

$$\sigma_1 = \begin{cases} \frac{\pi}{2} \left(\frac{d}{l}\right)^2 N X & X < 1 \\ 0 & X > 1 \end{cases} \quad (118)$$

2) $\frac{d}{l} X \gg \sqrt{1-X^2}$

$$\mathcal{J}_1 = \begin{cases} 1 - \frac{8}{\pi^2} \left(\frac{1}{X^2} - 1\right) & X < 1 \\ \frac{2}{\pi} \frac{d}{l} \sqrt{1 - \frac{1}{X^2}} & X > 1 \end{cases}$$

$$\sigma_1 = \begin{cases} \frac{2d}{\pi l} N \sqrt{1-x^2} & x < 1 \\ 0 & x > 1 \end{cases}$$

The above expressions were used to plot approximate curves of $\mathcal{J}_1$ and σ_1 in Figures 8 and 9 as functions of ω/N for various values of d/l .

6.2.3 Oscillating long vertical circular

Again closed forms for $\mathcal{J}_1$ and σ_1 were not obtained. The following approximations may be used where x is given by (116) and where the ratio a/l of the cylinder radius to length is small

$$\mathcal{J}_1 = \begin{cases} 0 & x < 1 \\ \frac{8a}{3\pi l} \sqrt{1 - \frac{1}{x^2}} & x > 1 \end{cases}$$

$$\sigma_1 = \begin{cases} \frac{8a}{3\pi l} N \sqrt{1-x^2} & x < 1 \\ 0 & x > 1 \end{cases}$$

(120)

6.3 Theoretical expressions for the viscous drag on an oscillating vertical cylinder

It is assumed that an infinitely long circular cylinder of radius a is submerged in water having kinematic viscosity ν and is oscillating along its axis with a relative displacement given by

$$Z_R = Z_{R0} e^{i\omega t}$$

where ω is the circular frequency of oscillation. From standard texts (Batchelor 1967) one can derive the drag force per unit length of the cylinder as

$$\pi a^2 \rho \left[-\omega^2 \mathcal{J}_3(\omega) + i\omega \sigma_3(\omega) \right] Z_{R0} e^{i\omega t} \quad (121)$$

where ρ is the water density and $\mathcal{J}_3$ and σ_3 are given by

$$\mathcal{J}_3 = \frac{2}{x} \frac{N_1(x)}{N_0(x)} \sin\left(\phi_1 - \phi_0 + \frac{3\pi}{4}\right)$$

$$\sigma_3 = \frac{2\nu x}{a^2} \frac{N_1(x)}{N_0(x)} \cos\left(\phi_1 - \phi_0 + \frac{3\pi}{4}\right) \quad (122)$$

where

$$X = a \sqrt{\frac{\omega}{\mu}}$$

and (N_0, ϕ_0) (N_1, ϕ_1) are the functions defined by (112). For a long neutrally buoyant cylinder the total drag can be expressed as (56) using (121). The functions $\mathcal{J}_3$ and σ_3 have been plotted in Figure 10 as functions of $a^2\omega/\mu$. Limiting forms are,

$$\left. \begin{aligned} \mathcal{J}_3 &= \frac{\frac{\pi}{2} X^2}{\left(\ln \frac{2}{X} - .577\right)^2 + \left(\frac{\pi}{4}\right)^2} \\ \sigma_3 &= \frac{2\mu}{a^2} \frac{\ln \frac{2}{X} - .577}{\left(\ln \frac{2}{X} - .577\right)^2 + \left(\frac{\pi}{4}\right)^2} \end{aligned} \right\} X \ll 1 \quad (123)$$

$$\left. \begin{aligned} \mathcal{J}_3 &= \frac{\sqrt{2}}{X} \\ \sigma_3 &= \frac{\sqrt{2} \omega}{X} \end{aligned} \right\} X \gg 1$$

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Figure 1. Photograph of a vertical current meter used in 1969 to measure vertical flow in a winter sinking region. Diameter of the glass sphere is approximately 41 cm. The sound projector is housed in the lower cylinder.

Figure 2. Photograph of three of the most recent vertical current meters lying horizontally in their rack on the ship's deck.

Figure 3. The viscous reactive mass ratio δ_θ and resistive damping σ_θ of a long neutrally buoyant circular cylinder undergoing rotational oscillations about its principal axis as a function of the circular frequency ω of the oscillation. Here, a is the cylinder radius and μ the kinematic viscosity. All quantities are defined by (8) and computed using (111) and (112).

Figure 4. The steady state pitch of the VCM in Figure 2 computed as a function of relative vertical velocity using (18).

Figure 5. The rotational pitch of the VCM in Figure 2 computed as a function of frequency using (23) assuming the relative vertical displacement is oscillatory with a fixed amplitude. Curves are shown for five different values for this amplitude.

Figure 6. The rotational pitch of the VCM in Figure 2 computed as a function of frequency using (23) for five values of the mean relative vertical speed.

Figure 7. The rotational phase lag of the VCM in Figure 2 computed as a function of frequency using (24) for five values of the mean relative vertical speed.

Figure 8. The reactive mass ratio due to pressure on a neutrally buoyant vertical prolate spheroid oscillating vertically in a stratified fluid as a function of the frequency ω of the oscillation. Here, N is the Vaisälä frequency, and l/d is the ratio of major to minor axis. Computations were made using (115), (118), and (119).

Figure 9. The radiation damping due to pressure on a neutrally buoyant vertical prolate spheroid oscillating vertically in a stratified fluid as a function of the frequency ω of the oscillation. Here, N is the Väisälä frequency, and l/d is the ratio of major to minor axis. Computations were made using (116), (118), and (119).

Figure 10. The viscous reactive mass ratio δ_3 and resistive damping σ_3 of a long neutrally buoyant circular cylinder oscillating along its principal axis as a function of the circular frequency ω of the oscillation. Here, a is the cylinder radius, and μ the kinematic viscosity. All quantities are defined by (56) and were computed using (122) and (112).

Figure 11. The low frequency vertical attenuation coefficient (59) of the VCM in Figure 2 computed as a function of depth for two oceanic locations: Site D at 39°10'N 70°00'W, and in the Sargasso Sea at 25°00'N 68°00'W.

Figure 12. The ratio of high to low frequency vertical attenuation computed as a function of frequency using (66) for the VCM in Figure 2 for five values of the mean relative vertical speed.

Figure 13. The vertical phase lag computed as a function of frequency using (67) for the VCM in Figure 2 for five values of the mean relative vertical speed.

Figure 14. Geometry and notation used in discussing the tilt of a neutral buoyant vertical cylinder due to vertical shear.

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