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ON THE MOMENTUM TRANSFER
AT THE SEA SURFACE

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R. B. MONTGOMERY

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I. ON THE FRICTIONAL FORCE BETWEEN AIR AND WATER AND ON THE OCCURRENCE OF A LAMINAR BOUNDARY LAYER NEXT TO THE SURFACE OF THE SEA

BY C.-G. ROSSBY

It is fairly generally assumed that the horizontal circulation of the ocean "troposphere" is maintained by the large scale permanent wind system of the atmosphere. Accurate knowledge of the tangential force exerted by the wind on the surface of the water should therefore be a prerequisite for every attempt to analyze the movements of the surface water. Very few data are available from which this shearing force may be accurately determined, although several, in principle simple, methods of attack are available. Recently some estimates of the wind force have appeared in studies not primarily concerned with the oceanographic aspects of the problem. In view of the conflicting results obtained and in view of the importance of the problem to physical oceanography it seems desirable to examine the question theoretically and to study critically the empirical determinations just referred to. The author is fully aware of the fact that the final answer to the problem must wait until the proper measurements have been made and until these measurements have been carefully analyzed.

There are some indications that under certain favorable conditions a laminar sub-boundary layer of air may exist next to the sea surface. Thus kinematic viscosity becomes one of the factors which control the motion in the frictional layer, contrary to statements sometimes made in investigations of atmospheric flow. For this reason it seems desirable to start the present study by listing and discussing the factors which may affect the motion of the air near the surface of the earth. These factors are:

1. f , the Coriolis parameter, or the double angular velocity of the earth around a vertical axis at the station in question,

$$(1) \quad f = 2\Omega \sin L,$$

where L is the latitude and Ω the angular velocity of the earth around its axis.

2. z_0 , the characteristic roughness measure of the earth's surface at the station. z_0 may be determined from simultaneous wind velocity determinations at two different levels within the first 50 or 100 meters above ground, since, in the absence of thermal stability,

$$(2) \quad W = \frac{1}{k_0} \sqrt{\frac{\tau_0}{\rho}} \ln \frac{z + z_0}{z_0}, \quad (k_0 = 0.38)$$

and consequently

$$(3) \quad \frac{W_2}{W_1} = \frac{\log(z_2 + z_0) - \log z_0}{\log(z_1 + z_0) - \log z_0}.$$

In these expressions W_2 and W_1 are the observed velocities at the elevations z_2 and z_1 , τ_0 the (constant) shearing stress and ρ the air density. Data on the value of z_0 for different types of surface have been given by Sverdrup^{1,2} and in B, Chapter II, 1.†

† References are indicated by elevated figures and listed at the end of the paper. For convenience the letter "B" is used in referring to the following paper: C.-G. Rossby and R. B. Montgomery, 1935, The Layer of Frictional Influence in Wind and Ocean Currents, *Papers in Physical Oceanography and Meteorology*, Vol. III, No. 3.

3. ν , the kinematic viscosity. This factor does not *directly* influence the motion except when the system under investigation is subject to true skin-friction along some part of its boundary. This probably does not occur in winds blowing over a rough land surface but appears to be of importance in air currents flowing over a smooth water surface, whether at rest or in undulating motion.

4. U_0 , the gradient wind velocity.

5. $\frac{g}{\theta} \frac{d\theta}{dz}$, the restoring force per unit vertical displacement due to thermal stability.

In this expression g means the acceleration of gravity and θ the potential temperature.

At a land station, f and z_0 are constants which may be determined once and for all.† At sea, the surface changes its character with the wind velocity. Thus, for high winds, the laminar boundary layer presumably disappears and the sea surface takes on the character of a rough boundary, the roughness of which will depend on the wind velocity.

On land, the kinematic viscosity does not directly affect the motion. Thus the only variable parameters at a given land station are the gradient wind velocity and the stability. Both these factors may vary with elevation, although the variation of U_0 within the frictional layer generally may be neglected, at least when the vertical stability is small.

The factors listed above, properly interpreted, would be the controlling ones also for the layer of frictional influence next to the bottom in those shallow ocean regions where a measurable gradient current extends all the way down. In this case, the restoring force may be written in the form $\frac{g}{\rho_w} \frac{d\rho_w}{dz}$, in which expression ρ_w represents the density of the water. The compressibility of the water has been neglected.

In the wind driven surface layer of the ocean, the corresponding parameters are:

1. f , the Coriolis' parameter.

2. z_{0w} , the characteristic roughness of the sea surface with respect to the motion just below it.

3. ν_w , the kinematic viscosity of the water.

4. τ_0 , the frictional force exerted by the wind on the water per unit area. For this force one may substitute a characteristic velocity w^* , defined by

$$(4) \quad w^* = \sqrt{\frac{\tau_0}{\rho_w}}.$$

In accordance with von Kármán's³ suggestion, one may introduce the name "friction velocity" for this quantity.

5. $\frac{g}{\rho_w} \frac{d\rho_w}{dz}$, the unit restoring force due to the stability of stratification in the water.

In this case, as in the case of the air above, the roughness factor (z_{0w}) presumably

† This statement should be taken *cum grano salis*. It is probable that the z_0 -value determining the velocity profile in the first few meters above the surface depends upon the character of the ground within, say, the nearest hundred meters, whereas the z_0 -value determining the velocity profile higher up and the general characteristics of the entire layer of frictional influence, presumably depends upon the character of the landscape over a distance of many kilometers. To some extent this point has been analyzed in B, Chapter II, 3 and will be brought up again further down.

varies with the wind force. It is conceivable that the surface may be regarded as smooth with respect to the water moving below it for low wind velocities (small τ_0). However, due to the absence of data on the detailed velocity distribution next to the surface in drift currents, little is known about this (B, Appendix).

Dimensional analysis shows that the five factors first listed permit the construction of three non-dimensional quantities:

$$(5) \quad R = \frac{U_g^2}{f\nu}, \quad N = \frac{U_g}{fz_0}, \quad P = \frac{g}{f^2\theta} \frac{d\theta}{dz}.$$

These quantities at first sight appear unfamiliar, but if we introduce an auxiliary height K , defined by

$$(6) \quad K = \frac{U_g}{f},$$

they take the form

$$(7) \quad R = \frac{U_g K}{\nu}, \quad N = \frac{K}{z_0}, \quad P = \frac{g}{f^2\theta} \frac{d\theta}{dz}.$$

K is approximately proportional to the height of the layer of frictional influence (B, Chapter II, 3). Thus R has the character of a Reynolds number and N is a roughness ratio of the type studied by Nikuradse.⁴ According to the theory for the motion in the atmospheric turbulent layer presented in B the rate of shear at some distance from the ground is proportional to f , and thus P has the same form as the non-dimensional stability ratio discussed by Prandtl,⁵ Richardson⁶ and others.

It is important to emphasize that both R and N contain the gradient wind velocity U_g . Thus it is impossible, by studying the variation with U_g of any particular characteristic quantity of the turbulent layer, to determine the effect on the system of variations in the Reynolds' Number. In the study of pipe line flow, both R and N are significant, but in that case K is the fixed radius of the pipe line and it is therefore possible to vary R without changing N . At a given land station, U_g is the only parameter producing changes in R and N , disregarding the slight variations in ν , resulting from temperature changes. Thus it is impossible to separate the effects of R and N except by comparison of conditions between different stations having different z_0 - and f -values.

However, the ground is generally quite rough; the resistance over land is due mainly to local pressure gradients and not to true skin friction. This type of flow does not noticeably depend on Reynold's Number but will depend on the roughness ratio N . This contention is supported by an analysis of velocity gradients near the ground (B, Chapter II, 1).

Over the sea, and possibly over smooth snow or ice surfaces, both R and N may be of significance. von Kármán³ has shown that a surface which is "smooth" for low velocities (small R) may act as a rough surface for high velocities. Conversely, for a given velocity, there is a critical roughness. Furthermore, the sea surface itself changes character with increasing wind, becoming increasingly corrugated. There are various indications that the air motion over the sea surface at low velocities may be controlled by Reynolds' Number but at higher velocities by the roughness ratio N .

To investigate this point, we restrict the discussion to the surface layer of approxi-

mately constant shearing stress and introduce "local" non-dimensional quantities, r , n and p instead of R , N and P . We define these local non-dimensional quantities through

$$(8a) \quad r = \frac{zW}{\nu}, \quad n = \frac{z}{z_0}, \quad p = \frac{g(z+z_0)^2}{\frac{\tau_0}{\rho}} \frac{1}{\theta} \frac{d\theta}{dz} \cdot \dagger$$

Here z is the height above the surface, W the wind velocity at the level z and τ_0 the constant shearing stress. The stability ratio assumes a more familiar form if one keeps in mind the fact that near the ground, and in the absence of stratification, the rate of shear is given by

$$(9) \quad \frac{dW}{dz} = \frac{1}{k_0(z+z_0)} \sqrt{\frac{\tau_0}{\rho}}.$$

Table 1 contains the results of an analysis of some wind velocity measurements made over shallow water in the vicinity of Woods Hole. The complete observations are presented by my collaborator, Mr. Montgomery, in the next section.

TABLE 1
WIND MEASUREMENTS ON BUZZARDS BAY, SEPTEMBER 1935

Date	Num- ber of obs	b (cm.)	W_b (m.p.s.)	$10^{-8} r_b$	$10^2 \gamma_b$	$\sqrt{\frac{\tau_0}{\rho}}$ (m.p.s.)	a	$\frac{10^{-8} a}{\nu} \sqrt{\frac{\tau_0}{\rho}}$	W_a comp.	W_a obs.	$10^{-8} r_a$ obs.	$10^2 \gamma_a$ obs.
20	3	505	5.95	2.00	3.03	18.03	105	12.62	5.24	5.35	3.75	3.37
22	2	452	1.78	0.54	3.35	5.96	52	2.08	1.46	1.40	0.49	4.26
23	3	466	3.07	0.95	3.20	9.82	66	4.31	2.59	2.62	1.15	3.75
24	2	473	1.88	0.59	3.30	6.20	73	3.02	1.58	1.63	0.79	3.80
23	1(1547)	473	4.2	1.33	3.12	13.10	73	6.37	3.59	3.5	1.70	3.74

If there is a laminar sub-boundary layer next to the surface, it seems reasonable to assume that the vertical velocity distribution above this layer should obey the general law established by von Kármán³ for flow over smooth plates, namely,

$$(10) \quad \frac{W}{\sqrt{\frac{\tau_0}{\rho}}} = 5.5 + \frac{1}{k_0} \ln \frac{z}{\nu} \sqrt{\frac{\tau_0}{\rho}} = 5.5 + 5.75 \log \frac{z}{\nu} \sqrt{\frac{\tau_0}{\rho}} \cdot \ddagger$$

If we introduce a non-dimensional coefficient of resistance γ defined by

$$(11) \quad \gamma = \frac{1}{W} \sqrt{\frac{\tau_0}{\rho}}, \quad \S$$

the preceding equation takes the form

$$(12) \quad \frac{1}{\gamma} + 5.75 \log \frac{1}{\gamma} = 5.5 + 5.75 \log r.$$

[†] It would be more rational to define r through the equation

$$(8b) \quad r = \frac{z}{\nu} W^* = \frac{z}{\nu} \sqrt{\frac{\tau_0}{\rho}}$$

but for the following applications the definition given in (8a) appears more convenient.

[‡] The numerical constant 5.75 implies that k_0 has the value 0.40. In connection with flow over rough surfaces it will be assumed as before that k_0 has the numerical value 0.38.

[§] It should be kept in mind that the coefficient of resistance thus defined is a function of the height at which the wind velocity is measured.

This equation is represented graphically in Figure 1. If we accept von Kármán's law we may apply it to the upper of the two levels at which the tabulated wind measurements were made (*b*) and thus, knowing W_b , b and ν , compute r_b , then γ_b from (12) and finally $\sqrt{\frac{\tau_0}{\rho}}$. This has been done and the resulting values are given in the table. Knowing $\sqrt{\frac{\tau_0}{\rho}}$, we are in a position to compute the velocity W_a at the lower level (*a*) from (10) and compare it with the observed value. The agreement is good and lends support to the assumption that a laminar boundary layer must exist next to the surface. Knowing $\sqrt{\frac{\tau_0}{\rho}}$, we may also compute γ_a and ν_a from the *observed* velocity at the lower level (*a*). These values have been entered in Figure 1 and are marked "M." They are all close to the theoretical line.

The numerical constant 5.5 in von Kármán's law signifies that the thickness (δ) of the sub-boundary layer is given by

$$(13) \quad \delta = \frac{11.5 \nu}{\sqrt{\frac{\tau_0}{\rho}}}.$$

Thus the thickness of the laminar sub-boundary layer decreases with increasing wind velocity.

TABLE 2A
SHOULEJKIN'S WIND MEASUREMENTS ON THE BLACK SEA

	<i>z</i> (cm.)	W_z obs (m.p.s.)	$10^{-5} r_z$	$10^2 \gamma_z$	$\sqrt{\frac{\tau_0}{\rho}}$ (m.p.s.)	$\frac{10^{-3} z}{\nu} \sqrt{\frac{\tau_0}{\rho}}$	W_z comp.	$10^{-5} r_z$ obs.	$10^2 \gamma_z$ obs.
Series I	220	2.22	3.26	3.47	7.703	—	—	—	—
	120	2.07	—	—	7.703	6.163	2.10	1.656	3.72
	70	1.95	—	—	7.703	3.595	2.00	0.910	3.95
	15	1.78	—	—	7.703	0.770	1.70	0.178	4.33
Series II	70	2.89	1.35	3.74	10.81	—	—	—	—
	15	2.48	—	—	10.81	0.162	2.48	0.25	4.35

Shoulejkin's⁷ observations in the Black Sea also seem to indicate the existence of a laminar sub-boundary layer for low wind velocities. Measurements were made at four different levels, on two different occasions, the wind velocities at the top level (2.2m.) being 2.22 and 3.54 m.p.s. respectively (Tables 2 A and 2 B)†. The data for the first series have been tabulated and treated in the same fashion as the data in Table 1, that is, the shearing stress has been computed from the observed wind velocity in 2.2 m. with the aid of (12). From this value the theoretical velocities at the lower levels have been computed and compared with the observed ones (Table 2 A). The agreement is good.

Knowing $\sqrt{\frac{\tau_0}{\rho}}$ from the 2.20 m. reading, we may also compute γ and r for the lower

† All numerical values given in this discussion are scaled off Shoulejkin's diagrams.

levels from the *observed* velocities. The values so obtained have been entered in Figure 1, and are marked "S." They fall close to the theoretical line.

FIG. 1.—Relation between coefficient of surface resistance and Reynolds number for a smooth sea surface. The full line represents von Kármán's theoretical relation. The points were obtained from wind gradient measurements by Shoulejkin (S) and Montgomery (M).

The second series seems to indicate flow over a rough surface with a characteristic roughness z_0 of about 0.6 cm. (Table 2 B). However, this interpretation of the velocity profile does not account for the surprisingly high wind velocity at the 15 cm. level. On the other hand, if we assume that also in this case a laminar sub-boundary layer exists

TABLE 2B

z (cm.)	W_z obs (m.p.s.)	W_z comp.	$\sqrt{\frac{\tau_0}{\rho}} \frac{z}{v}$ (m.p.s.)
220	3.54	—	22.74
120	3.17	3.17	22.74
70	2.89	2.85	22.74
15	2.48	(1.92)	22.74

Computation based on $z_0=0.6$ cm., $k_0=0.38$.

next to the sea surface, we may apply von Kármán's formula and compute the velocity at 15 cm. from the observed velocity at 70 cm. The agreement is excellent (Table 2 A). Using the shearing stress computed from the 70 cm. reading with the aid of (12), it is possible to determine, from the *observed* wind velocity, the value of γ and r at the 15 cm. level. The corresponding point would fall close to the theoretical line in Figure 1.

It was shown in B from theoretical considerations that a stable temperature stratification produces deviations from the logarithmic wind profile, which become more and more marked with increasing distance from the surface. Sverdrup¹ has improved this theory considerably and established its validity through an analysis of vertical wind and temperature gradients over ice and snow. Shoulejkin's second velocity profile with its relative increase of slope at higher levels suggests the presence of a stable lapse rate which

apparently is unable to affect the slope between 15 cm. and 70 cm. In the absence of temperature data it is impossible to verify this hypothesis. Another possible explanation is suggested below.

Wüst's⁸ observations from the Baltic have already been used in B to compute a characteristic roughness for the sea surface (Chapter II, 1). The mean values for 6.0, 2.5 and 1.0 m. obey the logarithmic law very closely and give a z_0 -value of 4 cm., but the individual observations show marked irregularities.

Prandtl⁵ states that for certain types of artificially produced roughness, obtained by gluing grains of sand to a smooth surface, the roughness parameter z_0 will be about 1/30 of the diameter of the grains. Using this factor and considering the normal wave height associated with the wind velocities prevailing during Wüst's observation period, a value of 4 cm. for z_0 appears quite reasonable and was therefore accepted as characteristic of the sea surface during moderate winds. However, the tangential shearing stress associated with so large a value for z_0 is greatly in excess of the value indicated by other meteorological and oceanographical evidence discussed below. For this reason, it appears desirable to re-analyze Wüst's individual observations.

TABLE 3
COMPUTATION OF z_0 FROM WÜST'S DATA

Sept. 7-8, 1919 Hour	W_{600} (m.p.s.)	W_{250}	W_{100}	W_{20}	z_0 (cm.) from W_{600} and W_{100}
10a	3.7	2.9	2.3	2.0	5
12m	4.3	2.6	2.1	1.8	17
2p	6.2	5.8	4.4	3.4	1.4
4p	3.8	2.8	1.8	1.2	20
6p	5.5	4.0	2.6	2.1	20
8p	5.5	4.4	3.5	3.0	4.3
10p	6.3	5.5	4.6	3.8	0.8
12n	5.8	5.1	4.3	4.1	0.6
[2a]	6.3	5.7	4.8	4.4	[0.3]
[4a]	6.2	5.9	5.1	4.8	[0.025]
[6a]	8.2	6.5	6.0	5.9	[0.8]
8a	7.0	6.5	5.1	4.5	0.9
10a	4.3	3.6	3.0	2.5	1.6

Of the thirteen observation series published by Wüst and reproduced in Table 3, the ten designated by him as "*einwandfrei*" have been plotted on a semi-logarithmic scale in Figure 2. It appears from an inspection of this diagram, that in each series there is a marked increase in slope at the 1 m. level. With few exceptions, the upper parts of the curves indicate a logarithmic wind distribution.

The slope between 0.2 m. and 1 m. in most cases agrees with the value predicted by von Kármán's formula, as may be seen from the computation in Table 4. The average value of the ratio between the computed and the observed wind difference between these two levels is 96 per cent.

It is tempting to attribute the increase in slope at and above the 1 m. level to the effect of thermal stability, but a calculation on the basis of Sverdrup's theory indicates that the temperature inversion required to bring about the observed pronounced increase in slope with elevation is entirely incompatible with Wüst's data on the mean temperature distribution during the observation period.

Disregarding conditions next to the surface, the upper part of the curves may be used to determine a roughness parameter for each series of observations. The values thus ob-

The horizontal eddy shearing stress prevailing at some height above the wave tops, say double the height of the waves themselves, must balance the horizontal components of *all* the forces acting on the air at the surface of the water. These forces consist of true shearing stresses and of the horizontal components of the normal pressures between air and water. In case of smooth waves this horizontal component of the resultant of all the normal pressures must be very small, as indicated by the successful application of von Kármán's formula to Montgomery's and Shoulejkin's data. In this case the eddy stress at higher levels balances the true shearing stress acting at the surface of the water. For higher wind velocities and higher shearing stresses the pressure distribution on the sea surface presumably becomes asymmetric. The normal pressures now have horizontal

[illegible]

components which must be added to the true shearing stress in order to give the eddy shearing stress at higher levels. In this case one should expect von Kármán's formula to apply only to the layer in the immediate vicinity of the water. In Wüst's observations, it appears that the wind gradient between 0.2 and 1m. gives a fairly good measure of the true shearing stress and thus only to a slight extent reflects the influence of the normal pressures whereas the readings at higher levels apparently give the total eddy shearing stress.

The preceding reasoning implies that there are at least two distinct types of eddies present over the water. The formula given by von Kármán is based on the assumption that the true shearing stress transmitted through the laminar sub-boundary layer is carried up to higher levels by eddies corresponding to a linear mixing length distribution, with a minimum value at the top of the laminar sub-boundary obtainable from the equation

$$(14) \quad l_{\min} \frac{W_{\delta}}{\delta} = \sqrt{\frac{\tau_0}{\rho}},$$

where W_{δ} represents the wind velocity at the top of the laminar sub-boundary layer. Since

$$(15) \quad \frac{\tau_0}{\rho} = \nu \frac{W_{\delta}}{\delta}$$

it follows that

$$(16) \quad l_{\min} = k_0 z_0 = \frac{\nu}{\sqrt{\frac{\tau_0}{\rho}}}.$$

To these eddies, then, must be added another group of larger eddies associated with the normal surface pressures. This latter group accounts for the major part of the effective stress some distance above the wave tops.

The reasoning outlined applies also to conditions over a landscape where open patches of land are broken by stands of trees. The wind profile over the meadows is determined by the shearing stress next to the ground and by the fine grain roughness of the latter. The effective shearing stress derived from the wind profile at a height of two or three times the height of the trees must include the effect of the normal pressures exerted on the groves. It is this total eddy stress which determines the character of the layer of frictional influence as a whole. This point has been brought out by Sutcliffe in a recent paper.⁹

The individual z_0 -values obtained from the upper parts of the curves in Figure 2 show a great deal of scattering. Of the ten values obtained, three are quite large and of those, two occurred with fairly light winds. Wüst reports that the wind during the observation period changed from an easterly to a north-northwesterly direction. It seems probable that the state of the sea and the prevailing wind cannot have been in equilibrium during the first part of the period. The z_0 -values obtained from the latter part of the observation period show a relatively small amount of scattering and, although the wind was stronger, are decidedly lower than the ones for the first part of the period. This result still holds when the three series at 2a, 4a and 6a are included, which on ac-

count of the difficulty of measurements with rough sea were characterized by Wüst as less accurate.

In Figure 3 are reproduced two mean wind curves obtained from all of Wüst's data. The first curve includes the seven observations for which $W_6 \leq 5.5$ m.p.s. The second group contains all the remaining observations ($W_6 \geq 5.8$ m.p.s.). Of these two groups the second presumably corresponds to fairly steady conditions. All but one of the observations included in this group were obtained during the latter part of the period. The three upper points satisfy the logarithmic law fairly well and give a z_0 -value of 0.6 cm. The first curve, corresponding to much lighter winds, also seems to satisfy the logarithmic law reasonably well but gives a much higher z_0 -value (10 cm.). It is probable that the observations included in this group do not correspond to a steady state relationship between the wind and the state of the sea. For this reason we shall disregard the corresponding z_0 -determination.

FIG. 3.—Variation of wind with height for light and for fresh winds (from Wüst's data).

It should be added that the particularly steep velocity gradients obtained in three of Wüst's series may in part have been the result of sudden accelerations or gusts, but it appears unlikely that the mean acceleration over a period of five minutes could reach such a value as to be the sole cause of these steep gradients.

In his theoretical computation of the evaporation from the ocean surface, Sverdrup¹⁰ assumes that the ocean surface has a characteristic roughness z_0 which may be computed from the normal maximum wave height corresponding to a given wind velocity at 8 m. through multiplication with a constant factor $1/40$ (Table 5). For light to moderate winds this assumption agrees with the results obtained through our original analysis of Wüst's data in B.

TABLE 5
COMPUTATION OF γ_8 FROM SVERDRUP'S VALUES FOR THE ROUGHNESS OF THE SEA SURFACE

W_8 (m.p.s.)	2.0	4.0	6.0	8.0	10.0	12.0
z_0 (cm.)	0.75	2.00	3.50	6.00	8.75	11.50
$\gamma_8 \cdot 10^2$	5.45	6.34	6.99	7.77	8.41	8.95

Using Sverdrup's values for z_0 , one may compute the non-dimensional coefficient of resistance γ_8 as a function of the wind velocity at the 8 m. level. This ratio, which was defined in (11), may be computed from (2), which after rearrangement takes the form

$$(17) \quad \gamma = \frac{1}{W} \sqrt{\frac{\tau_0}{\rho}} = \frac{k_0}{\ln \frac{z + z_0}{z_0}} = \frac{0.165}{\log \frac{z + z_0}{z_0}}$$

The results are given in Table 5. If one compares the γ -values thus obtained with the values computed from von Kármán's formula, it appears that they differ by a factor of about three, so that the total shearing stress corresponding to a given wind velocity will be about nine times as large when the surface is rough as when a laminar sub-boundary layer is present and there are no normal pressures to consider. Sverdrup's γ -values are about 50% larger than that computed from the z_0 -value of 0.6 cm. derived from Wüst's data. Sverdrup's successful computation of the evaporation from the sea surface is based on the z_0 -values tabulated above and must therefore be regarded as an indirect support for his assumption regarding the roughness of the sea surface, although the computed evaporation is relatively insensitive to large variations in z_0 . Furthermore, it should be kept in mind that in higher latitudes where the winds generally are quite variable, the effective roughness may be much greater than the steady-state value of 0.6 cm. obtained above.

Against this evidence, we have Sutcliffe's⁹ recent computation of $\gamma^2 = \frac{\tau_0}{\rho W^2}$ from kite

observations made on board the German research vessel *Meteor*.¹¹ Sutcliffe obtains, as a mean value for the North and South Atlantic, a γ^2 -value of 0.0004. This value is amazingly low and points to the existence of a laminar sub-boundary layer. The theoretical curve in Figure 1 indicates that γ decreases slightly with increasing r . The surface wind velocities observed on board the *Meteor* were measured at an elevation of 6 m. above sea level. With this value of z and assuming $\nu = 0.15$ one finds from Figure 1 that γ_6 has a value 0.03 for 6 m.p.s. and does not drop to 0.025 below a wind velocity of about 100 m.p.s. The average wind velocity for the cases analyzed by Sutcliffe was in the vicinity of 7 m.p.s. Thus Sutcliffe's mean value for γ_6^2 is less than one-half of the theoretical value obtained from von Kármán's well-established formula for the flow over a smooth plate. For one group of thirteen observations in the belt Lat. 10°–20°S., Sutcliffe actually obtains a mean value of 0.0001, or one-ninth of the value for smooth plates. Sutcliffe does not claim that his results are final but he does claim that they prove beyond a reasonable doubt that at sea γ^2 is of the order of magnitude of 1/10 of its value over land. It would seem from the comments made above that his γ^2 -value of 0.0004 is too small. A value of 0.0009, on the other hand, would agree well with the prediction of the laminar sub-boundary layer theory and still be of the order of magnitude indicated by Sutcliffe.

If we accept, temporarily, the result that γ^2 is of the order of magnitude of 0.0009, it means that we accept the existence of a laminar sub-boundary layer even at wind velocities averaging nearly 7 m.p.s. and attribute the total horizontal force at the sea surface exclusively to true shearing stresses. This conclusion certainly represents a contradiction of the indications furnished by Wüst's wind measurements as analyzed above. Furthermore, it would mean that the shearing force exerted by the wind on the sea surface, for normally occurring wind velocities, would be much smaller than hitherto assumed. Thus Sutcliffe's result, if accepted, reopens the entire question of the cause of the horizontal circulation of the ocean troposphere. The wind-produced convergence is proportional to the tangential force of the wind and the same applies to the gradient currents resulting from this convergence. Thus Sutcliffe's low value for γ^2 would make it very difficult to account for the circulation of the ocean troposphere on the basis of wind action.

There are various sources of errors in the data used which may conceivably account

for Sutcliffe's result. It is advisable to reproduce his formulae in order to demonstrate these possibilities.

If we place the x -axis in the direction of the surface wind and the y -axis normal thereto (pointing to the left of the x -axis), the equations of motion may, in the absence of acceleration terms, be written in the form

$$\begin{aligned} 0 &= \rho f(V - V_g) + \frac{\partial \tau_x}{\partial z} \\ 0 &= -\rho f(U - U_g) + \frac{\partial \tau_y}{\partial z}, \end{aligned} \quad (18)$$

where the gradient wind components V_g and U_g have been introduced instead of the pressure gradients. The first of these equations, integrated between the lower boundary and a level D , gives

$$\tau_{x0} - \tau_{xD} = \bar{\rho} f \int_0^D (V - V_g) dz, \quad (19)$$

where $\bar{\rho}$ represents the mean density. Sutcliffe extends the integration to an upper level D where τ_{xD} disappears.

Sutcliffe's method requires knowledge of the gradient wind both as to direction and to velocity, the latter to a considerable degree of accuracy, since the formula used involves the computation of the difference between the actual cross wind component V and the gradient wind component V_g normal to the surface wind, both of which are small in comparison with the total wind. If the x -axis be placed in the direction of the isobars,

$$\tau_{x0} = \bar{\rho} f \int_0^D V dz \quad (20)$$

and

$$\tau_0 = \frac{\tau_{x0}}{\cos \varphi_0}, \quad (21)$$

where φ_0 is the angle which the surface wind forms with the isobars. This modified formula does not require accurate knowledge of the gradient wind velocity but does assume that the gradient wind direction is independent of elevation within the layer D .

It is evident that the evaluation of either of the two expressions above involves knowledge of the small wind component V normal to the surface wind or the gradient wind and therefore requires accurate information concerning the *variation* of wind-direction from level to level. Inspection of the *Meteor* kite data shows that the wind directions at the surface and aloft are given only to compass points (32 points = 360° , 1 point = $11^\circ 25'$). Since the total change in direction between the surface and the gradient wind level normally is of the order of magnitude of 0-2 points, it appears that the *Meteor* kite data are unsuited for stress determinations by this method. Furthermore, to reach a level of 1000 m. it is generally necessary to make use of more than one kite. When more than one kite is used the determinations of the wind direction aloft become rather unreliable. This is particularly true of kite ascents made underway in light winds, since they require correction for the speed of the vessel.

TABLE 6
FREQUENCY DISTRIBUTION OF ANGLE FROM SURFACE WIND TO GRADIENT WIND
The angles are expressed in compass points and counted positive *cum sole*

Source	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	R	Average	N
Jeffreys	0	0	5	6	10	5	107	171	123	78	38	9	7	8	16.4°	566
Meteor	1	0	1	3	5	11	21	18	16	5	5	1	1	—	7.2°	88
Meteor	0	0	1	2	4	5	9	9	5	2	2	0	0	—	2.9°	39

In Table 6, three frequency distributions are given to illustrate the comments made above. The first is based on 565 observations from the North Sea and is taken from a paper by Jeffreys.¹¹ It represents the angle between the surface wind and the gradient wind, the latter determined from the synoptic charts. The second series is based on *Meteor* ascents made south of Lat. 10° S. and reaching a minimum altitude of 1000 m. A few additional restrictions have been introduced to insure a homogeneous material, reducing the number of available ascents to 88. This is a small number of observations but greater than the total number employed by Sutcliffe in his study. The table gives the frequency distribution for the angle between surface wind and wind direction 1000 m. above sea level. In each case the angle is expressed in compass points and counted positive *cum sole* from surface wind to wind aloft. The frequency maximum occurs at +1 for Jeffreys' data and at 0 for the *Meteor* data. The average values computed from these two sets of frequencies are 16.4° and 7.2°. It would appear that the explanation for Sutcliffe's low γ^2 -values must be sought in the abnormally small angles between surface winds and winds aloft recorded by the *Meteor*.

If the 39 observations recording a wind speed of less than 10 m.p.s. at 1000 m. be analyzed separately, the mean value for the angle between surface wind and wind aloft is found to have a value of 2.9°, while the frequency maximum occurs at 5.6°. For comparison there is reproduced in Table 7 some data from B, giving the average values of the surface wind deflection from the isobars as computed from Jeffreys' frequency data

TABLE 7
AVERAGE VALUES OF ANGLE FROM SURFACE WIND TO GRADIENT WIND AND THE CORRESPONDING
ROUGHNESS VALUES (FROM JEFFREYS' DATA)

U_0 (m.p.s.)	0-8	8.5-12	12.5-18	>18
Number of observations	166	164	160	71
Average ϕ_0	15.7°	18.2°	17.2°	16.3°
N	—	$1.3 \cdot 10^7$	$2.8 \cdot 10^7$	$6.2 \cdot 10^7$
Average U_0 (m.p.s.)	4.00	10.25	15.25	22?
z_0 (cm.)	—	0.70	0.49	0.32

for different gradient wind velocities. Since these data are free from the errors inherent to wind direction determinations based on kite observations and since they are based on a much larger number of observations, it seems reasonable to accept the values obtained from them in preference to the indications of the *Meteor* data.

In spite of the criticisms which thus may be directed against the observation material used in Sutcliffe's investigation, his study is of great importance since it emphasizes the relatively small value of the shearing stresses exerted on the ocean surface.

It is not without interest to investigate the effect of a laminar sub-boundary layer on the angle between surface wind and gradient wind. For the velocity distribution in the surface layer, one may accept von Kármán's distribution law. The exact character of

the motion above this layer of approximately constant stress is not known. If one accepts the solution derived by the author¹³ in 1932 with the aid of certain similarity considerations first introduced by von Kármán¹⁴ and used in B to compute the angle between surface wind and gradient wind for motion over a rough boundary, it is nevertheless possible to obtain an indication of the effect of the introduction of a laminar sub-boundary layer. We have, within the layer of constant stress next to the ground ($z < H$),

$$(22) \quad W = \sqrt{\frac{\tau_0}{\rho}} \left(5.5 + 5.75 \log \frac{z}{\nu} \sqrt{\frac{\tau_0}{\rho}} \right)$$

$$(23) \quad l = \text{mixing length} = k_0 \left(z + \frac{\nu}{k_0 \sqrt{\frac{\tau_0}{\rho}}} \right) \quad (k_0 = 0.40)$$

$$(24) \quad \varphi_0 = \text{angle between surface wind and gradient wind} = \text{constant.}$$

At the level $z = H$ we have, according to the previously derived solution (B, Chapter II, 1),

$$(25) \quad W_H = U_g \left(\cos \varphi_0 - \frac{1}{\sqrt{2}} \sin \varphi_0 \right)$$

$$(26, 27, 28) \quad h = \frac{9k^2}{2f} U_g \sin \varphi_0, \quad \sqrt{\frac{\tau_0}{\rho}} = \frac{fh}{3k}, \quad l = \frac{kh}{\sqrt{2}}.$$

Fitting the two solutions at $z = H$ and requiring continuity in mixing length, shearing stress, wind velocity and wind direction, we obtain

$$(29) \quad H = \frac{kh}{k_0 \sqrt{2}}$$

and, through combination of the two expressions for the velocity at H and simplifications,

$$(30) \quad \begin{aligned} \ln R = \ln \frac{U_g^2}{f\nu} = & -2 \ln \sin \varphi_0 + \frac{2k_0}{3k} \cotan \varphi_0 \\ & - \frac{k_0 \sqrt{2}}{3k} - 5.5 k_0 - \ln \frac{27k^4}{4k_0 \sqrt{2}} \end{aligned}$$

or, with $k_0 = 0.4$ and $k = 0.065$,

$$(31) \quad \log \sqrt{R} = 0.891 \cotan \varphi_0 - \log \sin \varphi_0 + 0.728.$$

For a rough boundary the corresponding relation is (B, chapter II, 1).

$$(32) \quad \log N = 1.694 \cotan \varphi_0 - \log \sin \varphi_0 + 1.441.$$

The two solutions are plotted in Figure 4.

We shall make use of the two solutions given above to study the deflecting angles obtained from Jeffreys' data and listed in Table 7. Assuming the surface to be rough, the appropriate values of N are found from (32) or Figure 4.

Using a value of $1.12 \cdot 10^{-4}$ for f it is possible to compute z_0 from N and from the mean

value of U_0 for each group. The results of the computation are given in Table 7. There is good agreement with the value of 0.6 cm. obtained from Wüst's wind measurements. There is some indication that z_0 decreases with increasing wind velocity.

The light wind group value is much below the N -curve in Figure 4, but only slightly in excess of the value to be expected from (31) on the assumption that a laminar sub-boundary layer exists next to the surface for these light winds. However, an even lower value was obtained by Taylor¹⁵ for light winds from an analysis of Dobson's pilot balloon data from Salisbury Plains. Velocity gradients measured in England and on the continent over meadows and open grass land indicate that there is no laminar sub-boundary layer present over land; thus it appears that the low value of φ_0 during light winds over the ocean cannot be regarded as a proof for the existence of a laminar sub-boundary layer. In

FIG. 4.—Theoretical angle between surface wind and gradient wind for a rough and for a smooth sea surface.

B, it was attempted to attribute this small value for the angle of deflection at low wind velocities to the presence of "residual" turbulence at higher levels. There is no reason, apparently, why the same explanation should not apply to the ocean data. The extremely low value indicated by the 39 *Meteor* light wind kite data in Table 6 cannot readily be explained on the basis of either assumption.

Jeffreys' investigation contains a table giving the frequency distribution for the ratio of the surface wind speed to the gradient wind speed. Out of a total of 565 observations, 10 give values for this ratio outside the interval 24% to 102%. Excluding these ten observations and subdividing the material according to gradient wind speed, we obtain

the data in Table 8. The average value varies from 64.4% for light winds to 59.1% for the strong winds. The average for all the groups is 61.9%.

According to the theory presented in B, this ratio is given by

$$(33) \quad \frac{W_z}{U_g} = \frac{3k}{2k_0} \sin \varphi_0 \ln \frac{z + z_0}{z_0},$$

where W_z is the surface wind velocity, measured at the height z above sea level. If this equation is combined with (32), φ_0 may be eliminated. If $\frac{W_z}{U_g}$, U_g , f and z are given, it

is possible to determine z_0 . In attempting to evaluate Jeffreys' data, one must keep in mind the great inaccuracy of wind speed estimates on the Beaufort scale. Furthermore, there is a great deal of uncertainty concerning the level to which the m.p.s. equivalent of the Beaufort scale refers. From Simpson's¹⁶ study of the latter, it appears reasonable to assume a value of 15 m. above sea level for z . Using the previous value of $1.12 \cdot 10^{-4}$ for f , it is possible to solve equations (32) and (33) for z_0 . This has been done graphically; the resulting z_0 -values are given in Table 8. If these z_0 -values and the known values for U_g

TABLE 8
AVERAGE VALUES OF THE RATIO BETWEEN SURFACE WIND AND GRADIENT WIND AND THE
CORRESPONDING ROUGHNESS VALUES (FROM JEFFREYS' DATA)

U_g (m.p.s.)	0-8	8.5-12	12.5-18	>18	All
Number of observations	164	161	159	71	555
Average $\frac{W}{U_g}$	0.644	0.605	0.620	0.591	0.619
z_0 (cm.)	—	0.85	0.36	0.64	—
φ_0 comp.	—	18.4°	16.8°	17.1°	—

and f are combined, one may work backwards, computing first N and then φ_0 from (32). The computed values of φ_0 are given in the last line of Table 8. They agree well with the observed values in Table 7.

Also this last determination leads to z_0 -values in good agreement with the results of our analysis of Wüst's data and again we find a slight indication of a decrease in z_0 with increasing gradient wind velocity.

A study of available data concerning the *wind factor*, or the ratio between surface drift speed and surface wind velocity seems to indicate that roughness parameters of the order of magnitude of 0.5 cm. or less characterize the motion of the surface air layer. However, in many cases wind factors have been obtained by methods so crude that nothing definite can be said regarding the levels to which the two speeds refer. Furthermore, it is often doubtful that current components other than the pure wind drift have been eliminated. Since, finally, the relation between surface wind drift speed and shearing stress depends on the total depth of the drift current layer, and since this depth may be controlled by other factors than the wind-produced turbulence (tidal currents, permanent current systems, thermal stability) it appears that the wind factor determinations are practically useless for stress computations except under ideal conditions.

The earliest, and in the oceanographic literature most generally quoted value for the surface shearing stress was given by Ekman¹⁷ in 1905. From some early observations by Colding on changes in water level produced by strong winds in the southern part of the Baltic, Ekman obtains,

$$(34) \quad \tau_0 = 32 \cdot 10^{-7} W^2. \quad (W \text{ in cm.p.s., } \tau_0 \text{ in dynes per cm.}^2)$$

This relation may be given proper dimensions through the introduction of a numerical value of $1.28 \cdot 10^{-3}$ for the air density. Thus,

$$(35) \quad \frac{\tau_0}{\rho W^2} = \gamma^2 = 25 \cdot 10^{-4}.$$

The original wind observations used by Ekman were Beaufort estimates, for wind velocities in the vicinity of 20 m.p.s. If one assumes that in Ekman's formula the wind velocity refers to an elevation of 15 m. the coefficient $\gamma^2 = 25 \cdot 10^{-4}$ would, according to (17), indicate a roughness of about 0.6 cm., in good agreement with our estimates above. However, Ekman's determination of γ^2 from Colding's data is based on several assumptions of a very restrictive character which somewhat diminish its significance.

TABLE 9
RÉSUMÉ OF ROUGHNESS DETERMINATIONS

Source of Data	Method	Roughness (cm.)	Gradient Wind (m.p.s.)
Ekman	Storage of Water	0.75	about 20
Wüst	Velocity Gradient near Surface	0.6	?
Jeffreys	Angle between Surface Wind and Gradient Wind	0.70	10.25
		0.49	15.25
		0.32	22?
Jeffreys	Ratio between Surface Wind Speed and Gradient Wind Speed	0.85	10.25
		0.36	15.25
		0.64	22?

Table 9 summarizes the z_0 -determinations discussed above. Only those values which refer to steady conditions in the open sea have been included. The results are surprisingly consistent. There is some indication of a slight decrease in z_0 with increasing gradient wind velocity but this trend is very uncertain. The average obtained from all the determinations listed in the table is 0.59 cm. In order to indicate the value of the shearing stress under equilibrium conditions in the open sea, the value of γ corresponding to 15 m. elevation has been computed. With $z_0 = 0.6$ cm. and $k_0 = 0.38$ it is found that γ_{15} has a value of $4.86 \cdot 10^{-2}$.

In conclusion, it appears that the interaction between air and water takes place partly through real tangential shearing stresses and partly through normal pressures. With light winds and fairly smooth water, the normal pressures appear to be insignificant. The wind velocity profile is then given by von Kármán's formula which implies the existence of a laminar sub-boundary layer next to the surface. With rough seas, normal pressures come into play. Within a distance from the surface which does not exceed the height of the waves, the velocity profile seems to be determined by the tangential forces mainly, but somewhat higher up the (logarithmic) velocity gradient is steeper, in response to the influence of normal pressures. In the case of rapidly changing conditions, the roughness may reach very high values ($z_0 = 20$ cm.), but with steady winds the sea surface appears to adjust itself in such a fashion as to permit the air to move over it in the most economical fashion. The roughness parameter corresponding to steady moderate to strong winds seems to be in the vicinity of 0.6 cm.

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II. MEASUREMENTS OF VERTICAL GRADIENT OF WIND OVER WATER

BY R. B. MONTGOMERY

Despite the great importance, in studies of the frictional forces between atmosphere and ocean, of wind gradient measurements next to the ocean surface, to my knowledge the only observations that have been published are those of Wüst and of Shoulejkin.† Both of these are so fragmentary that one would find it very difficult to draw general conclusions from them alone. Feeling that further measurements of any sort might prove useful, and lacking time and facilities for a large program, the author felt justified in making the following simple measurements during September 1935, which may be regarded as preliminary.

The anemometers available for the purpose were two quite dissimilar ones of the four cup type, each indicating distance on its dial. Before the measurements were made the anemometers were carefully compared by mounting them side by side on top of an eight meter tripod on a wharf; about forty readings of ten minute intervals were made at different wind speeds, always with onshore winds. Previously one of the instruments had been calibrated in a wind tunnel, after the measurements it was calibrated down to nearly 1 m.p.s. on a whirling arm; the two calibrations differed by 0.2 m.p.s. The final calibration curves were drawn up from the latter calibration and the comparison of the two anemometers.

FIG. 1.—Simultaneous observations of wind speed at two heights above the water surface.

Since the logarithmic distribution of wind near the surface has been definitely established (for lapse rates that are nearly adiabatic) from measurements over land, it was considered necessary to study the wind at only two heights in order to define the wind gradient completely.

The measurements were made from a row boat on the Woods Hole side of Buzzards Bay. One anemometer was mounted at a constant distance of 4 meters above the other on the outer pole of a fish trap, where the water was some six meters deep. The lower anemometer was supported by a bracket 1 meter to windward of the pole, so that the instruments were completely unobstructed by nearby obstacles. By means of strings the indicator of the upper anemometer was started and stopped simultaneously with reading the lower one, the upper being lowered for readings between measurements.

A scale bearing 5-cm. divisions projected from the lower anemometer into the water.

† References 7 and 8 on page 20.

The lowest and highest positions of the water surface on this scale during half minute intervals were noted; the mean of these gave the height (a) of the lower anemometer, and their difference the wave amplitude. The approximate frequency of major wave crests was also noted.

The individual observations at heights a and b are shown in Figure 1 and Table 1.

Each observation is for an interval of about 10 minutes. The wind was always on-shore. No noticeable surface current occurred during the measurements. Unfortunately the air and water temperatures were not recorded; however, there is no reason to believe that the lapse rates were far from the adiabatic.

TABLE 1
INDIVIDUAL OBSERVATIONS

Day	Time	Wave		Heights		Wind		
		Amplitude cm.	Frequency min. ⁻¹	a cm.	b cm.	W_a m.p.s.	W_b m.p.s.	
20	1530	40	25	98	498	4.9	5.65	WSW
	55	40		108	508	5.3	5.8	WSW
22	1613	37	26	110	510	5.85	6.4	WSW
	1150	13	36	56	456	1.05	1.7	WSW
	1205	8		53	453	1.4	1.75	WSW
	18	11		51	451	1.4	1.8	WSW
23	1547	47	26	73	473	3.5	4.2	NW
	1601	36	26	67	467	2.8	3.25	NW
	14	38		66	466	2.5	2.95	WNW
	26	33		64	464	2.55	3.0	WNW
24	1118	19	31	70	470	1.7	1.95	N
	43	16	39	75	475	1.55	1.8	WNW

Table 2 gives averages of consistent measurements. The observation for the lightest wind (1150 on the 22nd) is omitted from the averages because it is inconsistent with the others, the one at 1547 on the 23rd because it stands alone.

TABLE 2
AVERAGES

Day	Time	Wave amplitude		Heights		Wind		Observations
		Observed	Computed	a	b	W_a	W_b	
20	1530-1613	39	108	105	505	5.35	5.95	3
22	1205-18	10	10	52	452	1.40	1.78	2
23	1601-26	36	29	66	466	2.62	3.07	3
24	1118-43	18	11	73	473	1.63	1.88	2

The fetch of the wind over water for these measurements was about nine miles. Hence, for the comparatively light winds observed, the surface roughness was probably about the same as occurs in the open ocean for the same winds. Included in Table 2 are the wave heights computed from the formula

$$(B\ddagger 202) \quad \epsilon = \frac{G}{g} W_b^2,$$

where g is the acceleration of gravity and $G=0.3$. The agreement is good except in the first group, where the indication is that the surface was not as rough as would be found in the open ocean for this wind of 6 m.p.s.

The results of these measurements are further discussed on page 6.

† As in the preceding section, the letter "B" refers to C.-G. Rossby and R. B. Montgomery, 1935: The Layer of Frictional Influence in Wind and Ocean Currents, *Papers in Physical Oceanography and Meteorology*, Vol. III, No. 3.

III. TRANSPORT OF SURFACE WATER DUE TO THE WIND SYSTEM OVER THE NORTH ATLANTIC

BY R. B. MONTGOMERY

The results of Section I above and of our previous paper (B†) are here applied to the specific computation of the transport of the drift current system over the North Atlantic. Only the computation is given here, but the authors of this paper expect to discuss in later articles its significance in connection with the surface salinity distribution and with the Equatorial Current.

The component of the steady surface current which is produced directly by the wind, or drift current, has a net mass transport which, in the absence of stress at its bottom, is directed $\pi/2$ *cum sole* from the surface wind. The magnitude (T) of the mass transport is easily computed in terms of the surface stress (τ_0), for the deflecting force on the drift current as a whole must balance the surface stress, thus:

$$\tau_0 = fT \quad (f = 2\Omega \sin L).$$

The first difficulty met with in attempting to compute the mean field of the transport over the North Atlantic is that the mean field of the surface wind is not known with sufficient accuracy. It is best therefore to work from the gradient wind computed from the mean pressure field.

Use of the gradient wind for computing the surface wind and surface stress might perhaps introduce a rather large error due to a possible stability of the air below the gradient wind level. However, the available evidence indicates that the departures from the adiabatic lapse rate are not large enough to introduce appreciable error in this computation. Schott¹ gives a chart showing the difference (mean for the whole year) between air and water temperatures. Over practically the whole North Atlantic the water is warmer than the air (for the most part the difference is between 0.5° and 1.5°), showing that at least in the surface layer the atmosphere is not usually stable over the major part of the area covered by this computation. The air is warmer than the water only in the region of the Grand Banks and Gulf of Maine, and in the region of upwelling along the west coast of Africa.

There is also aerological evidence, which is essentially limited to the eastern part of the trade winds. This region is characterized in general by a steep lapse rate from the surface up to a distinct inversion, called the "trade wind inversion." According to von Ficker,² in the central Atlantic this inversion sinks from *ca.* 1400 meters at 30°N. to *ca.* 1000 meters in the zone of the strongest trades (about 15°N. ?) and then rises again equatorwards. This may be compared with the theoretical total height of the layer of frictional influence ($H+h$), corresponding to the mean gradient wind for each latitude from Table 1 below, computed from (B 32b):

L (N.)	$12\frac{1}{2}^\circ$	$17\frac{1}{2}^\circ$	$22\frac{1}{2}^\circ$	$27\frac{1}{2}^\circ$
U_g (m.p.s.)	9.3	8.0	7.4	5.2
$H+h$ (meters)	1790	1140	850	510

These values of $H+h$ seem to intersect the trade wind inversion at about $17\frac{1}{2}^\circ\text{N.}$, indicating that stability might affect the calculation of surface stress south of this latitude

† As in the preceding sections, the letter "B" refers to C.-G. Rossby and R. B. Montgomery, 1935: The Layer of Frictional Influence in Wind and Ocean Currents, *Papers in Physical Oceanography and Meteorology*, Vol. III, No. 3.

only. More specific information is given by Sverdrup³ for summer over the Atlantic east of 40°W. and between 10°N. and 30°N. His Figure 33 shows that the decrease in temperature from surface to 500 meters is between 4° and 5° over the entire region. His Figure 35 shows in the layer 500–1000 meters a temperature decrease of 3° in the northwestern half of the region, little or no change in the southern part, and an increase of 2° near the African coast north of 20°N. This indicates that, south of 22½°N. in the western half of the region and south of 27½°N. in the eastern half, stability might become effective, so that the surface stress computed from the gradient wind would be too large. However, it seems likely, even where the actual homogeneous layer is considerably lower than the theoretical height of frictional influence, that the surface wind has nearly its normal adjustment to the gradient wind.

Accepting this assumption that the atmosphere below the gradient wind level may be considered free from stability, it is possible to compute the surface stress quite accurately from the theoretical relations derived in B.

The angle, φ_0 , between surface wind and gradient wind is given by

$$(B\ 31a) \quad N \equiv \frac{U_g}{fz_0} = \frac{2\sqrt{2}k_0}{9k^3} \cdot \csc \varphi_0 \cdot e^{\frac{2k_0}{3k}(\cot \varphi_0 - \frac{1}{\sqrt{2}})}, \quad (k_0 = 0.38, \quad k = 0.065)$$

where U_g is the gradient wind speed and z_0 the roughness parameter of the sea surface with reference to the medium above it. It is shown by Prof. Rossby in the first section that, for steady surface winds greater than a critical value of about 3–5 m.p.s., $z_0 = 0.6$ cm. Although z_0 is much smaller than this for light winds, I have used this constant value throughout. This is partially justified because, although the vectorial mean wind velocity in certain regions is light, the average of the magnitude of the velocity would be less than the critical value only in much smaller regions.

The surface stress is given in terms of U_g , φ_0 , and the air density (ρ) by

$$(B\ 34) \quad \tau_0 = \frac{9k^2}{4} \rho U_g^2 \sin^2 \varphi_0,$$

so the transport of water is

$$(I) \quad T = \frac{9k^2}{4} \frac{\rho}{f} U_g^2 \sin^2 \varphi_0.$$

These expressions imply that there are no volume forces (hence stress constant with elevation) within the lower layer of frictional influence, where the wind increases as the logarithm of height. The case where pressure gradient and deflecting force are taken into account within the lower layer was treated in B under the heading *Second Approximation*. Since the corrections introduced by this more complete treatment are small, it seems unnecessary to consider them here where other sources of error are much larger.

The mean pressure distribution used in this computation is that from Defant⁴ for the month of July, which is the month when the Azores high pressure area reaches its maximum development. His values are given for the intersections of each five degrees of latitude and longitude, and are the means of the pressures read off at the intersections by interpolation from the isobars on daily synoptic maps for the period 1881–1905. The pressures are reduced to sea level and to standard gravity at latitude 45°.

I have plotted Defant's pressure values and drawn smooth isobars, following as closely as possible a linear interpolation of the plotted values. The gradient wind velocity was computed for the center of each five degree square from the distance between the isobars, from their curvature and from their direction. The gradient wind speed is given in Table 1.

The angle between surface wind and gradient wind, corresponding to (B 31a) above, is easily found from Figure 4 on page 17. This angle also is given in Table 1.

TABLE 1
 U_θ IN m.p.s. AND φ_0

$L \backslash \lambda$	80°	70°		60°		50°		40°		30°		20°		10°		0°
60°				.9 21.9°	1.0 21.7°	2.3 20.3°	2.1 20.4°	1.8 20.6°	2.1 20.4°	2.8 19.9°	3.0 19.8°	3.2 19.7°	3.4 19.6°	3.1 19.8°	2.9 19.9°	2.4 20.2°
50°					2.7 20.2°	3.1 20.0°	3.6 19.7°	3.8 19.6°	4.0 19.6°	4.6 19.4°	5.0 19.3°	5.1 19.3°	4.6 19.4°	4.3 19.5°	3.8 19.6°	3.5 19.8°
					4.8 19.2°	5.1 19.1°	5.7 19.0°	5.0 19.1°	5.8 18.9°	6.1 18.9°	6.2 18.8°	5.8 18.9°	4.8 19.2°	3.3 19.8°	2.6 20.1°	2.2 20.4°
40°		3.1 19.8°	3.8 19.4°	4.6 19.1°	5.3 18.9°	5.5 18.9°	6.0 18.8°	5.5 18.9°	5.3 18.9°	5.2 19.0°	4.8 19.1°	4.5 19.2°	3.4 19.6°	3.3 19.6°	3.4 19.6°	3.2 19.7°
	2.1 19.9°	3.4 19.2°	4.4 18.8°	4.6 18.7°	5.3 18.5°	5.4 18.5°	5.4 18.5°	4.6 18.7°	3.7 19.1°	0	0	3.7 19.1°	5.1 18.6°	5.8 18.4°	3.8 19.0°	
30°	2.5 19.4°	3.6 18.9°	4.4 18.6°	4.3 18.6°	3.6 18.9°	2.3 19.6°	3.6 18.9°	0	0	0	4.3 18.7°	5.7 18.3°	5.8 18.3°	5.9 18.2°	3.9 18.8°	
	3.7 18.6°	3.0 18.9°	3.6 18.7°	3.6 18.7°	4.0 18.5°	3.5 18.7°	4.1 18.5°	5.8 18.0°	6.5 17.9°	6.2 17.9°	7.4 17.7°	7.4 17.7°	6.9 17.8°	7.2 17.8°		
20°		8.5 17.3°	7.1 17.5°	6.8 17.6°	6.4 17.7°	6.6 17.6°	6.8 17.6°	7.0 17.6°	8.1 17.4°	7.7 17.5°	8.0 17.4°	7.7 17.5°	7.7 17.5°			
			8.2 17.1°	8.0 17.1°	9.3 17.0°	8.4 17.1°	8.1 17.1°	8.4 17.1°	8.6 17.0°	8.4 17.1°	7.7 17.2°	7.9 17.1°	5.3 17.6°			
10°				10.3 16.4°	9.5 16.5°	9.9 16.5°	9.7 16.5°	9.9 16.5°	9.7 16.5°	7.5 16.8°	9.7 16.5°	8.4 16.7°	8.6 16.6°			

It seems worthwhile at this point to compare the computed gradient winds with observed surface winds. For this comparison have been chosen the values at $47\frac{1}{2}^\circ\text{W.}$, from $12\frac{1}{2}^\circ\text{N.}$ to $47\frac{1}{2}^\circ\text{N.}$ In Table 2 are listed first the computed speed and direction of the gradient wind (U_θ). Next are given the gradient winds reduced to surface values by means of the relation

$$(B\ 35) \quad W_z = \frac{3k}{2k_0} U_\theta \sin \varphi_0 \ln \frac{z}{z_0},$$

which for $z=15$ meters (see page 19) and for $z_0=0.6$ cm. becomes

$$(2) \quad W_{15} = 2.0 U_\theta \sin \varphi_0.$$

It is understood that this relation, as well as the one for surface stress (1), does not apply exactly in the trade wind region where the gradient wind decreases rapidly with elevation, but the approximation should be good. Finally the table lists, under W_s , the re-

sultant wind velocity computed by means of the Meteorological Office Beaufort scale† from the July wind roses for five degree squares of the K. Nederlandsch Meteorologisch Instituut.⁵ In this computation I have followed C. E. P. Brooks:⁶ "The resultants were calculated from the wind roses . . . using the mean Beaufort force of the wind, converted to metres per second, as well as the frequency, from each direction."

It is seen that the directions of computed and observed velocities agree well except near the center of the high pressure area, but that the observed magnitudes are smaller in the prevailing westerlies and larger in the trade winds. The difference between observed and computed values is, however, in each case less than one Beaufort unit. In the region of prevailing westerlies the winds blow from all directions with any force, so that the expected error of the resultant of the Beaufort estimates is nearly as great as the difference from the computed velocities. The discrepancy in the trade wind region is no greater than the difference between the trade wind velocities stated by various sources. Thus for the region between 10°N. and 30°N. and between Africa and 30°W. Shaw⁷ states that the mean for July is 4.6 m.p.s. From Table 1 the 13 squares in this region give a mean of

TABLE 2
COMPARISON OF SURFACE WIND (W_{15}) DERIVED FROM COMPUTED GRADIENT WIND
(U_g) WITH OBSERVED SURFACE WIND (W_s)

L	U_g		W_{15}		W_s	
	m.p.s.	DIRECTION	m.p.s.	DIRECTION	m.p.s.	DIRECTION
47½°	5.7	239°	3.7	220°	2.2	225°
42½°	6.0	240°	3.9	221°	2.0	243°
37½°	5.4	240°	3.4	221°	2.1	268°
32½°	3.6	197°	2.3	178°	1.1	132°
27½°	4.1	116°	2.6	97°	3.5	87°
22½°	6.8	93°	4.1	75°	5.1	82°
17½°	8.1	93°	4.8	76°	5.8	69°
12½°	9.7	90°	5.6	73°	6.8	73°

$U_g = 7.7$ m.p.s. and $\varphi_0 = 17.3^\circ$, which leads to $W_{15} = 4.6$ m.p.s.—in this case the agreement is exact. This is just the region for which Schott's and Sverdrup's charts indicate that the computed surface wind and stress might be too large due to stability. Hence the assumption above, that the lower atmosphere may be considered free from stability, seems to be justified.

It seems reasonable to conclude from this digression that, for the present purpose, winds computed from the pressure gradient are not only more consistent and easier to handle, but probably also equally as correct as those based on Beaufort estimates. This is especially true if observations are meagre, because reports from relatively few ships can define the pressure system over the whole ocean with fair accuracy, whereas it would require many more ship reports to directly define the wind system.

The magnitude of the transport of the drift current is found from (1). Using ρ and f in c.g.s. units and U_g in m.p.s., T is found in tons per second across a vertical surface normal to the net drift and 1 meter wide. The direction of transport is the direction of flow of the gradient wind, less φ_0 , plus $\pi/2$ (measuring angles *cum sole*). Magnitude and direction of transport are given in Table 3.

For ρ , which enters into this calculation of transport and into the calculation of gradient wind, I have used the July surface air temperature distribution given in the atlases of the K. Nederlandsch Meteorologisch Instituut.⁵ The variation in ρ over

† Reference 16 on page 20.

the region covered by this investigation is small, being from $1.17 \cdot 10^{-3}$ to $1.24 \cdot 10^{-3}$.

On the accompanying chart (Figure 1) are shown the pressure distribution and the transport vector for the center of each five degree square.

From these transport vectors I have computed the transport across each side of the five degree squares (the mean of the normal component of the transport in the squares

TABLE 3
MAGNITUDE (tons per meter per sec.) AND DIRECTION (*cum sole* FROM NORTH) OF T , AND TOTAL CONVERGENCE IN EACH FIVE DEGREE SQUARE (10^6 tons per sec.)

$\lambda \backslash L$	80°	70°	60°	50°	40°	30°	20°	10°	0°						
60°			.01 184°	.01 111°	.06 104°	.05 111°	.04 144°	.05 168°	.09 161°	.10 161°	.11 149°	.13 138°	.11 138°	.09 146°	.06 167°
50°				.09 138°	.13 118°	.15 122°	.17 134°	.18 150°	.24 152°	.28 154°	.29 156°	.23 153°	.21 148°	.17 163°	.14 167°
40°				.27 143°	.31 136°	.38 130°	.29 139°	.39 143°	.43 149°	.43 155°	.39 161°	.27 169°	.14 170°	.09 185°	.06 186°
30°		.13 151°	.19 143°	.27 143°	.34 141°	.37 137°	.44 131°	.37 136°	.34 140°	.33 147°	.29 164°	.26 180°	.15 208°	.15 247°	.14 319°
20°	.07 159°	.16 136°	.26 131°	.28 134°	.36 132°	.38 133°	.38 131°	.28 142°	.19 142°	0 142°	0 142°	.19 251°	.34 256°	.44 263°	.20 256°
10°	.10 120°	.19 121°	.28 116°	.27 116°	.20 103°	.09 118°	.20 88°	0 88°	0 88°	.28 309°	.46 298°	.48 292°	.50 277°	.23 279°	
0°	.23 40°	.16 49°	.22 46°	.22 21°	.27 33°	.21 7°	.28 7°	.54 342°	.67 330°	.61 321°	.86 318°	.86 306°	.77 306°	.82 302°	
		1.28 12°	.92 3°	.85 359°	.76 5°	.81 358°	.85 345°	.91 337°	1.18 331°	1.09 317°	1.17 318°	1.09 308°	1.10 307°		
			1.49 359°	1.42 1°	1.89 4°	1.56 355°	1.47 346°	1.58 336°	1.63 327°	1.58 323°	1.34 315°	1.39 306°	.66 309°		
				2.97 6°	2.58 10°	2.81 356°	2.69 343°	2.81 336°	2.69 325°	1.66 330°	2.69 306°	2.06 311°	2.15 317°		

separated by the side, times the length of the side), and hence the total convergence within each square. This also, in mass per unit time, is given in Table 3. Dividing by the area of the square, one gets the unit convergence in mass per unit area per unit time. In the case of the coastal squares the total convergence is the net computed inflow across the water part of the periphery, and unit convergence is total convergence divided by water area in the square. Hence the strong negative convergence along the African coast for instance, which is actually limited to an intense band of upwelling adjacent to the coast, is averaged over a much larger area in this method of representation.

On the chart the convergence is shown by the figures within the circles, the unit

FIG. 1.—July isobars for each mm. over the region of the North Atlantic Ocean. The vectors represent the transport of the drift current at the center of each five degree square. The figures in circles give convergence of drift current in cms. per day. The broken lines are lines of equal convergence.

being grams per square cm. per day, which is equivalent to cms. per day. Lines of equal convergence are drawn for 0, 5, 10, 15 grams per square cm. per day.

Other sources of error involved in the calculation by this method of the drift current transport are discussed by the author in another article.⁸

Superimposed on the drift current are other currents which may be called, for brevity, the gradient current. In order to maintain continuity the convergence of the gradient current must everywhere be equal and opposite to the convergence of the drift.† While the drift is usually limited to the surface 50 meters, the gradient current, and presumably also its convergence, nearly always extends to several hundred meters. Hence only a small portion of the convergence of the gradient current occurs in the drift current layer, so the convergence of the drift, as here computed, represents convergence of surface water within the drift current layer. This convergence necessitates sinking, which at the bottom of the drift current layer nearly equals the convergence, and which extends downward with decreasing intensity to the lowest extent of the negative convergence of the gradient current. This does not imply that the surface water necessarily sinks throughout this whole distance; it will be shown below that in one important region the surface water sinks only a few meters. In so doing, however, it displaces deeper water which in turn must sink. This whole argument rests on the assumption that the gradient current is essentially negatively convergent at all depths; there is no reason to doubt this if the gradient current has the same direction at all depths.

Continuity must be observed, not only in regard to the whole water column as above, but also in regard to the surface water alone. The convergence of the drift current may be balanced in some regions partly by mixing of surface water with deeper water. If mixing is absent, then the net convergence of surface water must be zero (as in the case below referred to above), and the convergence of the drift current is balanced by a deepening of the surface water layer in the direction of the gradient current. It is important to bear in mind this distinction in regard to the term *convergence*, as applied on the one hand to a definite current, and on the other hand to a definite water mass.‡ Thus the convergence computed in this paper is that of the surface water in the drift current layer, *i.e.* the layer of frictional influence due to the wind, which layer does not everywhere include all the surface water.

The region where the drift current convergence is maximum lies between 40°W. and the West Indies, at or south of $17\frac{1}{2}^{\circ}\text{N.}$ Unfortunately the computation does not extend sufficiently far south to determine the exact latitude. But since 10°N. is usually stated as roughly the southern limit of the trade winds, the drift across this parallel must be small, so the maximum convergence could not occur at $12\frac{1}{2}^{\circ}\text{N.}$ Hence it must occur within a few degrees of $17\frac{1}{2}^{\circ}\text{N.}$

It is at first surprising to find the maximum convergence so far south, for the Sargasso Sea is the region where surface water collects to a great depth and where the principal thermocline is deep. The depth to which surface water extends depends, however, on the intensity of mixing with the deeper water and on the strength of the gradient current. Both of these effects tend to decrease the depth to which pure surface water will accumu-

† Evaporation and precipitation, and the unsteady state, are not considered. While important for the discussion when applied to certain regions, they merely form additional terms which may easily be taken into consideration in a qualitative way where necessary.

‡ While discussing the term *convergence*, it might be emphasized that it is used here in the hydrodynamic sense, which has no relation to the use that Seiwel⁹ makes of it with regard to regions of close spacing of iso-lines in horizontal projections.

late. In the Sargasso Sea both are small. At $17\frac{1}{2}^{\circ}\text{N}$. the principal thermocline is very steep and here again there is little mixing with the deeper water, as shown by Seiwel.⁹ But the gradient current (North Atlantic Equatorial Current) is here very strong, we may say some 10 miles per day.[†] It is easy to compute the increase in thickness of surface water in the Equatorial Current during its course from 40°W . to the West Indies. This distance of 1400 miles is traversed in 140 days. The convergence of the drift being about 20 cm. per day, the thickness of surface water would increase by only 28 meters. This is so small as to be undetectable in the sparse hydrographic data available for this region, and shows why the region of maximum convergence is not directly observed.

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[†] There is no exact information concerning the velocities of the North Atlantic Equatorial Current, and the available information is conflicting. This value is based on Krümmel's¹⁰ statement that south of 20°N . the average stream strength is 15 to 17 miles per day. This is of course the surface current, which, following Sverdrup's¹¹ analysis of the equatorial currents of the Pacific, is supposedly the sum of the drift current and of a gradient current (the true Equatorial Current). The drift current corresponding to the gradient winds computed for this region is roughly NW 6 miles per day; for a surface current of 15 the gradient current must be 10 if we assume it to flow westward (B p. 96).